Distance Geometry in Quasihypermetric Spaces

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Let (X, d) be a compact metric space and let $M(X)$ denote the space of all finite signed Borel measures on X. Define $I : M(X) \to \mathbf{R}$ by

$$I(\mu) = \int_X \int_X d(x, y) d\mu(x) d\mu(y)$$

and set $M(X) = \sup I(\mu)$, where μ ranges over the collection of measures in $M(X)$ of total mass 1. This talk will focus mainly on the following three questions:

1. Under which conditions is $M(X)$ finite?
2. If $M(X)$ is finite are there measures μ (we call them maximal measures) such that $M(X) = I(\mu)$?
3. If no maximal measures exist is there a characterisation of sequences of measures μ_n (we call them maximal sequences) such that $I(\mu_n)$ tends to $M(X)$?
4. What are concrete examples illustrating the first three questions?

I will report on answers to these questions which I worked out together with Peter Nickolas.