

On value sets of off-diagonal Carlitz-type polynomials

Let F be a field and $f \in F[x_1, \dots, x_n]$ be a polynomial of the form

$$f(x_1, \dots, x_n) = f_1(x_1) + \dots + f_n(x_n) + g(x_1, \dots, x_n),$$

where each f_i is a nonconstant polynomial in $F[x]$ and $\deg g < \deg f_i$ for every i . For nonempty finite sets $A_i \subseteq F$ we give a sharp lower estimate on the size of the value set $\{f(a_1, \dots, a_n) \mid a_1 \in A_1, \dots, a_n \in A_n\}$ in the spirit of a result of Sun. Our proof depends on the generalized Combinatorial Nullstellensatz and yields an extension of an old theorem of Carlitz.