Introduction to mathematical cryptography
Homework problems
Week 3

5. Assume G is a group, and H_1, H_2 are subgroups of G. Prove that if $H_1 \cup H_2$ is also a subgroup of G, then $H_1 \subseteq H_2$ or $H_2 \subseteq H_1$ (or equivalently, if $H_1 \not\subseteq H_2$ and $H_2 \not\subseteq H_1$, then $H_1 \cup H_2$ is not a subgroup of G).

6. Let G be an abelian group, and assume that a, b are elements of G such that the order of a is 3 and the order of b is 4. What is the order of ab?

Note: Please, provide complete arguments everywhere, and explain how you arrived at your answer/solution. Giving the result without explanation leads to score deduction.