

Introduction to number theory
summary of notions, definitions and theorems

General notions. Divisibility, units.

Structure of \mathbf{Z} . Euclidean division, euclidean algorithm, gcd, lcm. Primes, irreducibles, fundamental theorem of arithmetic, canonical form, square-free numbers. Residue classes, remainders, congruence, Chinese remainder theorem. Number-theoretic functions: $\tau_s, \omega, \Omega, \varphi, \mu$, (total) additivity, (total) multiplicativity. Multiplicative groups, Lagrange's theorem, Euler-Fermat theorem, Fermat's theorem, structure of multiplicative groups. Legendre symbol, Jacobi symbol, law of quadratic reciprocity.

Primes. Number of primes, reciprocal sum of primes, Dirichlet's theorem, Chebyshev's theorem, prime number theorem. Mersenne primes, Fermat primes, perfect numbers.

Approximation of irrational numbers. Dirichlet's approximation (two forms), Liouville numbers.

Pell's equation. Pell's equation. Structure of solutions.

Number theory of polynomials. Polynomials over a field: euclidean division, euclidean algorithm, gcd, primes, irreducibles, fundamental theorem of arithmetic, residue classes, remainders, congruence. Polynomials over \mathbf{Z} : primitive polynomials, product of primitive polynomials is primitive, irreducible polynomials over \mathbf{Z} are irreducible over \mathbf{Q} , fundamental theorem of arithmetic.

Algebraic and transcendental numbers. Algebraic numbers, algebraic numbers form a field. Transcendental numbers, Liouville numbers are transcendental.

Quadratic forms. Sum of two squares: gaussian numbers, gaussian integers, conjugate, norm, euclidean division, euclidean algorithm, gcd, primes, irreducibles, fundamental theorem of arithmetic, description of primes representable as the sum of two squares, description of numbers representable as the sum of two squares. Sum of four squares: quaternions, conjugate, norm, Hurwitz quaternions, description of numbers representable as the sum of four squares. Sum of three squares: description of numbers representable as the sum of three squares.