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Title: Cohomology of oriented Grassmannians

Abstract: The cohomology ring of complex Grassmannians is well-understood, by means of Chern classes or Schubert calculus. For the real Grassmannians of k -planes in \mathbb{R}^n , the same descriptions hold by replacing Chern classes with Stiefel-Whitney classes and by reducing the coefficients mod 2. With integer coefficients, the description is a bit more complicated due to the presence of two-torsion, but the situation is still manageable.

It might be surprising then that by simply taking the double cover of real Grassmannians, very little is known: already with mod 2 coefficients, the $k = 3$ cases have only been settled in 2024. The main issue is that Stiefel-Whitney classes no longer generate the cohomology ring; classes that are not expressible in terms of SW classes are called *anomalous classes*. Of course, in the limit we just get the cohomology of a classifying space: these anomalous classes do not survive in the limit. Nevertheless, already in small dimensions they contain important geometric information related to octonions and calibrated geometries.

In this talk I will discuss results, questions and conjectures related to the following topics:

- What's the structure of the mod 2 cohomology?
- What order of torsion can appear?
- What is the geometry of these anomalous classes?

The talk is based on joint work in progress with Milica Jovanović and Matthias Wendt.