Two complexity results on parametric degree sequence realizability

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Abstract

We present two complexity results on parametric degree sequence realizability: one concerning sparse hypergraph degree sequences and the other addressing the bipartite degree realization problem under extremal density constraints.

The first result, presented in [1], establishes a complete dichotomy theorem for the parameterized sparse t-uniform hypergraphic degree sequence problem. For fixed $t \geq 3$, we consider degree sequences of length n whose degrees lie between $n^{\alpha'}$ and $6n^{\alpha}$, where $0 < \alpha' \leq \alpha < t-1$. We show that when

$$\alpha' \leq \frac{t(\alpha-1)+1}{t-1},$$

deciding whether a t-uniform hypergraph exists with the prescribed degree sequence is NP-complete. In contrast, if

$$\alpha' > \frac{t(\alpha-1)+1}{t-1},$$

the realizability problem admits a linear-time algorithm. The proof relies on a detailed structural analysis of sparse realizations, exploiting balancing hinge-flip operations and rigid decomposition properties reminiscent of the Tyshkevich product.

The second paper [2] investigates the computational complexity of the Bipartite Degree Realization (BDR) problem, that is, deciding whether a given degree sequence has a bipartite graph realization, under extremal density constraints. For constants $0 \leq c_1 \leq c_2 \leq 1$, we define BDR_{c_1,c_2} as the restriction of BDR to degree sequences of length n whose degrees lie in the interval $[c_1 n, c_2 n]$. We prove that BDR_{c_1,c_2} is solvable in polynomial time whenever

$$0 \leq c_1 \leq c_2 \leq \frac{\sqrt{c_1(c_1+4)}-c_1}{2},$$

as well as for all $c_1 > \frac{1}{2}$. Furthermore, assuming the NP-completeness of unrestricted BDR, we show that BDR_{c_1,c_2} remains NP-complete for all $0 < c_2 < \frac{1}{2}$ and

$$c_1 < 1 - c_2 - \sqrt{1-2c_2}.$$

Although the two problems appear quite different, in both cases the main technical tools are balancing hinge-flip operations and the existence of Tyshkevich-product-like rigid structures in the realizations of certain

extremal degree sequences. These structural properties play a crucial role in both the algorithmic tractability results and the NP-completeness proofs.

References

[1] I. Miklós, M. Ruszinkó, and B. Zavalnija, "A complete dichotomy theorem on the sparse t-Uniform Hypergraphicality Problem," *arXiv:2512.15356*, 2025.

[2] I. Miklós, "On the Complexity of Bipartite Degree Realizability," *arXiv:2512.17709*, 2025.