Introduction to mathematical cryptography
Homework problems
Week 5

9. Assume N is the product of two different prime numbers. Prove that if N and $\varphi(N)$ are given, then you can compute the prime factors of N in polynomial time.

10. ('multiplication without modulus' cipher) Now the cryptosystem is the following: $\mathscr{M}, \mathscr{C}, \mathscr{K} = \mathbf{N}$, and for $m \in \mathscr{M}, k \in \mathscr{K}$, $e_k(m) = km$. Alice and Bob agree on a large number $k \in \mathscr{K}$, and start to communicate. Eve intercepts the messages
$$c_1 = 10302619, \qquad c_2 = 5277099287.$$
How can Eve decrypt the messages? (Recall that Eve has the ability that if she reads a message, she recognizes it.)

Note: Please provide complete arguments everywhere.