

MIDTERM EXAM

In the exam, there are four problems, each is worth six points. Note that complete arguments are required. Please make sure that you write your name on every page.

1. Compute $\varphi(2016)$. **(6 points)**

2. Prove that for any positive integer n , there exists a positive integer $x \in [n, n+99]$ satisfying $x \equiv 2 \pmod{4}$ and $x \equiv 3 \pmod{25}$. **(6 points)**

3. Prove that if $n > 2$, then $\varphi(n)$ is even. (6 points)

4. Characterize by their canonical form the positive integers $n > 1$ satisfying $\tau_1(n) < n + \sqrt{n}$. (Note: you have to state and prove an 'if and only if' statement.) **(6 points)**