

P-stability and fully-graphicality in sparse and dense t -uniform hypergraphs

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Abstract

The degree sequences of uniform hypergraphs exhibit a striking connection between realizability, computational complexity, and the stability of realization counts. We investigate this connection in both sparse and dense degree ranges.

A degree-sequence class is called fully graphic if every admissible sequence in the class is realizable by a uniform hypergraph. P-stability, informally, means that the number of realizations of a degree sequence changes by at most a polynomial factor under small perturbations of its degrees. Previous work established a sharp dichotomy between fully graphic classes, where realizability can be decided in linear time, and classes where the problem is NP-complete for sparse t -uniform hypergraphs and dense 3-uniform hypergraphs [4, 2]. We show that, in both settings, the same boundary separates P-stable from non-P-stable degree-sequence classes.

For dense t -uniform hypergraphs, we prove a general conditional result for every t : whenever the threshold function for full graphicality is continuous, every degree range above that threshold is P-stable. Although the threshold is explicitly known only in the 3-uniform case, our stability result requires only the stated continuity assumption and holds for every t .

We establish P-stability in both regimes using probabilistic methods. In the dense case, McDiarmid's bounded-differences inequality provides a concentration argument for induced subhypergraphs [3]. For sparse t -uniform hypergraphs, the key condition in our probabilistic proof is $\Delta^t = o(\Sigma^{t-1})$, where Δ is the maximum degree and Σ is the sum of the degrees. This condition also appears in asymptotic enumeration results for hypergraphs [1]. We also show that this condition holds if and only if a sparse t -uniform degree sequence class is fully graphic.

In contrast, we prove non-P-stability in every non-fully-graphic range, for every uniformity, using embedding constructions that turn obstructions to realizability into large discrepancies between realization counts.

Our results show that the computational complexity phase transition for hypergraphicality is accompanied by a corresponding transition in the stability of the number of realizations.

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References

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