

MIDTERM EXAM

1. (a) State the law of quadratic reciprocity. **(2 points)**
(b) Compute the value of the Legendre symbol

$$\left(\frac{15}{2027}\right).$$

(Take for granted that 2027 is a prime.) **(4 points)**

2. (a) Define prime numbers. (Note: you have to give the definition of primes that we used in the class, not the equivalent definition of irreducibles.) **(2 points)**
- (b) Give those primes $p \geq 2$ that can be written as the sum of two consecutive integers. **(4 points)**

3. (a) Define the number-theoretic function φ . **(2 points)**
(b) Solve the equation $2\varphi(n) = n$ in $n \in \mathbf{N}$. **(4 points)**

4. (a) State our proposition about $\gcd(a, b)$ and integer combinations of a and b . **(2 points)**
- (b) Assume $a, b, n \in \mathbf{N}$ such that $n > ab$ and $\gcd(a, b) = 1$. Prove that there exist $x, y \in \mathbf{N}$ such that $ax + by = n$. **(4 points)**