

1. Are the following congruences solvable:
  - (a)  $x^2 \equiv 66 \pmod{191}$ ;
  - (b)  $x^2 \equiv 7! \pmod{83}$ ;
  - (c)  $x^2 \equiv 30 \pmod{77}$ ;
  - (d)  $x^2 \equiv 38 \pmod{187}$ ;
  - (e)  $2x^2 + 3x + 5 \equiv 0 \pmod{101}$ ?
2. For which primes  $p > 2$  are the following congruences solvable:
  - (a)  $x^2 \equiv 3 \pmod{p}$ ;
  - (b)  $x^2 \equiv 5 \pmod{p}$ ?
3. Prove that if  $1997 \mid a^2 - 2b^2$  (for  $a, b \in \mathbf{Z}$ ), then  $1997 \mid a, b$ . (Note that 1997 is a prime.)