Miklós Eper

Title: On a hyper-Kähler isometry between two moduli spaces

Abstract: A Hodge moduli space carries two distinguished complex (and Kähler) structures: one parametrizes meromorphic connections (the de Rham space), while the other parametrizes meromorphic Higgs bundles (the Dolbeault space), both of rank r over a complex curve X. We consider the case $r = 3$ and X equal to the Riemann sphere, where the corresponding moduli spaces have complex dimension two. In this situation, six combinatorial cases arise, each related to 3×3 Lax representations of connections associated with Painlevé systems. On the other hand, the natural representations of these Painlevé systems are of rank 2, and there is well-known biregular correspondence between the moduli spaces of rank 2 and rank 3 representations. Our main result is that this correspondence also preserves the hyper-Kähler structures of the corresponding Hodge moduli spaces.

In the talk I will outline some details of the proof and describe the construction of the rank 3 Dolbeault spaces via blow-ups of certain pencils on the Hirzebruch surface of index 1. This is joint work with my supervisor, Szilárd Szabó.