

On set addition with restricted restrictions

Let A_1, A_2, \dots, A_n be subsets of a field and Γ a graph with vertex set $V(\Gamma) = \{1, 2, \dots, n\}$. Denote by $\sum_{\Gamma} A_i$ the set of all elements that can be written in the form $a_1 + \dots + a_n$, where $a_i \in A_i$ and $a_i \neq a_j$ for $ij \in E(\Gamma)$. Extending a recent result of Wang and Sun, we provide lower bounds on $|\sum_{\Gamma} A_i|$ in the case when Γ is either a path or a cycle, and the cardinalities of the sets A_i are nearly equal. Our proof depends on a quantitative variant of the Combinatorial Nullstellensatz known as the Coefficient Lemma.