A geometric approach to Wigner-type theorems

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Let H be a complex Hilbert space and let $\mathcal{P}(H)$ be the associated projective space (the set of rank-one projections). Suppose that $\dim H \geq 3$. We prove the following Wigner-type theorem: if H is finite-dimensional, then an arbitrary orthogonality preserving transformation of $\mathcal{P}(H)$ (i.e. sending orthogonal projections to orthogonal projections without the assumption that the orthogonality relation is preserved in both directions) is induced by a unitary or anti-unitary operator. In the case when H is infinite-dimensional, this fails.

The problem is reduced to a description of orthogonality preserving lineations. Lineations are maps between projective spaces which send lines to subsets of lines. In general, the behavior of lineations is complicated; they are not injective and can send lines to parts of lines only. Our version of Wigner's theorem is a consequence of the following result: every orthogonality preserving lineation of $\mathcal{P}(V)$ to itself is induced by a linear or conjugate-linear isometry (now, we do not require that H is finite-dimensional).

As an application, we describe (not necessarily injective) transformations of Grassmannians preserving some types of principal angles.

This is a joint work with Thomas Vetterlein.

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