

# Control theory and harmonic analysis

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We review some recent results in control theory of partial differential equations:

- The Hilbert uniqueness method of J.L. Lions. Based on the duality between observability and controllability, exact controllability problems can be deduced from observability estimates for the solutions of a dual system. The latter having homogeneous boundary conditions, this approach led to many simplifications. In many important problems concerning the vibrations of mechanical systems such estimates were obtained by the powerful multiplier method.
- Feedback stabilization has been an important engineering tool since Watt's regulator for the steam engine. It is more realistic and easier to implement than to achieve exact controllability. For many hyperbolic systems the efficiency of natural feedback laws could be established in an elegant way by the multiplier method.
- Natural feedback laws, although easy to implement, only lead to limited decay rates. A more general construction based on the relations between observability and stabilizability (in a sense a variant of the Hilbert uniqueness method) makes it possible to construct feedback leading to arbitrarily high decay rates for a large class of linear systems. Its surprising efficiency has been proved by numerous computer and physical experiences.
- Many particular observability problems, which could not be solved by the multiplier method, were successfully treated by generalized Parseval type inequalities. These improvements of a classical theorem of Ingham were motivated by special applications. Their investigation also revealed a formerly hidden connection between generalized Parseval type inequalities and eigenvalue problems for the Laplacian operator.