Corrigendum to my paper on the Erdos-Posa theorem

Miklos Simonovits

February 26, 2025

The paper is connected to the following theorem of Erdos and Posa:

Theorem. There exist two positive constants, $c_1 > c_2 > 0$ for which If for some fixed k a graph G does not contain k + 1 vertex-independent cycles, then we can delete $c_1 k \log k$ vertices of of G to obtain a tree or a forest. On the other hand, there exist infinitely many graphs G_n without k + 1independent vertex-independent cycles in which one needs to delete at least $c_2 k \log k$ vertices to obtain a tree or a forest.

The Erdős-Pósa paper provides an upper bound and a lower bound on the number of vertices to be deleted to ruin all the cycles. I would think that my upper bound is one of the simplest possible one, and completely algorithmic, on the other hand, I completely overlooked the other part, where I wished to provide a construction of the graphs G_n . (The original paper contained a "random graph construction".)

Slightly later I learned the strongly related works of H.J. Voss, and a little later G. Margulis constructed a 4-regular graph G_n with girth $\geq c_3 k \log k$ which also was a good – but highly non-trivial– construction for our problem. The Margulis-Lubotzky-Phillips-Sarnak graphs were also good constructions.