

## Introduction to number theory

summary of notions, definitions and theorems

**General notions.** Divisibility, units.

**Structure of  $\mathbf{Z}$ .** Euclidean division, euclidean algorithm, gcd. Primes, irreducibles, fundamental theorem of arithmetic, canonical form, square-free numbers. Residue classes, remainders, congruence, Chinese remainder theorem. Number-theoretic functions:  $\tau_s, \omega, \Omega, \varphi, \mu$ , (total) additivity, (total) multiplicativity. Euler-Fermat theorem, Fermat's little theorem. Legendre symbol, law of quadratic reciprocity.

**Primes.** Number of primes, reciprocal sum of primes, Dirichlet's theorem, Chebyshev's theorem, prime number theorem. Mersenne primes, Fermat primes, perfect numbers, description of even perfect numbers.

**Approximation of irrational numbers.** Dirichlet's approximation (two forms).

**Pell's equation.** Pell's equation. Structure of solutions.

**Number theory of polynomials.** Polynomials over a field: euclidean division, euclidean algorithm, gcd, primes, irreducibles, fundamental theorem of arithmetic. Polynomials over  $\mathbf{Z}$ : primitive polynomials, product of primitive polynomials is primitive, irreducible polynomials over  $\mathbf{Z}$  are irreducible over  $\mathbf{Q}$ , fundamental theorem of arithmetic.

**Quadratic forms.** Description of numbers representable as the sum of two, three and four squares. Geometry of numbers: Minkowski's convex body theorem. Gaussian numbers, gaussian integers, conjugate, norm, euclidean division, euclidean algorithm, gcd, primes, irreducibles, fundamental theorem of arithmetic. Quaternions, conjugate, norm, Hurwitz quaternions.