On Bollobás-type theorems

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Date of the talk: 2025.9.25

Let $\mathcal{P} = \{(A_i, B_i) \mid i \in [m]\}$ be a family of pairs of sets (spaces), such that $A_i \cap B_i = \emptyset$ ($\{0\}$). We say $\mathcal{P}$ is a Bollobás system if $A_i \cap B_j \neq \emptyset$ ($\{0\}$) for every $i \neq j$, and a skew Bollobás system if $A_i \cap B_j \neq \emptyset$ ($\{0\}$) for every $i < j$. Note that a Bollobás system is always a skew Bollobás system.

In 1965, Bollobás [1] proved that for a Bollobás system $\mathcal{P} = \{(A_i, B_i) \mid i \in [m]\}$, we have

$$\sum_{i\in[m]} \binom{|A_i| + |B_i|}{|A_i|}^{-1} \leq 1,$$

which later attracted extensive interest from mathematicians, and became one of the most important topics in extreme set theory.

In this talk, we will give an introduction of different generalizations and variations of Bollobás type theorems, and show some new results on this topic [2, 3, 4].

References

[1] B. Bollobás, On generalized graphs, Acta Math. Acad. Sci. Hungar. 16 (1965) 447-452.

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[4] E. Yue, B.Lv, P. Sziklai, K. Wang, A Bollobás-type theorem on singular linear spaces, https://arxiv.org/abs/2408.05630, 2024-8.