1. Prove that for any $n \in \mathbf{N}$, the number $\underbrace{11\ldots11}_{n}\underbrace{22\ldots22}_{n}$ can be written as the product of two consecutive integers.

2. Prove that for any $n \in \mathbf{N}$, the number $\underbrace{44\ldots44}_{n}\underbrace{88\ldots8}_{n-1}9$ is a square.

Explain how the two problems are related.