QUANTIFIED BEURLING'S UNCERTAINTY PRINCIPLE FOR FOURIER TRANSFORMS

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The uncertainty principle in harmonic analysis states that a non-zero function and its Fourier transform cannot be simultaneously too sharply localized. There are numerous precise mathematical formulations of this meta-theorem.

Beurling's own version of the uncertainly principle for Fourier transforms is the following clean and elegant statement: Given $f \in L^1(\mathbb{R})$,

$$\iint_{\mathbb{R}^2} |f(x)\widehat{f}(\xi)|e^{|x\cdot\xi|}\mathrm{d}x\mathrm{d}\xi < \infty \implies f = 0.$$

The goal of this talk is to present a general quantified version of Beurling's uncertainty principle. We will characterize in several ways those $f \in L^1(\mathbb{R}^n)$ such that

$$\iint_{\mathbb{R}^{2n}} \frac{|f(x)\widehat{f}(\xi)|e^{|x\cdot\xi|}}{W(|x|+|\xi|)}\mathrm{d}x\mathrm{d}\xi < \infty,$$

where $W : [0,\infty) \to [1,\infty)$ is an unbounded non-decreasing function subject to certain natural regularity conditions.

The talk is based on collaborative work with Lenny Neyt (University of Vienna).

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