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## I. Main duties of the research unit in 2013

The fundamental goal of the Alfréd Rényi Institute of Mathematics is to pursue research of high international standing in pure mathematics. The institute is an important center of mathematics internationally. In 2013 it further improved its reputation both in Hungary and abroad, thanks to the outstanding achievements of its researchers. Researchers of the institute have been awarded by several international prizes in 2013: the Shannon Prize, the Dobrushin Award, the Coxeter-James Prize; three academicians were elected to Academia Europaea. One researcher has obtained a five-year support grant of the European Research Council in the Consolidator Grant category. This will be the fifth research group in the institute supported by an ERC grant. Quite a number of national awards were also received by members of the institute (including emeriti): the Grand Cross, the Commander's Cross and the Officer's Cross of the Order of Merit of Hungary, the Széchenyi Prize, the Academy Prize, the Eötvös Wreath, and the Paul Erdős Prize of the Section of Mathematics of the Hungarian Academy of Sciences.

The scientific tasks of the institute concentrate on fundamental research. However, significant efforts were devoted to some topics of applied mathematics as well. The main applied areas investigated in the institute are bioinformatics and cryptography, but applications of mathematical statistics are also relevant, and a new research topic in financial mathematics was started. The institute is organized in the framework of nine scientific departments. The four research groups supported by the Academy's Momentum programme (Cryptography since 2009, Low Dimensional Topology since 2010, Group Theory since 2012, Limit Structures since 2013), as well as the research groups with ERC grants (Number Theory between 2008-2013, Geometry since 2010, Topology since 2012, Regularity since 2013) work on their own independent research projects within the relevant scientific departments. The research topics are continuously adjusted to the most recent developments in mathematics.

## II. Outstanding research and other results in 2013

### a) Outstanding research and other results

*Low Dimensional Topology "Momentum" Research Group*

It was known which weighted homogeneous complex surface singularities can be smoothened so that the rational homology becomes equal to the rational homology of the ball. Based on algebraic geometric calculations they showed that every singularity possessing such a smoothing is weighted homogeneous, so the earlier results yield a complete classification.

The existence of Stein fillable contact structures on high dimensional manifolds has been studied, and as a corollary of this study it has been verified that if a manifold admits a contact structure then so does its product with the 2-dimensional sphere. Using surgery methods an obstacle element was defined for odd dimensional almost contact structures, and it vanishes

exactly when the manifold carries a Stein fillable contact structure. Using this element it was proved that any simply connected 7-dimensional manifold whose second homotopy group is free can always be equipped with a contact structure.

Based on the Floer homology groups of knots, the researchers found some numerical invariants that can be used for estimating the first Betti number of those (not necessarily orientable) surfaces embedded into the 4-dimensional space whose boundary is the given knot. Such estimates were previously known only in the oriented case.

*Groups, Graphs and Ergodic Theory "Momentum" Research Group*

Define the matching measure of a finite graph as the uniform measure on the roots of its matching polynomial. This is weakly convergent for a sequence of graphs that is convergent in the Benjamini-Schramm sense, therefore the asymptotic number of matchings can be bounded using the moments of the limit measure. This gives the best currently known bounds for 3-dimensional grids, an important special case for statistical physics. Sharp estimates on monomer-dimer entropies for Euclidean lattices were given.

The Benjamini-Schramm convergence of bounded Riemann manifolds was studied. It was proved that an invariant random hyperbolic 3-manifold that is finitely generated must be doubly degenerate.

They showed that every expander graph contains a Lipschitz subgraph not involving any short cycle, where the Lipschitz constant depends only on the rate of expansion. They generalized the known algorithm for the local lemma to infinite graphs. Using this tool they improved the dynamical solution of the von Neumann problem by a Lipschitz constraint. From this it follows that a group is not amenable if and only if it contains a non-commutative free geometric subgroup.

*Department of Algebra*

A new direction has been started in the investigation of Morita equivalence of semigroups, without assuming the presence of any kind of local units. Among others, an example has been constructed of semigroups which are Morita equivalent but not strongly Morita equivalent.
It has been shown that the *m* generated Grassmann algebra can be embedded into a matrix algebra of size depending on *m* over a factor of a commutative polynomial algebra in *m* indeterminates. Polynomial identities for the matrix algebras over the *m* generated Grassmann algebra are derived from this embedding.

Several new results have been obtained on the behaviour of radicals of polynomial rings.

In 1960 Graham Higman conjectured that for each fixed *n* the number of conjugacy classes in the Sylow *p*-subgroup of the *n*-by-*n* invertible matrices over the *p*-element field is a polynomial in *p*. In trying to generalize the conjecture, the concept of equa-pattern group was introduced, and it was shown that du Sautoy's famous groups can be represented as equa-pattern groups of degree 13, but the number of conjugacy classes in these groups is not a polynomial in *p*.

Hermitian matrices with a bounded number of distinct eigenvalues constitute a real algebraic subvariety in the space of all hermitian matrices. The vanishing ideal of this naturally occurring real algebraic variety has been investigated by applying the representation theory of the unitary group. They proved that in the smallest nontrivial case, namely for degenerate three by three hermitian matrices, the vanishing ideal is generated by its lowest degree homogeneous component, which is 20-dimensional.

It is well known that to any normal lattice polytope there corresponds a projective toric variety together with an embedding into some projective space. It was shown that in the case of flow polytopes the ideal of this projective variety is generated by relations of degree at most three. Some results on the classification of toric varieties defined by flow polytopes were established.

If $G$ is a finite group acting faithfully on a finite vector space $V$, then the minimal base size of $G$ is at most 2, provided that the action of $G$ is co-prime. This recent theorem has been generalized to the case when $G$ is $p$-solvable and $p$ is at least 5, where $p$ is the characteristic of the underlying field. If $p$ is equal to 2 or 3 then the minimal base size of $G$ is at most 3.

*Department of Algebraic Geometry and Differential Topology*

Using purely topological methods they proved the semicontinuity of the mod 2 spectrum of local isolated hypersurface singularities. In another article, they generalized these results for analytic germs defined on normal surface singularities. For this, they had to define and develop the "fractured Seifert matrix" and its properties.

For a normal surface singularity with rational homology sphere link they recovered the Seiberg-Witten invariants of the link as certain coefficients of an equivariant multivariable Ehrhart polynomial. For this they constructed the corresponding polytope, and developed the equivariant Ehrhart theory for them.

They proved a "Reduction Theorem" for the lattice cohomology of negative definite plumbed 3-manifolds. This reduces the rank of the lattice in the definition of the cohomology to the number of "bad" vertices. In parallel, they determined the corresponding graded roots as well.

Using the path lattice cohomology they provided an explicit topological characterization of the geometric genus of hypersurface Newton non-degenerate and superisolated singularities of dimension two.

They developed the theory of holomorphic arcs. They showed that the connected components of local "short arcs" can be characterized topologically (using the fundamental group) in the case of quotient and normal surface singularities. This is the holomorphic analogue of the famous Nash correspondence.

They obtained significant results on nets of conics and related singularities using equivariant cohomology (calculating Thom polynomials, determining the multiplicities of the determinant map. They proved that holomorphic maps between projective spaces are maximally singular.

They have significantly rewritten their paper “Growth in finite simple groups of Lie type”, as requested by the referees. As a continuation of the subject, they significantly improved the theorem of Breuillard, Green, and Tao on the description of approximate groups: for linear groups they obtained polynomial bounds.

They computed lattice cohomologies of certain three-manifolds constructed similarly as the links of superisolated singularities. They also examined the computability of lattice cohomologies of superisolated singularities from different combinatorial characterizations of local singularities. They generalized known results for singularities constructed from curves with locally reducible singularities. They also made computer-aided searches helping in the classification of complex projective plane curves.

They studied the failure of the local-global principle for points on principal homogeneous spaces of tori defined over the function field of a curve over a *p*-adic field. They have shown that the said failure is fully controlled by an obstruction located in a third cohomology group of the space. They also have succeeded in extending these results to a fairly large class of reductive groups.

They studied the regular homotopy type of the immersions of spheres which can be realized as the link of a complex surface singularity. They identified the Smale invariant – the complete invariant of the regular homotopy classes – with an algebraic invariant of the singularities, namely the number of the complex Whitney umbrellas appearing in the course a generic complex perturbation. This is really a significant new result in the theory of surface singularities.

*Department of Algebraic Logic*

They applied definability theory of mathematical logic to spacetime theory. They proved that, in the spacetime of special relativity, an observer can “map” spacetime just by the use of light-signals, observers can define coordinate systems, and they can check whether they indeed are in a special relativistic spacetime. In these investigations it is possible to require that information/causality along each light signal flows only in one direction (i.e., light-signals can be directed), and the proofs carry through even in this more difficult setting.

Seeing the many examples in the literature of causality violations based on faster-than-light (FTL) signals, one naturally thinks that FTL motion leads inevitably to the possibility of time travel. They showed that this logical inference is invalid. Namely, they constructed a model for special relativity – one for each spacetime dimension – which satisfies Einstein’s Relativity Principle, in which information can propagate along each line (timelike and spacelike alike), and in which there is no information-cycle.

They proved that some natural dynamical assumptions imply that, if there are massive faster than light particles, then their relativistic masses, energies and momenta decrease with their speed.

They provided modal logic axiom systems for relativistic kinematics and dynamics which allow distinguishing between actual and potential objects.

They provided a proof for the undecidability of the intuitionistic theory of real closed ordered fields, by using a weak but reasonable version of randomized Kripke's structure in the intuitionistic metatheory.

*Department of Analysis*

The approximation properties of the so called incomplete polynomials (polynomials with many missing coefficients) have been widely investigated in the univariate case. The density of multivariate incomplete polynomials in the space of continuous functions was known only on the convex bodies. It was shown that multivariate incomplete polynomials are dense in the space of continuous functions on certain star-like domains, and the rate of best approximation was also found.

The multivariate Bernstein inequality was extended from convex domains to star-like domains. It was discovered how the pointwise upper estimate for the derivatives depends on the distance to the boundary and geometric properties of the boundary of the domain.

The Wiener-Ikehara method, which generalizes the prime number theorem, is related to the deduction of the asymptotic behaviour of monotone increasing functions from the properties of their Laplace transforms. The relaxation of the monone increasing property is an important problem in this area. In this respect the above question was resolved for different classes of slowly or moderatly decreasing functions, and Wiener-Ikehara type theorems were obtained with an effective remainder term.

The Goldston-Pintz-Yildirim (GPY) method which brought a breakthrough in the study of differences between consecutive primes is based on construction of sieve coefficients with the help of a weight function which is in some sense optimal for the given problem. Using methods of operator theory an optimal weight function was found for this problem. Application of this weight function leads to improvements of results obtained by the GPY method.

The classical results of Carathéodory and Fejér give the maximal value of the leading coefficient of a nonnegative trigonometric polynomial with fixed constant term. Generalizing this result the above problem was solved on arbitrary locally compact Abel group for the class of normalized positive definite functions vanishing outside a given set. It was shown that the general question can be reduced to the proper Carathéodory-Fejér type extremal problem.

Methods of Fourier Analysis were applied for the upper estimate of independence number of Paley graphs. A sharper upper bound was given for this problem which has been investigated for several decades. In addition, methods of Fourier Analysis were used in order to improve upper bounds for the density of measurable sets on the plane whose distances do not equal one.

The Kantorovich operators are generalizations of the classical Bernstein linear operators which are designed to be applicable for non continuous integrable functions. In this respect the saturation problem of weighted Kantorovich operators was solved.

*Department of Discrete Mathematics*

It should be mentioned before the concrete research results that the research fellows of the department played a leading role in organizing Erdős Centennial, the conference to celebrate the 100th anniversary of the birthday of Paul Erdős. About 600 mathematicians participated in this monumental event, due to not only the importance of it but the high level results of the speakers, most of them coauthors or followers of Paul Erdős.

They finished their research about paths in hypergraphs, determined the maximal number of edges in an $r$-uniform hypergraph not containing any path of given length. Since paths can be defined in hypergraphs in various ways, they proved several results.

They studied secret sharing schemes and proved a general upper bound by extending an earlier result on graphs to hypergraphs.

They studied conflict-free colorings of graphs. They answered a question from an earlier paper on the number of colors needed in the worst case to conflict-free color the neighborhoods of an $n$ vertex graph.

The researchers of the department studied the communication complexity of sparse set disjointness and related problems and established exact trade-offs between the number of rounds and the number of bits exchanged.

They developed the metric limit theory of trees. They proved that the full group of a sofic equivalence relation is sofic. Also, they proved that the topological full group of a Cantor system has the LEF property.

Like in the last few years, one of the main topics was the limit theory of graphs. There are partial results about the "weak limit", (the statistics of densities of subgraphs with given number of vertices and edges, instead of considering the densities of every graph), on quasirandom properties, based on Janson's method, and in problems on graph colorings, on the statistics of the densities of monochromatic copies of fixed graphs.

They finished the writing and posted a 160 page long proof of the asymptotic version of the conjecture on embedding trees into simple graphs.

For every positive integer $k$, they have found a construction of a finite set of simple curves in the plane, where every two curves cross at most $k$ times, and two additional points $a$, $b$, such that every curve from $a$ to $b$ must cross some of the original curves at least $2k$ times. This is tight due to an earlier result from them.

They gave a deterministic algorithm to solve the Approximate Closest Vector Problem in exponential time for an arbitrary norm. This improves the known probabilistic algorithms and has many possibilities of applications from Integer Programming to code signaling.

They managed to give nontrivial lower and upper bounds on the length of paths covering given points on the plane.

Several versions of the “bin packing” optimization problem have been studied. Closing a question which was open for four decades - they found the exact formula for the worst-case behavior of the classical “First Fit Decreasing” algorithm in terms of the optimum.

Graphs and set systems have been studied in which the domination number (the minimum number of elements from which every element can directly be reached) is sufficient also for covering all members of the set system. In graphs, this equality can be tested by an efficient algorithm (despite that both parameters are NP-hard to compute), while in general set systems the problem is in a class slightly larger than NP. Restricting attention to set systems in which every member has at most $k$ elements for a fixed $k$, and the equality of the two parameters is required to hold in every subsystem, the property can be characterized by finitely many forbidden subsystems, for every finite $k$.

Motivated by coding theory, a new extremal problem has been studied concerning orderings of the members of set systems. The goal is to find an order in which the union of members in every initial segment is not much larger than the number of those members. The results give estimates on this difference, in terms of the cardinality of the underlying set.

They simplified the proof of a very difficult theorem on non-growing subsets of linear groups (still 65 pages!). The results for infinite groups in the new version are also explicit. They also succeeded in better understanding the general case by means of reduction theorems for arbitrary groups and found some new examples of non-growing sets (one of them gives a negative answer to a question of Helfgott).

They confirmed and generalized the conjecture of Forrester through $q$-analogues, and presented a general method extending the combinatorial Nullstellensatz.

The study of the graph parameter called local chromatic number was continued. The undirected version is a natural upper bound on the oriented version for any possible orientation of the graph at hand. Now it was shown that the value of the undirected version can be strictly larger than the value of the oriented version maximized over all orientation of the same graph. It was also shown that this is not so for the fractional values of these two parameters.

They investigated random processes defined on the vertices of a $d$-regular tree that are invariant under the automorphisms of the tree. An important question is to characterize the so called factor of i.i.d processes. Their importance is rooted in the fact that they can be modeled on an arbitrary finite large girth $d$-regular graph, using local algorithms. They gave various necessary and sufficient conditions. They investigated the automorphism group of graphons and showed that this is a compact group, and they gave descriptions of it in terms of graph algebras.

The existence of graph homomorphisms is an essential part of the Constraint Satisfaction Problem, which is one of the central themes of theoretical computer science. Here the problem is to find dual set pairs in the partial order of finite, directed graphs. Refuting an earlier conjecture it turned out that such pairs indeed exist. Even more, they gave a complete description of such dual pairs in cases, where both sets in the dual pair form antichains.

They studied a series of problems in connection with degree sequences. They introduced a new problem class called “restricted degree sequence” problem. This is a special case of Tutte's *f*-factor theorem. Tutte's question is solvable in polynomial time, but the solution of the new problem is much faster (it is quadratic in time). It is an important difference however that the simpler problem has a practically useable polynomial time uniform sampling method. Furthermore, the theorem extends the applicability of several existing sampling methods showing that they provide effective approximate counting processes for the number of solutions.

In the line version of the classic Erdős-Szekeres theorem almost precise upper and lower bounds have been proved, much better ones than in the point case.

*Department of Geometry*

It is known that the square cannot be tiled by an odd number of equal area triangles. On the other hand, it was shown that every simple polygon can be triangulated with finite many equal-diameter triangles as well as into an infinite number of equal-perimeter triangles. Similar results can be proved on the sphere.

The case of equality in the Orlicz-Petty projection body inequality, which is related to problems in tomography, was an open problem. They settled this question by providing a stronger, stability version of the inequality.

They gave a new proof to the characterisation of tight frames consisting of unit vectors and provided new methods to construct extremal systems of unit vectors by means of solving energy minimising problems. They proved inverse Bernstein type inequalities about complex polynomials with roots on the unit circle, and generalised the related results.

They continued the study of geometric and topological graphs. An important aspect of these problems is that they are closely related to the following classic conjecture of Erdős and Hajnal: Every $n$-vertex graph that does not contain a fixed induced subgraph $G$, has a complete subgraph or an independent set of size at least $n^{c(G)}$, where $c(G)$ is a positive constant. They managed to verify this conjecture in some special cases and also established an approximate version of this conjecture for string graphs (intersection graphs of connected compact sets in the plane). They proved similar statements for semi-algebraically defined graphs and hypergraphs.

Let $f(n)$ be the smallest number with the property that $n$ points in the plane in general position can always be separated by $f(n)$ convex sets. In previous work they gave almost sharp bounds for $f(n)$. This has been generalized in several directions in higher dimensions, using a more general concept of separation, with some other condition on the separating sets, instead of convexity.

They constructed non-extendable $k$-simple topological graphs with $n$ vertices and $c_k n$ edges. The values of $c_k$ and the constructions are different for different values of $k$. They also proved that there is no such topological graph with less than $1.5n$ edges.

They translated into English the 1953 German edition of the classical book on packing and covering and added a survey of about 70 pages on the 800 items of new literature closely related to the subject of the book.

*Department of Set Theory and General Topology*

The study of resolvability properties of topological spaces was carried on. They produced a technically very complicated and lengthy proof of the fact that every regular space whose extent is smaller than its dispersion character (i.e. the supremum of the sizes of closed discrete subsets is smaller than the size of any non-empty open set) is $\omega$-resolvable. This is a significant improvement of a deep result of O. Pavlov concerning an old problem of V. Malychin. They also proved that every compact Hausdorff space is maximally $G_\delta$-resolvable, i.e. has as many $G_\delta$-dense subsets as the least size of a non-empty $G_\delta$ set.

It is known that any non-isolated point in a compact Hausdorff space is the accumulation point of a discrete subset. They proved that this statement is no longer valid if compactness is weakened in any way, e.g. it is replaced by the property of initial $\kappa$-compactness for any fixed cardinal $\kappa$.

More than 80 years ago, Kolmogorov raised the following question. Can every measurable planar set be mapped onto a polygon by a contraction such that the area of the polygon is at least the area of the set minus epsilon? They constructed a bounded, simply connected, open counter example.

They solved a more than 20-year old question first asked by Mycielski concerning Haar null sets in non-locally compact groups, e.g. in infinite dimensional Banach spaces. Namely, they showed that in every non-locally compact Polish group with an invariant metric there is a Haar null set that is not contained in any $G_\delta$ Haar null set.

In 1961 Vaught published the following conjecture: A theory has either at most countably many or exactly continuum many countable models. Some partial results have been obtained in connections with certain variants and generalizations of Vaught's conjecture. Studying the automorphism groups of countable, homogeneous structures, they proved that these groups always contain dense Abelian subgroups. The proof depends on topological and descriptive set theoretical methods.

Exploring properties of homogeneous structures is among the frequently studied and important problems in mathematical logic. By introducing a new kind of amalgamation property of the finite submodels they managed to describe those homogeneous structures in which Craig's interpolation theorem and Robinson's joint consistency theorem can be reproduced.

*Department of Number Theory*

They investigated differences of primes with analytic methods. In 2013 Y. Zhang, then by different methods J. Maynard proved the bounded-gap conjecture (slightly weaker than the twin prime conjecture) that the difference of consecutive primes is infinitely often under a given bound. Both methods are based on the Goldston-Pintz-Yildirim sieve. The paper (in preparation) involves results by members of the department.

Research in combinatorial number theory was carried on. They have improved the known estimate for the number of residues modulo a prime such that each difference is a quadratic residue. Combinatorial methods have been applied to a distant problem in functional analysis (hypercyclic operators). It was shown that every finite subset of the positive real numbers can be split into two parts, such that one part shows very little additive, the other part shows very little multiplicative behaviour.

They considered the class number one problem in a special family of real quadratic number fields. This is more general than the well-known Yokoi-family of quadratic fields. This work is still in progress, but there are promising partial results.

*Department of Probability Theory and Statistics*

One of the research objects was the theory of the dynamical systems. The main point of this research was the theory of non-smooth hyperbolic systems, because this plays an important role in the study of several physical systems. They generalized a coupling technique which has been developed by Chernov and Dolgopyat not long ago. This enabled them to study higher dimensional, piecewise smooth hyperbolic systems. This research was made in cooperation with the researchers of the Vienna and Helsinki University. This method also enabled them to work out rigorous models for some heat conduction systems.

The researchers of this group proved probabilistic results in which the number theoretical properties of the coefficients appearing in the problem played an important role. By giving a precise condition for the almost sure convergence of the averages of the functions $f(kx)$ they closed the investigation of a conjecture of Khinchin formulated in 1924. They obtained new results in the extremal theory of dependent and independent random variables and in the metric theory of continued fractions.

A good estimate was given about the speed of information transmission of two independent sources through a noisy channel with small probability of error. This result shows that the information theory of multi source channels is much more complex than that of the one source case. New results were proved in financial mathematics where information theory plays an important role. The gain of a portfolio was determined in the worst case when the effect of the environment is known only with some error.

Some of the researchers took part in the investigation of the independence ratio of graphs. In this research they could improve the best results known in this field. This research was connected to the investigation of the mixing time of Markov processes, where they could understand the reason for the speedup in the case of non-reversible Markov chains.

Also some applied work was carried out. They performed the statistical analysis for some medical papers on congenital cardivascular abnormalites, and made some improvements and additions to the vehicle rooting problem algorithm.

The estimation of the tail distribution of multiple integrals with respect to a direct product of a normalized empirical distribution with itself was investigated together with an estimation of the supremum of such integrals. The results obtained in this field are useful in the study of hard statistical problems. Such problems would be very hard to investigate without this theory. Also a Lecture Note was published about this subject by the Springer Verlag.

JDM (Joint Degree Matrices) were investigated, and their finer properties were studied. Also such Markov chains were considered whose phase space is a graph which has a JDM realization. Good estimates were given on the mixing properties of such Markov chains. The solution of these problems turned out to be useful in genetical investigations.

Some natural problems of financial mathematics were investigated. The optimal investment was studied for such investors whose behaviour follows the usual behaviour patterns; they are sometimes risk-averse in other cases risk-seeking, and subjectively magnify the probability of rare events. These problems led to non-convex optimization where even dynamic programing fails. As a result of this research it was possible to work out such a theory with the help of the methods in game theory which can be applied for discrete time market models.

To work out new statistical methods computer experiments were made in order to determine the distribution of the so-called Ajtai-Komlós-Tusnády statistic.

*Applied research*

The major part of the research carried out at the Rényi Institute is generated by questions raised by the inner development of mathematics. Nevertheless, together with the exploratory (theoretical) research that the institute conducts, these new results and other fundamental methods of mathematics are applied in other disciplines as well. These include the use of the results on representations of graphs in software development, the use of higher order Fourier analysis for time series analysis in economics, and writing the necessary software for this application, the use of information theory in economics, and answering a mathematical question coming from neurobiological investigations of a Momentum research group at the Institute for Experimental Medicine.

The Cryptology research group that started in the framework of the first Momentum project has been given a permanent status. As their most important result in 2013 they obtained the almost exact asymptotics for the complexity of secret sharing schemes determined by graphs and hypergraphs. They succeeded in estimating the asymptotic value of the complexity in question up to a factor of 2. This result is not only relevant to cryptography, it is also an important achievement in graph theory. Their provably secure new algorithm for attribute based multi-server encrypting received significant interest. Continuing their earlier research they studied analogues and particular cases of problems in extremal graph theory that are motivated by investigating the mathematical properties of unique, copy-proof stamps. A member of the research group was invited to an information theory symposium in Hong Kong. It can be considered as an acknowledgement of the achievements of the group that they were selected to obtain the right to organize the "Central European Conference on Cryptology" in 2014.

In the framework of research in bioinformatics the Rényi Institute has finished its part in the European consortial project "Comparative Genomics and Next Generation Sequencing". Together with other consortium members they developed a software package that can efficiently manage the enormous amount of genomic data supplied by the new generation sequencing techniques, and carries out the automatic annotation of the data as well. This software will be part of the software package of CLCbio, the largest European bioinformatics enterprise. The majority of the software development was done in the Rényi Institute. The institute also took part in an international collaboration, supported by the American DARPA, working on theoretical question on computational statistics in networks.

*Career advancement of researchers*

2013 two researchers of the institute were elected for full membership in the Hungarian Academy of Sciences, one person became a corresponding member of the Academy, another one received the DSc title. Six young researchers got PhD's. At the end of the year 8 members of the Academy (7 according to the statistical number of employees), 29 doctors of MTA (stat. num. 26), 38 researchers with PhD or CSc (stat. numb. 36) worked at the institute, 18 researchers (stat. num. 16) have not yet obtained a degree. Besides the regular employees 12 emeritus research professors (7 academicians, 5 with DSc title) take part in the scientific work of the institute.

The institute puts great emphasis on involving young talents – working towards their PhD or just obtaining the degree – into the research work of the institute. In 2013 further five young researchers were employed in the new or vacant positions offered by the Academy. Altogether 18 young researchers worked in the institute in 2013. The institute has an agreement with the Central European University. In this framework 13 doctoral students were supervised by members of the institute.

## b) Dialog between science and society

Unfortunately, most of the research topics in pure mathematics are not suitable for discussions aimed at the general public. On the other hand, the international success of the researchers has underlined the importance of the research conducted in the institute even in the media. Several interviews with researchers of the institute have been published during the year in Hungarian popular science magazines.

The researchers of the institute play an important role in popularizing mathematics, giving lectures for high school and university students. The institute regularly organizes an open house during the Festival of Hungarian Science, where high school students and their teachers can get information about the mathematics profession. Members of the institute take part in fostering mathematical talents by organizing several mathematical camps and other events for students interested in the subject. The institute plays a role in giving scientific background for the teachers of specialized mathematics classes in high schools.

## III. A presentation of national and international relations

*National relations*

Researchers of the institute teach part time in many universities both in Budapest in other cities (Eötvös University, Budapest University of Technology, University of Szeged, Pannon University, etc.). They play an important role in doctoral schools and in Master programs. 18 members of the institute are core members of doctoral schools in various universities, they supervise 42 doctoral students. Especially important is the collaboration between the institute and the Department of Mathematics and its Applications of the Central European University. The lecturers and the supervisors of the Masters and doctoral programs of CEU mainly belong to the institute, including the department chair and the leader of the doctoral program. Also a large part of lecturers of the Budapest Semesters in Mathematics English language study abroad program for North American students belongs to the institute. This program helps to bring the fame of Hungarian mathematics to American universities, and serves as a role model

for other international programs (e.g., the Aquincum Institute of Technology). For the institute the close contact with the new generation of mathematicians is of foremost importance. In this spirit 54 members of the institute (64 percent of all researchers) were active in teaching at universities in 2013, that included supervising 5 student research projects, 13 BSc and 22 MSc dissertations.

As part of the renewal program of the Academy, the institute restarted its guest researcher program, which enables university professors and lecturers to spend one or two semesters in the institute freed from their teaching duties. As part of this program three people from the Budapest University of Technology and one from Eötvös University joined the research teams of the institute in 2013.

The weekly seminars in the institute are attended regularly by researchers from other institutions, among them some people from universities outside Budapest as well. This way these seminars influence the whole mathematical scene in Hungary.

Members of the Rényi Institute traditionally take part in various Hungarian scientific committees well over proportion. In particular, the Section of Mathematics of the Hungarian Academy of Sciences (MTA) and its committees, the Hungarian Research Fund (OTKA), and the János Bolyai Mathematical Society (BJMT) can be mentioned. The president of the Section of Mathematics of MTA, the chairman and the secretary of the Mathematics Committee, the secretary of the Mathematics Doctoral Committee, one of the vice-chairmen and the secretary of the Bioinformatics Committee, the chairman of the Mathematics and Natural Sciences subcommittee of the Council of the Academy's Research Units, the chairman of the mathematics panel of OTKA, the president of the BJMT, the chairman and the secretary of the Scientific Section of BJMT, the vice-president of the Applied Mathematics Section of BJMT, the secretary general of the Hungarian Society for Bioinformatics are all researchers of the Rényi Institute.

*International relations*

The researchers of the institute have very extensive international relations. Among the coauthors of the members of the institute one finds mainly foreign mathematicians. Joint projects and jointly organized conferences are also typical.

In 2013 thirty-five people from the institute were involved in organizing international conferences, twelve researchers took part in setting up a national event, some of them at more occasions. By far the most important was the Erdős Centennial conference, one of the major mathematical events in 2013 worldwide. It attracted more than six hundred participants. The greatest share of the organizational work was done by members of the institute. It should also be mentioned that in the most important mathematical conference center in Europe (Mathematisches Forschungsinstitut Oberwolfach) three conferences were held in 2013 where the exclusive organizing committee (consisting of only 2–3 members) included a researcher from the institute; incidentally three Momentum group leaders. This underpins the topicality of their research areas. The young researchers in the institute have organized the fifth "Emléktábla Workshop" this year.

The visits in the framework of the bilateral exchange programs between the Hungarian Academy of Sciences and its partner institutions successfully contributed to the cooperation with foreign partners. With the help of these programs fruitful joint research projects, useful exchange of information, and conference participations were made possible.

Researchers of the institute took part in altogether ten international scientific committees. For example, the vice-president of the European Set Theory Society is a member of the institute. Names of the institute's researchers can be found 140 times on the list of editorial board of various international journals. In 2013 the researchers gave altogether 211 talks at international meetings, many of these were given as an invited or plenary lecture.

In 2013 eight researchers spent more than half a year abroad at the following institutions: University of Chicago (USA), Auburn University (USA), City University of New York (USA), National Science Foundation (USA), École Polytechnique Fédérale de Lausanne (Switzerland), Lancaster University (England), University of Edinburgh (Scotland).

Financed by the ERC and Momentum grants or from other sources 29 foreign researchers worked in the institute for 1–8 months (altogether that makes up 72 months). They came from Austria, the Czech Republic, France, Finland, Germany, Spain, UK, USA, Australia, Mongolia, China and India. The number of foreign visitors of the institute – not counting the conference participants, nor the foreign employees – was near to 120 in 2013.

## IV. Brief summary of national and international research proposals, winning in 2013

*National grants*

The Rényi Institute, similar to the practices of the previous years, successfully participated – both in terms of the applications and the winning projects in 2013 in the Hungarian OTKA (Hungarian Scientific Research Fund) project proposals. In 2013 further 3 research and 3 postdoc projects won support. The overall OTKA project support of the institute has increased significantly again.

The projects for the "Momentum" call for special projects and the postdoc projects of the Hungarian Academy of Sciences remain very important and valuable to the institute. After the three winning Momentum projects of the previous years another young researcher returning from Toronto was awarded a Momentum project to develop the mathematical tools needed to analyze the statistics of very large structures. Apart from the above Momentum grants only the two new positions won on the postdoc projects of the Academy contributed significantly to the total income of the national grants.

*International grants*

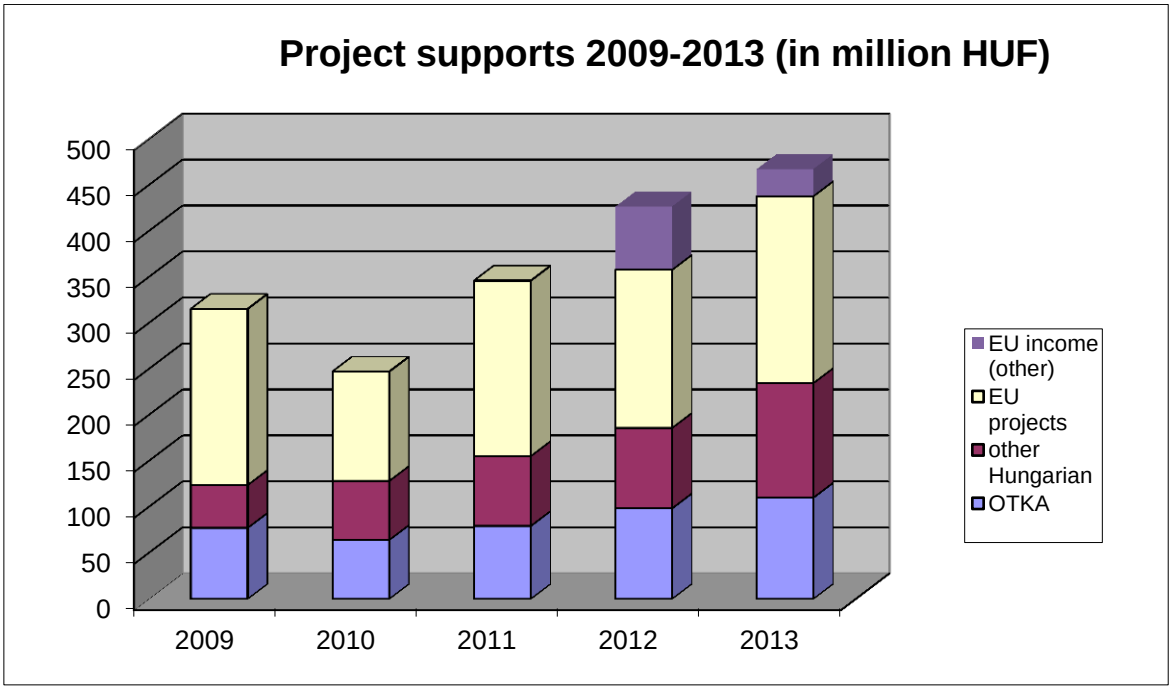
In the framework of the more and more known European Research Council (ERC) research projects – supporting small research groups run by the principal investigator (starting independent researcher or the advanced investigator) for a longer period with a significant, sometimes several million euros grants – a young researcher of the institute awarded a Momentum project in 2013 won a new ERC Consolidator Grant entitled „Limits of discrete structures". The project will start early 2014. The subject of the research is related to another, earlier Momentum project, and so the financial resources from these different projects allow the development of a research group of significant size, with real "momentum".

This fifth ERC project following the 4 ones of the previous years brings the Rényi Institute among the most successful ERC applicants of Hungary.

Furthermore, two individual mobility project proposals were submitted in FP7 by Hungarian researchers residing in or returning from foreign countries, both funded and starting 2014.

Altogether, the total research grant income of the institute in 2013 exceeded the grant income of any previous years (beyond the projects mentioned above the COGANGS project mentioned in chapter of Applied research brought further significant income). Both the support of OTKA and Momentum and the support of international projects from the EU accounted for 2013 were significantly higher than those of the previous years. The running Momentum, OTKA and EU projects together with the Momentum project awarded in 2013 and the new ERC project starting in 2014 will jointly ensure that there will be no significant decrease in the total research grant income of the institute in 2014, despite the fact that the first ERC Advanced grant project of the institute ended in 2013.

The following diagram shows the amount of project support received during the last 5 years.



## V. List of important publications in 2013

1. Abért M, Weiss B: Bernoulli actions are weakly contained in any free action. Ergod Theor Dyn Syst, 33:(2) 323-333 (2013) http://real.mtak.hu/10014

2. Bárány I, Itoh J-I, Vîlcu C, Zamfirescu T: Every point is critical. Adv Math, 235: 390-397 (2013) http://real.mtak.hu/10060

3. Böröczky KJ: Stronger versions of the Orlicz-Petty projection inequality. J Differ Geom, 95(2): 215-247 (2013) http://real.mtak.hu/10062

4. Böröczky KJ, Lutwak E, Yang D, Zhang G: The logarithmic Minkowski problem. J Am Math Soc, 26: 831-852 (2013) http://real.mtak.hu/10063

5. Domokos M, Fehér L, Rimányi R: Equivariant and invariant theory of nets of conics with an application to Thom polynomials. J Singul, 7: 1-20 (2013) http://real.mtak.hu/10061

6. Erdős PL, Tardif C, Tardos G: Caterpillar dualities and regular languages. SIAM J Discrete Math, 27:(3) 1287-1294 (2013) http://real.mtak.hu/9900

7. Füredi Z, Ruszinkó M: Uniform hypergraphs containing no grids. Adv Math, 240: 302-324 (2013) http://real.mtak.hu/7997

8. Gill N, Pyber L, Short I, Szabó E: On the product decomposition conjecture for finite simple groups. Groups Geom Dyn, 7:(4) 867-882 (2013) http://real.mtak.hu/9450

9. Goldstern M, Ságí G, Shelah S: Very many clones above the unary clone. Algebra Univ, 69:(4) 387-399 (2013) http://real.mtak.hu/10049

10. Goldston DA, Pintz J, Yildirim CY: Primes in tuples IV: Density of small gaps between consecutive primes. Acta Arith, 160:(1) 37-53 (2013) http://real.mtak.hu/10064

11. Harcos G, Templier N: On the sup-norm of Maass cusp forms of large level. III. Math Ann, 356:(1) 209-216 (2013) http://real.mtak.hu/10046

12. Hiai F, Kosaki H, Petz D, Ruskai MB: Families of completely positive maps associated with monotone metrics. Linear Algebra Appl, 439:(7) 1749-1791 (2013) http://real.mtak.hu/10059

13. Juhász I, van Mill J, Weiss W: Variations on $\omega$-boundedness. Israel J Math, 194:(2) 745-766 (2013) http://real.mtak.hu/10065

14. Kalmár B, Stipsicz AI: Singular maps on exotic 4-manifold pairs. Algebr Geom Topol, 13:(3) 1709-1731 (2013) http://real.mtak.hu/9949

15. Major P: On the estimation of multiple random integrals and U-statistics. Lect Notes Math, 2079: Berlin: Springer-Verlag, 298 (2013) http://real.mtak.hu/4125

16. Marton K: An inequality for relative entropy and logarithmic Sobolev inequalities in Euclidean spaces. J Funct Anal, 264:(1) 34-61 (2013) http://real.mtak.hu/10066

17. Mohar B, Simonyi G, Tardos G: Local chromatic number of quadrangulations of surfaces. Combinatorica, 33(4): 467-494 (2013) http://real.mtak.hu/9875