Lattice isomorphisms between projection lattices of von Neumann algebras

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A von Neumann algebra is a weak operator closed *-subalgebra of B(H), whose study was initiated by Murray and von Neumann in 1930's. The collection of projections of a von Neumann algebra forms a lattice, and its geometry has played a very important role in understanding the structure of von Neumann algebras for more than 80 years.

In this talk, we consider the following fundamental question: What is the general form of lattice isomorphisms between projection lattices of von Neumann algebras? Von Neumann gave an answer to this question for type II_1 factors. He proved that a lattice isomorphism can be described by means of a ring isomorphism between the algebras of affiliated operators. However, apparently no answer to this question has been given for the general case (in particular for type III von Neumann algebras) until now. In this talk, we begin with a brief recap of the classical theory of von Neumann algebras, and then give an answer to our question for general von Neumann algebras (save type I_1 and I_2) using ring isomorphisms between the algebras of locally measurable operators. We also consider a better description of ring isomorphisms between locally measurable operator algebras