# Goal-directed proofs and diagrams suitable for applications in philosophy of science


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*Thus, we can start applying goal-directed proof theory to philosophy of science.*

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**Result:** the consequence operation may be non-Tarskian, where Tarskian consequnence operation $\vdash$ is:

- reflexive ($A \vdash A$),
- transitive (*if* $\Gamma \vdash B$ and $\Delta \cup \{B\} \vdash A$, then $\Gamma \cup \Delta \vdash A$),
- monotonic (if $\Gamma \vdash A$, then $\Gamma \cup \Delta \vdash A$).

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- if the question $?A$ cannot be answered directly, obtain partial questions and proceed to answer $?A$ indirectly.

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This new format aimed at making the search patch (heuristics) formally explicit.

# Our project: motivation

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We focus on providing a reliable tool for making relevant inferences which can be used with ease and also giving the reasoner an enhanced understanding of the inference and the relation between the precedent and the consequent. This in turn leads to improving the strategies of our everyday reasoning.

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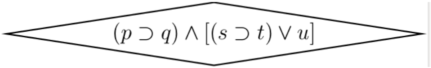

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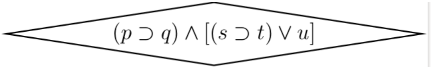


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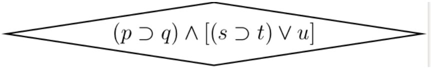


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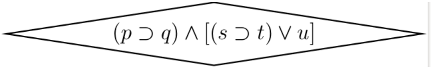


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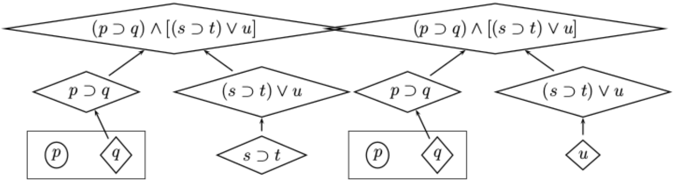

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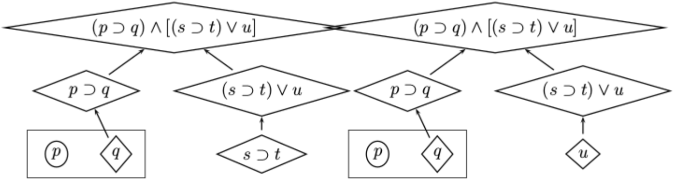


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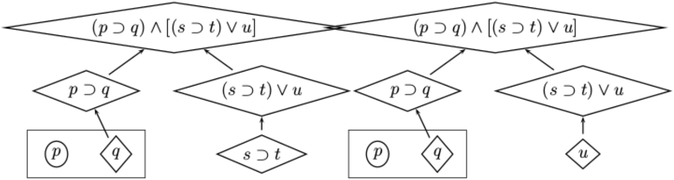


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We introduce premise $((r \supset \neg p) \wedge t)$, which grounds the rightmost part of the diagram. This part is now completed:

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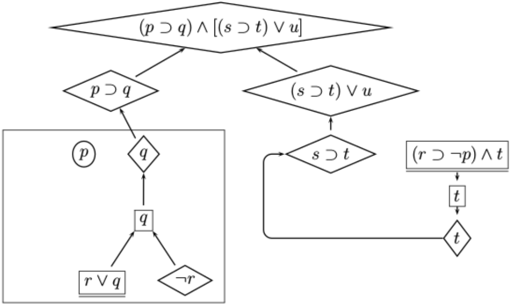

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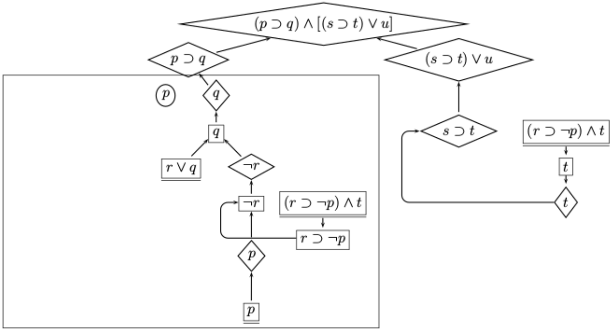

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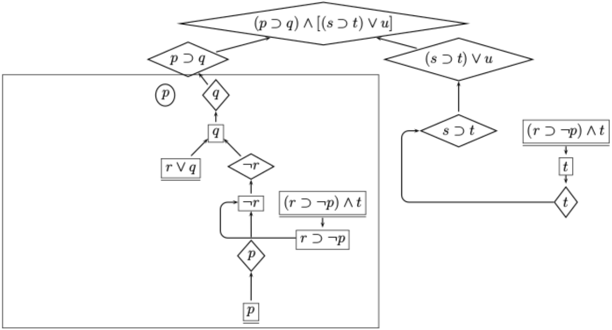


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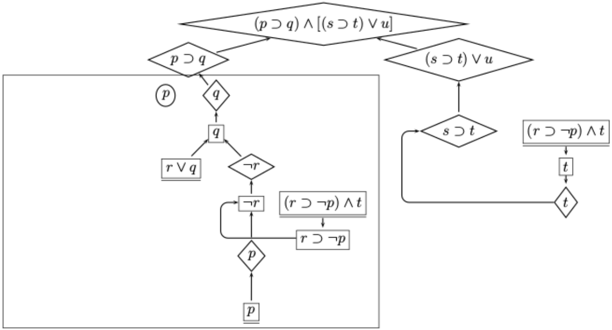


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Note that we handled the two implications differently: one was grounded via a conditional proof, one via the grounding of the consequent.

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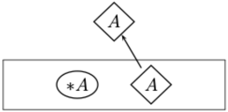
**HYP** allows us to write down a formula in a circular node; in doing so a new conditional proof is commenced in which the hypothesis may be used as an extra premise. The boundaries of the conditional proof are depicted by a box.

# Reductio ad absurdum



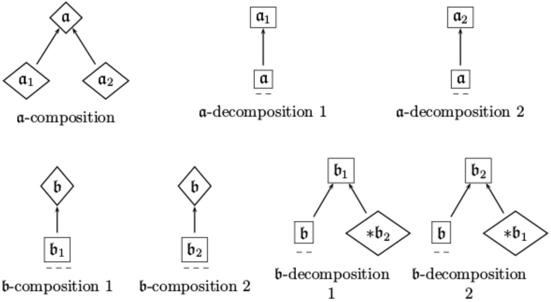

This allows us to ground any formula in a hypothesis consisting of its negation and itself. Given that the hypothesis functions as an extra premise inside the box, this means we can ground the formula if we can derive it from its negation.

# $\mathfrak{a}$ and $\mathfrak{b}$ rules

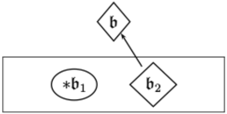
We use the following table to abbreviate formulas with the same meaning:

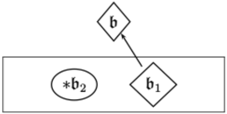
| $\mathfrak{a}$ | $\mathfrak{a}_1$ | $\mathfrak{a}_2$ | $\mathfrak{b}$ | $\mathfrak{b}_1$ | $\mathfrak{b}_2$ |
|---|---|---|---|---|---|
| $A \wedge B$ | $A$ | $B$ | $\neg(A \wedge B)$ | $*A$ | $*B$ |
| $\neg(A \vee B)$ | $*A$ | $*B$ | $A \vee B$ | $A$ | $B$ |
| $A \equiv B$ | $A \supset B$ | $B \supset A$ | $\neg(A \equiv B)$ | $\neg(A \supset B)$ | $\neg(B \supset A)$ |
| $\neg(A \supset B)$ | $A$ | $*B$ | $A \supset B$ | $*A$ | $B$ |

We use $*\mathfrak{b}$ to denote the complement of $\mathfrak{b}$. We can define the rules as follows:



# Conditional proof



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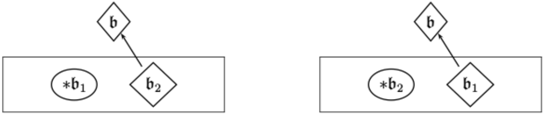
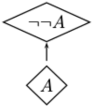
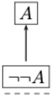



These rules allow us to ground a $\mathfrak{b}$-formula, such as an implication, in a hypothesis (the implicans) and the implicandum. The hypothesis functions as an extra premise in the entire box. Nodes that are part of the tree with the implicandum as root node also fall inside the box.

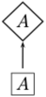
# Remaining rules



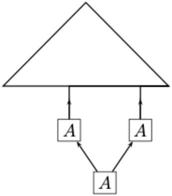

double negation 1




double negation 2

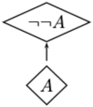



box to diamond




Contraction

# Remaining rules

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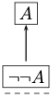
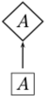



double negation 2

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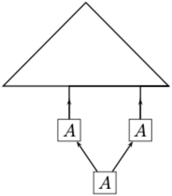


Contraction

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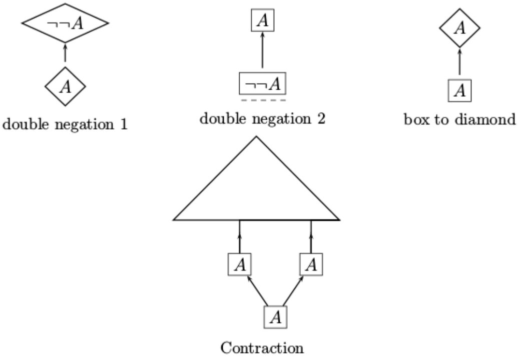


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- The contraction rule is expressed in a different way than the other rules. The triangle expresses that there is diagrammatic proof to which these nodes belong.

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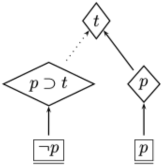

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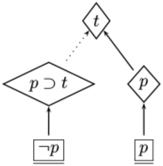


Why? $\mathfrak{b}$-decomposition 2 cannot be applied to a diamond-shaped node here and so $t$ cannot be deduced.

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- Our proof method allows human reasoners to use a specific proving method (without ECQs) without letting go of the Tarskian consequence operation in the *underlying* logic.
- In this way, our approach is closer to the original motivations of Gabbay and Olivetti.

Thank you for your attention!

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