

„Beyond the event horizon” Weyl’s forgotten cosmology

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- My name is George Szondy
- I have been dealing with the fundamentals of General Relativity for more than 15 years
- My presentation 2 years ago was about everything
 - Form the generalized 4-force by Karoly Novobatzky – which should have been explained by Gyula Dávid on the 24th of August
 - Till a geometry based model of quantum gravity
- This time My presentation is about „Weyl’s forgotten cosmology”

Hermann Weyl (1885-1955)

- German mathematician, theoretical physicist and philosopher
- He was one of the first trying to combine general relativity with electromagnetism
- He was the one who introduced frames into general relativity in 1929.
- He said: „*You can not apply mathematics as long as words still becloud reality*”



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Possibly most of you know the work of Hermann Weyl better than me...
He was ...

I brought a quote from Weyl, which is important: „*You can not apply mathematics as long as words still becloud reality*”

Means: Mathematics is just a tool, we need to understand physics behind.

It is especially true for General Relativity

Well known Problems of General Relativity

- **Nonlinearity**
„General relativity describes the gravitational field by curved space-time; the field equations governing this curvature are nonlinear and therefore difficult to solve in a closed form.”
- **Local frames instead of coordinate system**
 - Spacetime = frame field defined on a Lorentzian manifold
- **No quantum theory**

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GR > 100 years old...
...but still have some unresolved problems

Real problem with General Relativity

- We believe in it
- We know its mathematics
- But – in general – we still don't understand its **physical meaning**
 - *In 1957 at Chapel Hill "Conference on The Role of Gravitation in Physics" – **John Archibald Wheeler** spoke on the need to better understand the physical meaning of general relativity. (Peebles, 2016)*
 - Wheeler started teaching relativity 4 years earlier in 1953.

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But the real problem...

Without understanding it's hard to select what is important piece of knowledge, and what is not.

So several really important thoughts and results of the last 100 years has been forgotten.

...One such example is Weyl's forgotten cosmology

On the Theory of Gravitation By Hermann Weyl (1917)

Contents:

- Appendix to General Relativity
- Theory of the static, axial symmetric field
 - **Point-mass** with and **without electric charge**.

Motivation:

- To understand better the geometry of Schwarzschild solution

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This cosmology was founded and published by Weyl in 1917, less, then 2 years after the born/appearance of GR.

The article was

- mainly an „Appendix to General Relativity”, but
- had a short chapter about the point mass in static, axial symmetric field – practically the Schwarzschild solution

The motivation was: To understand better the geometry of Schwarzschild black hole

About the Schwarzschild solution

- ...uses standard coordinates
- The form of the line element:

$$ds^2 = A(r)dt^2 - B(r)dr^2 - r^2(d\theta^2 + \sin^2 \theta d\varphi^2)$$

- The metric:

$$ds^2 = \left(1 - \frac{r_s}{r}\right) dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2 \theta d\varphi^2)$$

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Weyl started from the Schwarzschild metric, which uses standard coordinates.

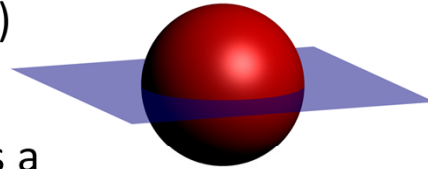
Standard metric has a property, that

- only the time and the radial coordinate has a coefficient which is different from 1
- the circumference of a circle with the center in the origin is always $2\pi r$

Here is the Schwarzschild metric, where the parameter r_s is called Schwarzschild radius

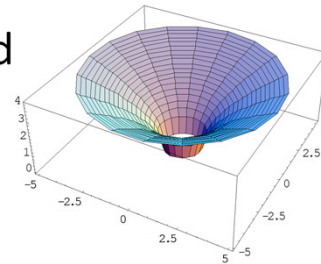
To understand better the geometry of Schwarzschild solution

Geometry of the plane surface through the equator ($\varphi=0$)



Line-element characterizes a geometry, which is valid for the following rotation ellipsoid in Euclidean space

$$z = \sqrt{8a(r - r_s)}$$



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Weyl's motivation was to understand better the geometry of Schwarzschild solution, or Schwarzschild black hole.

1. He examined the geometry of the plane surface through the equator of the black hole ($\varphi=0$)
2. He found that geometry of the surface is the same as the rotation ellipsoid defined by the following equation in Euclidean space
3. Which is like a FUNNEL

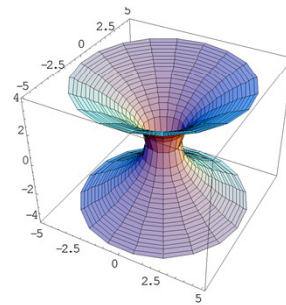
This funnel has a DISCONTINUITY at the event horizon at $z=0$.

Natural analytic continuation

Rotation ellipsoid $z = \sqrt{8a(r-r_s)}$ \Rightarrow $z = \pm\sqrt{8a(r-r_s)}$



- The projection covers
 - the outer part of the sphere ($r > r_s$) twice
 - the inner part ($r < r_s$) not at all



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His next step was to extend/continue this solution beyond this discontinuity.

The natural continuation of the rotation ellipsoid was to allow both +/- signs for the z coordinates

This projection covers

- the outer part of the sphere ($r > r_s$) twice
- And does not cover the inner part ($r < r_s$) at all

Is it familiar? (Seems like an Einstein-Rosen bridge – 18 years earlier than it was published)

To make it obvious: Change the coordinate system

Schwarzschild solution in standard coordinates

$$ds^2 = \left(1 - \frac{r_s}{r}\right) dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

Coordinate transformation

$$r = r' \left(1 + \frac{r_s}{4r'}\right)^2$$

Schwarzschild solution in isotropic coordinates

$$ds^2 = \left(\frac{1 - \frac{r_s}{4r'}}{1 + \frac{r_s}{4r'}}\right)^2 dt^2 + \left(1 + \frac{r_s}{4r'}\right)^4 (dr'^2 + r'^2 d\theta^2 + r'^2 \sin^2 \theta d\varphi^2)$$

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To make the geometry more obvious Weyl suggested a change of coordinate system
→ to isotropic/conform-euclidean coordinates

The transformation is this, where
r is the radius in standard coordinates
r' (r-prime) is the radius in the new, isotropic coordinate system

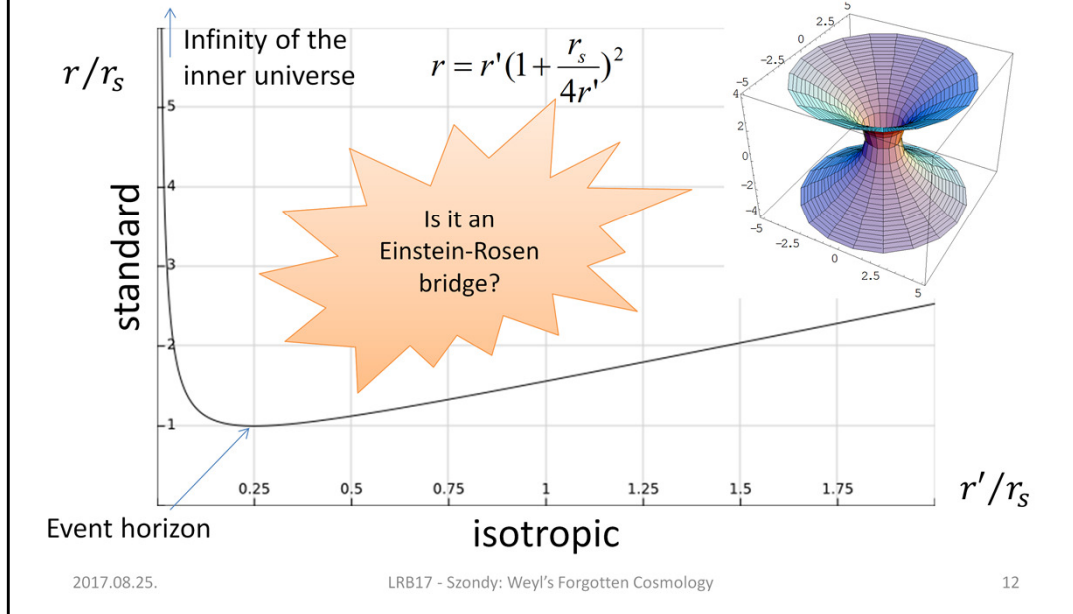
The metric tensor of Schwarzschild solution transformed to isotropic coordinates is this. Isotropic means that all the spatial coordinates have the same coefficient in the metric tensor.

This transformation can also be found in textbooks, like Landau, the transformed is also available there.

If we have a closer look at the coordinate transformation we will find interesting things:
→

The coordinate transformation

To understand Schwarzschild metric in isotropic coordinates



The diagram shows the transformation of the radial coordinate:

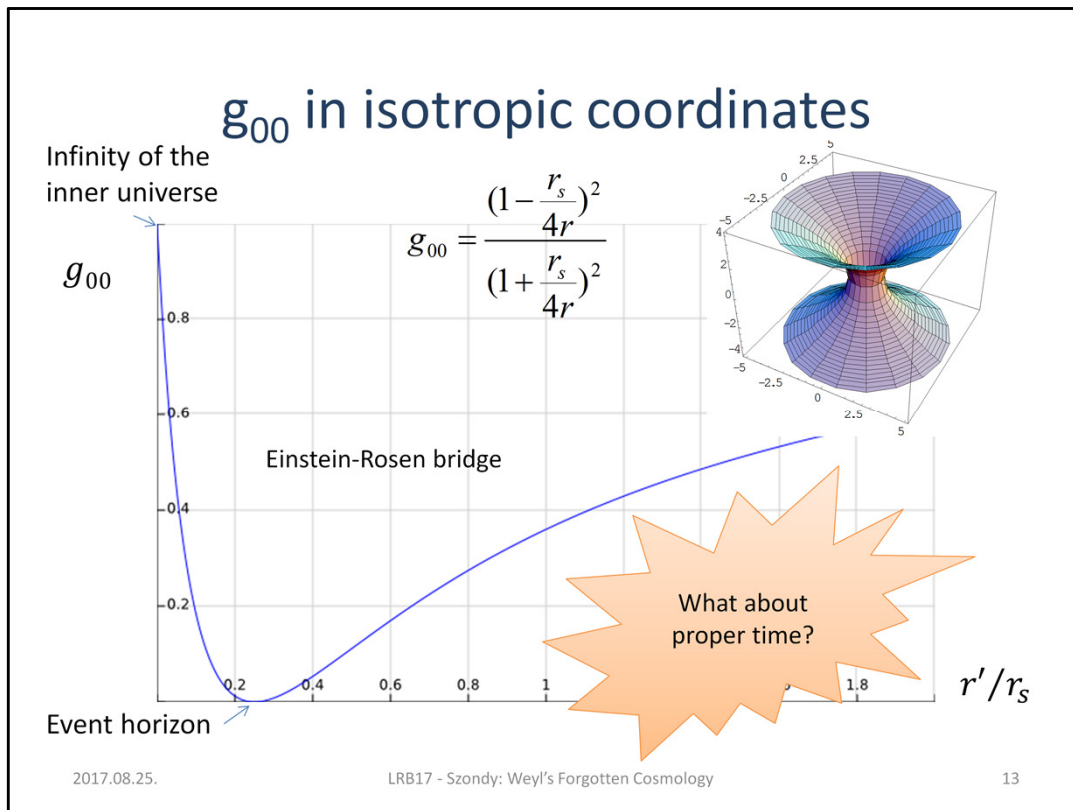
- the horizontal axis is r' , the radius in the new, isotropic coordinates
 - the vertical axis is r the radius in standard coordinates
- Both scales are relative to the Schwarzschild radius r_s

The standard radius (r) has a minimum at $r'=r_s/4$. This is the event horizon.

- $r' > r_s/4$ is the outer part of the solution – same as in standard coordinates
- when $r' < r_s/4$ and r' approaches to 0 the standard radius increases again with no limit
- this is the other funnel – or the second universe
- In isotropic coordinates the this 2nd universe can be considered an „inner universe“

Is the inner universe really an identical universe? So is it really and Einstein-Rosen bridge?

We should also check g_{00} to answer this question.



Here is the diagram of g_{00} in isotropic coordinates – scaled with the Schwarzschild radius again

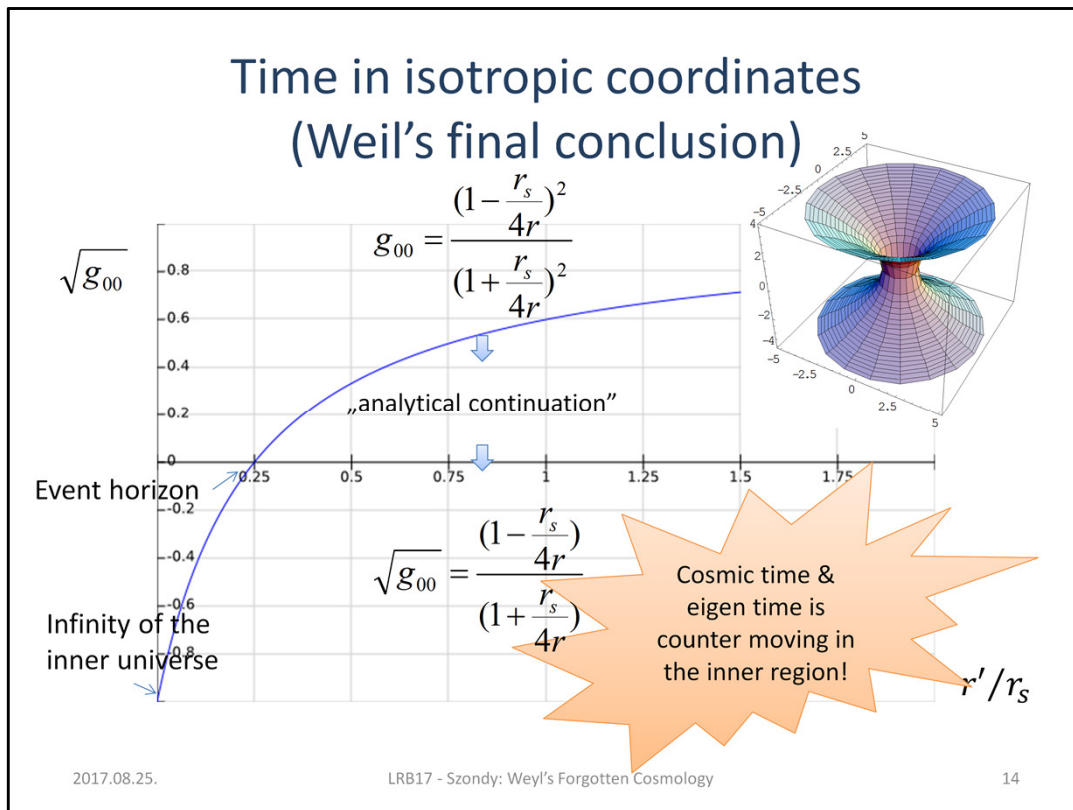
- It has a 0 minimum at $r'=r_s/4$ – at the event horizon
- It's value is 1 at $r'=0 \rightarrow$ same as in the positive infinity

So g_{00} behaves the same in the inner universe as the outer universe

So we can admit, that if we use isotropic coordinates instead of standard, we get an Einstein-Rosen bridge.

Kruskal and Szekeres had similar conclusion using hyperbolic coordinates in 1962 (45 years later)

One more question: what about proper time in isotropic coordinates?



Proper time is characterised via square root of g_{00} ...

- Weyl again tried to use analytical continuation – therefore he just left out the square signs from g_{00}

- with this the coefficient of proper time is

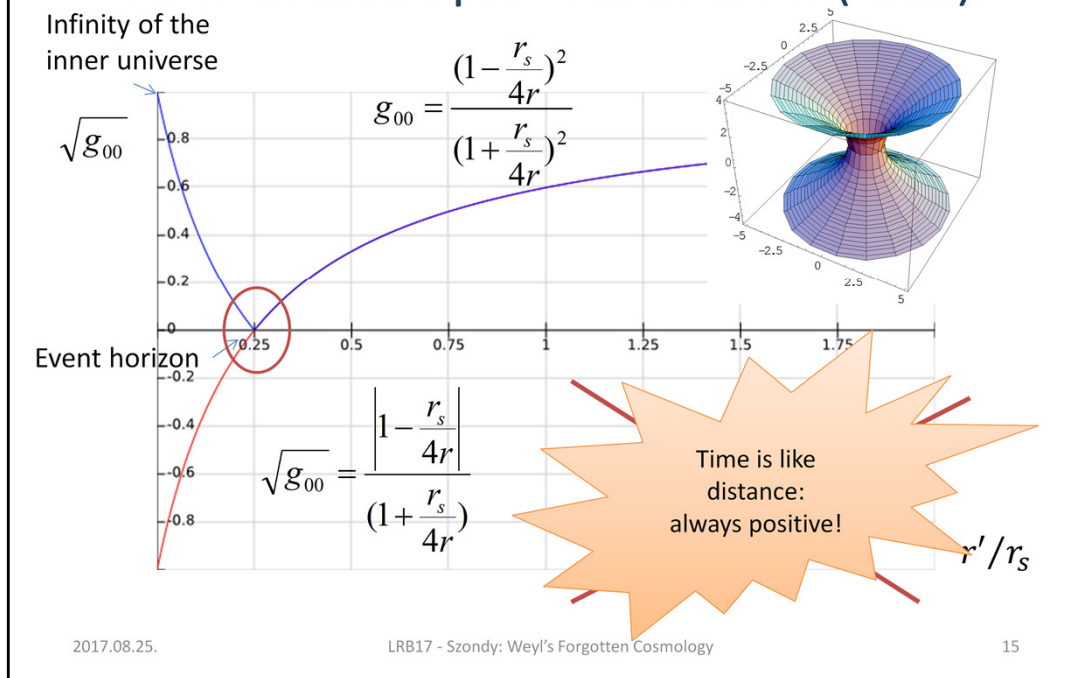
- 0 at the event horizon

- -1 at the inner infinity

- His final conclusion was that cosmic time and proper time flows in opposite direction in the inner universe.

From this point I will add my own comments to the comments of Weyl

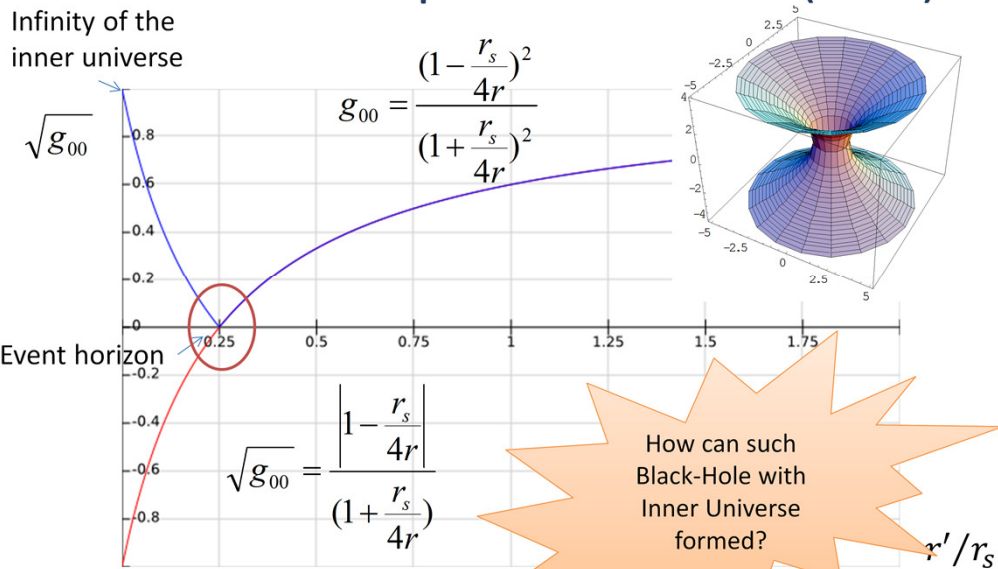
Time in isotropic coordinates (Real)



- My first comment is that squareroot of g_{00} is different:
- In this case it is always positive
 - Flow of time is like distance: it is always positive
 - Someone said yesterday, that „Time is a one-way street”

Of course, there is something strange at the event horizon – but we will get back to it later

Time in isotropic coordinates (Real)



How can such Black-Hole with Inner Universe formed?

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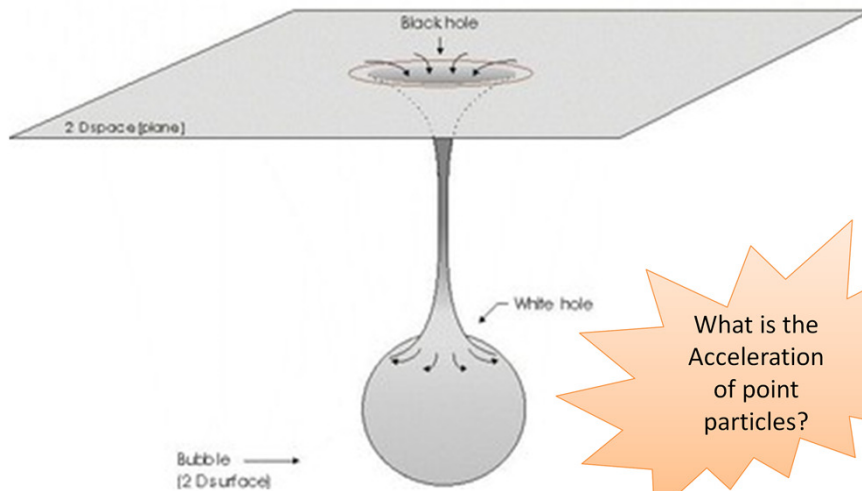
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The second comment is: that in this coordinate system we can think about how this kind of black hole with inner universe is formed. What is the evolution of such universe?

Evolution of Black-Hole (Lee Smolin – Loop Quantum Gravity)

Model in 2 dimensions



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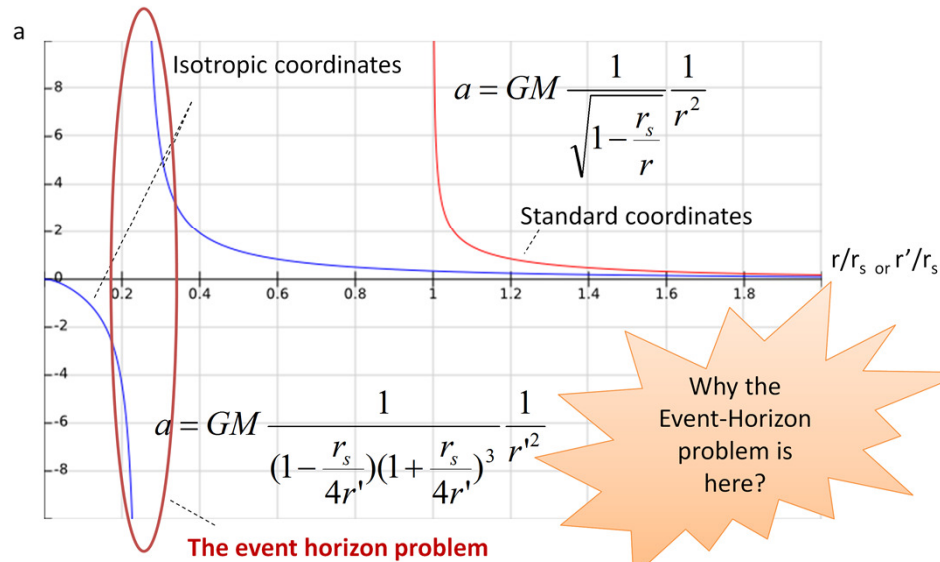
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The best illustration for this scenario is from Lee Smolin...

We can imagine, but can we calculate this evolution?

What is the acceleration of the infalling particles in the inner universe?

Gravitational acceleration



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We can calculate the acceleration from the metric tensor in both standard and isotropic coordinates

- RED line – 3D gravitational acceleration in standard coordinate system.
- BLUE line – the gravitational acceleration in isotropic coordinate system.

It shows, that in the outer universe the acceleration increases till the event horizon – where it becomes infinity.

But the acceleration in the inner universe is opposite

- We can say it is a repulsive force in isotropic coordinates
- If we would like to be more precise: it points towards the event horizon as well

At the event horizon it changes sign/direction – which is a problem in the description – as indicated before

We call it „The event horizon problem” (EHP)

Why this EHP is here?

The event horizon problem

"Einstein equivalence principle,,

- *The outcome of any **local** non-gravitational experiment in a freely falling laboratory is independent of the velocity of the laboratory and its location in spacetime.*
- *„Here "local" has a very special meaning: [...] it must [...] be **small** compared to **variations** in the **gravitational** field”*
- *EEP can not be applied near the event-horizon!*

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The reason can be found in the foundation of GR, the EEP

Free falling laboratory is locally an inertial system – gravitation force can be inhibited via an accelerating coordinate system (frame)

... but only if it is small compared to the variations in the gravitational field.

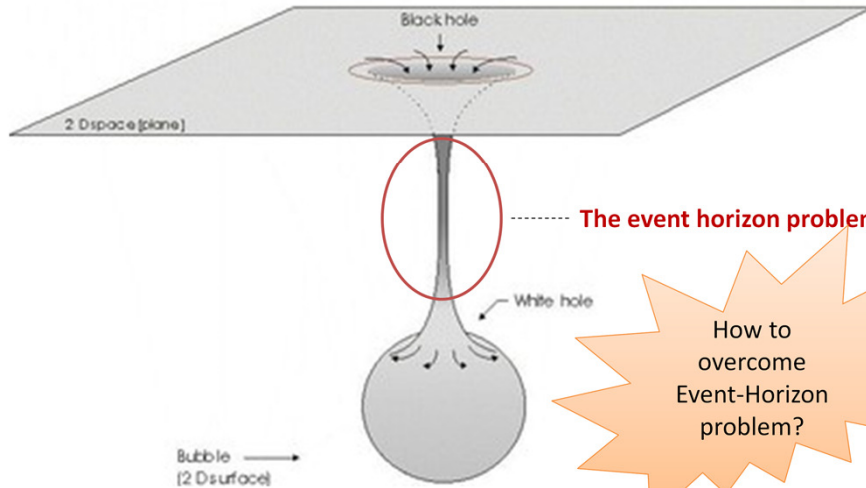
It states that GR can be used locally as an approximation...

But this assumption/condition is not satisfied at the event horizon

So GR can not be used there. :(not valid

The event horizon problem

Model in 2 dimensions



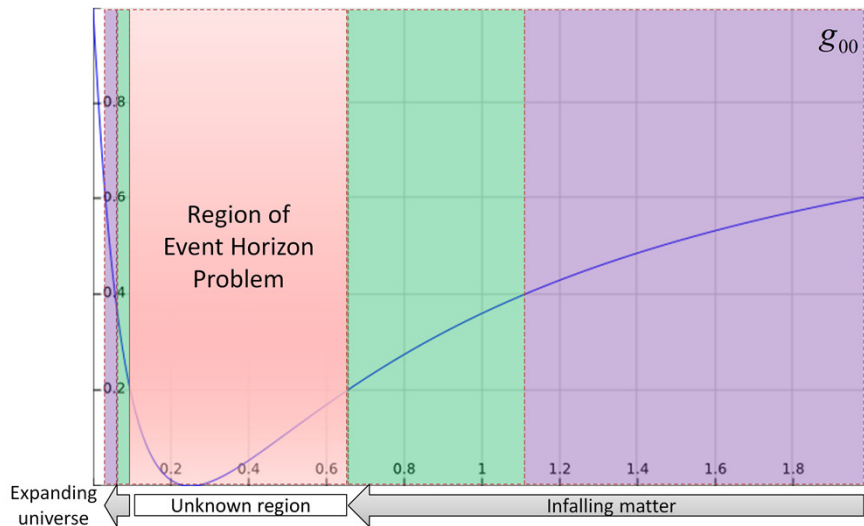
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Still – How?

Evolution of Black-Hole & Inner Universe



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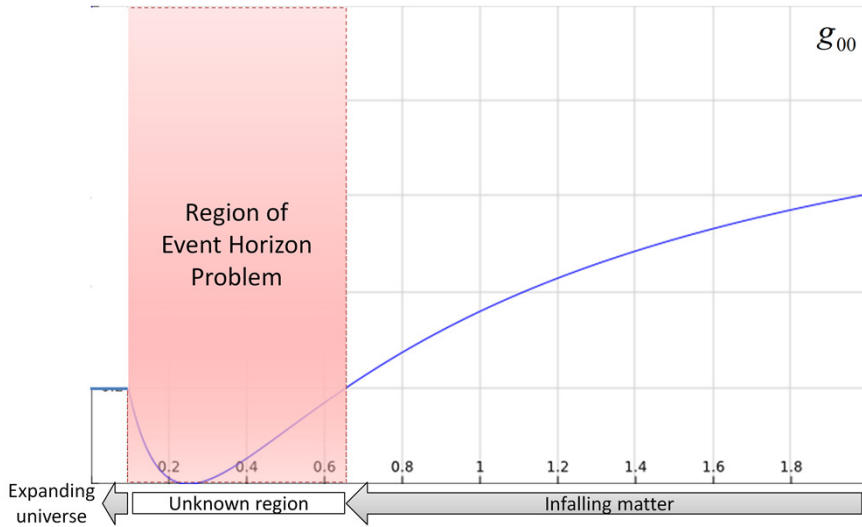
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Near the event horizon there is a region, where GR can not be applied. We have no clue what happens here, but assume that infalling material will cross this region in a finite time.

Fortunately the inner and outer regions are symmetric to each other, therefore the speed of particles will be the same at the mirror-points.

Collapse of a real object, like neutron star would be hard... but

Collapsing sphere



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Collapse of a sphere is much easier.

Inside the sphere the gravitation potential is uniform...

As the r' of the infalling sphere decreases, the apparent r increases...

Conclusions

- There is nothing inside a black-hole ($r < r_s$) – except an inner universe
- GR can not be used near the event-horizon
- Modelling of evolution of universe with well-founded parameters at t_0 is possible
- (Conform-euclidean \rightarrow coordinates superposition & possible analytic solution of 2-bodies problem)

Conclusions II. (phylosophy)

- There are lot of forgotten, important knowledge about GR
- We should understand the physical meaning of GR
- Might have been a better choice to explain the physical meaning of GR ...
- Perhaps next time?