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Title: Random Gems
Abstract:

We give three gems of the Probabilistic Method. These problems have been worked on for decades but here we give some beautiful arguments – illustrating the increasingly sophisticated applications of probability.

1. From Erdős in 1963: How big can k be so that given *any* collection of less than $m = 2^{n-1}k$ sets, all of size n, there exists a two-coloring of the underlying vertices so that none of the sets are monochromatic. Erdős randomly colored but others have done better, using an ingenious randomized algorithm.

2. From the speaker in 1985, while visiting the Rényi Institute. Given n vectors $\vec{v_1}, \ldots, \vec{v_n} \in R^n$, all with L^∞ norm at most one, some signed sum $\pm\vec{v_1} \pm \cdots \pm \vec{v_n}$ has L^∞ norm at most $K\sqrt{n}$, K constant. The original proof was "ugly", we give our choice for the Book proof, using a restricted Brownian motion.

3. From Bender, Canfield and McKay, 1990. Asymptotically, how many *connected*, labelled graphs are there with n vertices and (say) $2n$ edges. We prefer our own proof, with van der Hofstad, using a Brownian Bridge.