

1. Prove that if n is odd, then $n \mid 2^{n!} - 1$.
2. Prove that $n^{20} + 4n^{44} + 8n^{80}$ is divisible by 13 for any integer n .
3. Prove that $n^6 + 13$ and $n^2 + 21$ cannot be simultaneously prime for $n \in \mathbf{Z}$.
4. Prove that, for integers a, b, c ,
 - (a) $11 \mid a^{30} + b^{30} + c^{30}$ implies $11^{30} \mid a^{30} + b^{30} + c^{30}$;
 - (b) $9 \mid a^{30} + b^{30} + c^{30}$ implies $9^{15} \mid a^{30} + b^{30} + c^{30}$.