Overconvergence and zero distribution of Fourier series

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Given a finite Borel measure μ with a compact support K in the complex plane $\mathbf{C}$, let $\{p_n\}_{n=0}^{\infty}$

$$p_n(z) = \sum_{k=0}^{n} \gamma_{n,k} z^k, \quad \text{with } \gamma_{n,n} := \gamma_n$$

be the sequence of orthogonal polynomials with respect to μ. The basic assumption is that K is regular with respect to the Dirichlet problem and

$$\limsup(\max_{z \in K} |p_n(z)|^{1/n}) \leq 1.$$

Given a function f, analytic on the polynomial hull $P_c(K)$ of K, let

$$\sum a_n p_n, \; a_n = \int f \bar{p_n} d\mu$$

be the Fourier series associated with f, and

$$S_N := \sum_{n=0}^{N} a_n p_n$$

be the $N - th$ partial sum.

In the present talk, we will be interested in the phenomenon of overconvergence of partial sums. The relation to Hadamard-Ostrowski gaps will be revealed, and sufficient conditions for an occurrence of overconvergence will be proved. Special attention will be given to sufficient conditions in terms of zero distribution of partial sums. Results dealing with the density of a sequence of overconvergent series $\{S_{n_k}\}$ will be established. Finally, results characterizing the asymptotic distribution of the zero points of overconvergent sequence of partial sums will be derived.