Final Exam

1. (a) Define the number-theoretic function φ. **(2 points)**

   (b) Solve the equation

$$\varphi(n) = \frac{n}{2}$$

   over the positive integers (i.e. describe the set of solutions). **(4 points)**

2. (a) What is Pell's equation? **(2 points)**

   (b) Solve the equation

$$x^2 - 7y^2 = 1$$

   over the integers (i.e. describe the set of solutions). **(4 points)**

3. (a) Describe those positive integers which are representable as the sum of two squares, and also those which are representable as the sum of three squares. **(2 points)**

   (b) Prove that there are infinitely many positive integers which are representable as the sum of three squares but not represenatble as the sum of two squares. **(4 points)**

4. (a) State Minkowski's convex body theorem. **(2 points)**
   (b) Assume Λ is a lattice of covolume 1 in the plane. Prove that the minimal distance between two distinct points of Λ cannot be more than $2/\sqrt{\pi}$. **(4 points)**