Introduction to number theory
summary of notions, definitions and theorems

General notions. Divisibility, units.

Structure of Z. Euclidean division, euclidean algorithm, gcd, lcm. Primes, irreducibles, fundamental theorem of arithmetic, canonical form, square-free numbers. Residue classes, remainders, congruence, Chinese remainder theorem. Number-theoretic functions: $\tau_s, \omega, \Omega, \varphi, \mu$, (total) additivity, (total) multiplicativity. Multiplicative groups, Lagrange's theorem, Euler-Fermat theorem, Fermat's theorem, structure of multiplicative groups. Legendre symbol, Jacobi symbol, law of quadratic reciprocity.

Primes. Number of primes, reciprocal sum of primes, Dirichlet's theorem, Chebyshev's theorem, prime number theorem. Mersenne primes, Fermat primes, perfect numbers, description of even perfect numbers.

Approximation of irrational numbers. Dirichlet's approximation (two forms).

Pell's equation. Pell's equation. Structure of solutions.

Number theory of polynomials. Polynomials over a field: euclidean division, euclidean algorithm, gcd, primes, irreducibles, fundamental theorem of arithmetic. Polynomials over $\mathbf{Z}$: primitive polynomials, product of primitive polynomials is primitive, irreducible polynomials over $\mathbf{Z}$ are irreducible over $\mathbf{Q}$, fundamental theorem of arithmetic.

Quadratic forms. Description of numbers representable as the sum of two, three and four squares. Geometry of numbers: Minkowski's convex body theorem. Gaussian numbers, gaussian integers, conjugate, norm, euclidean division, euclidean algorithm, gcd, primes, irreducibles, fundamental theorem of arithmetic. Quaternions, conjugate, norm, Hurwitz quaternions.