On the extension property

Alexander Goncharov
(Bilkent University, Ankara, Turkey)

Given compact set $K \subset \mathbb{R}^d$, let $\mathcal{E}(K)$ denote the space of Whitney jets on K, that is the space of traces on K of functions from $C^\infty(\mathbb{R}^d)$, with its natural quotient topology. The compact set K is said to have the extension property if there exists a linear continuous extension operator $L : \mathcal{E}(K) \longrightarrow C^\infty(\mathbb{R}^d)$. We review the methods of extension of Whitney functions suggested by H.Whitney, R.T.Seeley, B.S.Mityagin, E.Bierstone, W.Pawłucki & W.Pleśniak and others. As well we consider a topological characterization of the extension property, given by M. Tidten. We touch such related subjects, as the basis problem (for the class of $C^\infty-$ functions), Markov's type inequality for compact sets, and the application of linear topological invariants for splitting of certain exact short sequences of Fréchet spaces. We discuss a longstanding problem of a geometric characterization of the extension property and give a review of such characterizations for the known cases. We show that, at least for the case of Cantor-type sets, there are interconnections of the extension problem and potential theory. In particular, the criterion of the extension property of the Cantor-type set K can be given in terms of the rate of growth of the values of the minimal discrete energy of compact sets that locally form K.

We are planning to give a variety of illustrating examples.