NEW RESULTS IN CLASSICAL ENUMERATIVE GEOMETRY AND EQUIVARIANT COHOMOLOGY OF COINCIDENT ROOT STRATA

Joint work with András Juhász.

We study a class of enumerative problems. The first examples in this class are due to Plücker:

- The number of tangent lines to a generic degree d plane curve through a point is $d(d-1)$.
- The number of flex lines to a generic degree d plane curve is $3d(d-2)$.

If we move to higher dimensions then other orders of tangencies also appear. For example:

- What is the number of lines in $\mathbb{P}(\mathbb{C}^n)$ having a 4-flex (order of tangency is 4) and two more tangent points with a generic degree d hypersurface $Z_d \subset \mathbb{P}(\mathbb{C}^n)$, intersecting a projective subspace of codimension 5?

In this talk I explain how to calculate these numbers, and some general patters. For example the numbers are always polynomials as a function of the degree, as in the Plücker examples above.

To find these numbers first we need to calculate the cohomology class $[\mathcal{T}_\lambda(Z_d)]$ of the tangential variety

$$\mathcal{T}_\lambda(Z_d) \subset \mathrm{Gr}_2(\mathbb{C}^n) = \{\text{lines in } P(\mathbb{C}^n)\},$$

where λ is a partition encoding the orders of the points of tangency.

It turns out that these cohomology classes do not depend on n: Let $\mathrm{Pol}^d(\mathbb{C}^2)$ denote the vector space of binary forms of degree d—homogeneous polynomials of 2 variables of degree d. Let Y_λ denote the stratum of polynomials with root multiplicities encoded by the partition λ. Then the *equivariant cohomology class* $[Y_\lambda] \in \mathbb{Z}[c_1, c_2]$ has the propery that

$$[\mathcal{T}_\lambda(Z_d)] = [Y_\lambda],$$

where $c_i = c_i(S^2 \to \mathrm{Gr}_2(\mathbb{C}^n))$, the Chern classes of the tautological plane bundle over the Grassmannian $\mathrm{Gr}_2(\mathbb{C}^n)$. In fact this property can be *the definition* of $[Y_\lambda]$. The equivariant cohomology classes of coincident root loci were calculated 15 years ago by Fehér-Némethi-Rimányi and, independently, by Kőműves. However these methods don't show the polynomiality. For this reason we designed a new recursive method. We also calculate the expected degree of these polynomials, and give a condition (in terms of Kostka numbers) when the expected degree is obtained.

This project is part of a more ambitious one, to calculate various genera (Euler characteristics, Todd genus, motivic χ_y-genus) of the tangential varieties, and show their polynomiality in d. If time allows I say some words on these.

The lecture is meant to be introductory, I try to explain how equivariant cohomology helps to solve classical and less classical enumerative problems. A basic level of understanding of Chern classes and projective varieties should be enough to enjoy the talk.