Functional Analysis and Operator Theory
Webinar

The set of quantum states analyzed by numerical range and numerical shadow of an operator

Karol Życzkowski
Jagiellonian University and Polish Academy of Sciences, Poland

The set Ω_N of density matrices - positive hermitian matrices of order N with trace equal to unity - plays a key role in the theory of quantum information processing. It is a convex set embedded in $\mathbb{R}^{N^2-1}$ with an involved structure, which for $N = 2$ reduces to the 3-ball.

Numerical range $W(X)$ (also called field of values) of an operator X of size N can be considered as a projection of Ω_N into a 2-plane. Further structure of the set Ω_N of quantum states is revealed by the numerical shadow of an operator - a probability measure on the complex plane, $P_X(z)$, supported by the numerical range $W(X)$. The shadow of X at point z is defined as the probability that the inner product (Xu, u) is equal to z, where u stands for a normalized N-dimensional random complex vector. In the case of $N = 2$ the numerical shadow of a non-normal operator can be interpreted as a shadow of a hollow sphere projected on a plane.

Studying joint numerical range of three hermitian operators, $W(H_1, H_2, H_3)$, one can analyze projections of Ω_N into a 3-space. A classification of possible shapes of 3D numerical ranges of three hermitian operators of order three is presented.

---

16 Dec 2020 12:00 London time

You can join the event via this link:
https://istaustria.zoom.us/j/97256950873?pwd=bWd6U1kyVXZFQk1wNll5ZTlXTE1ZQT09
Meeting ID: 972 5695 0873
Passcode: 582736

More information about the webinar series can be found at the following homepages
https://www.renyi.hu/~titkos/faot.html
https://researchseminars.org/seminar/FAOT

The slides and video recordings of all previous talks are available in this google drive folder:
https://drive.google.com/drive/folders/1Vo6musdFZwhsMmoD3OKuJQ35Q_IWSZwd