Midterm Exam

1. (a) State the fundamental theorem of arithmetic. **(2 points)**
   (b) Let a, b, c be digits in base 10. Prove that the number $\overline{abcabc}$ is divisible by 91. **(4 points)**

2. (a) State the Chinese remainder theorem (with arbitrary many moduli). **(2 points)**

   (b) I have a few apples, not more than 200. If I try to share them between 3 kids, 2 ones are left; if I try to share them between 5 kids, 4 ones are left; if I try to share them between 7 kids, 6 ones are left. How many apples do I have? **(4 points)**

3. (a) State the Euler-Fermat theorem. **(2 points)**
   (b) Prove that $3^{2000} - 1$ is divisible by 10. **(4 points)**

4. (a) State the quadratic reciprocity. **(2 points)**

   (b) Describe the prime numbers $p > 2$ which satisfy that 35 is a quadratic residue modulo p. **(4 points)**