Connection between neutron star observables and the quantum nature of nuclear matter

arXiv:1610.03674

(Submitted to Phys. Rev. C)

[1] G.G. Barnafoldi, A. Jakovac, P. Posfay, Phys. Rev. D 95, 025004

[2] P. Pósfay , G. Barnaföldi, A. Jakovác, PoS(EPS-HEP2015) 369

[3] A. Jakovác, A. Patkós and P. Pósfay, Eur. Phys. J C75:2

[4] G. Barnaföldi, P. Pósfay, A. Jakovác, The proc. of SQM 2016

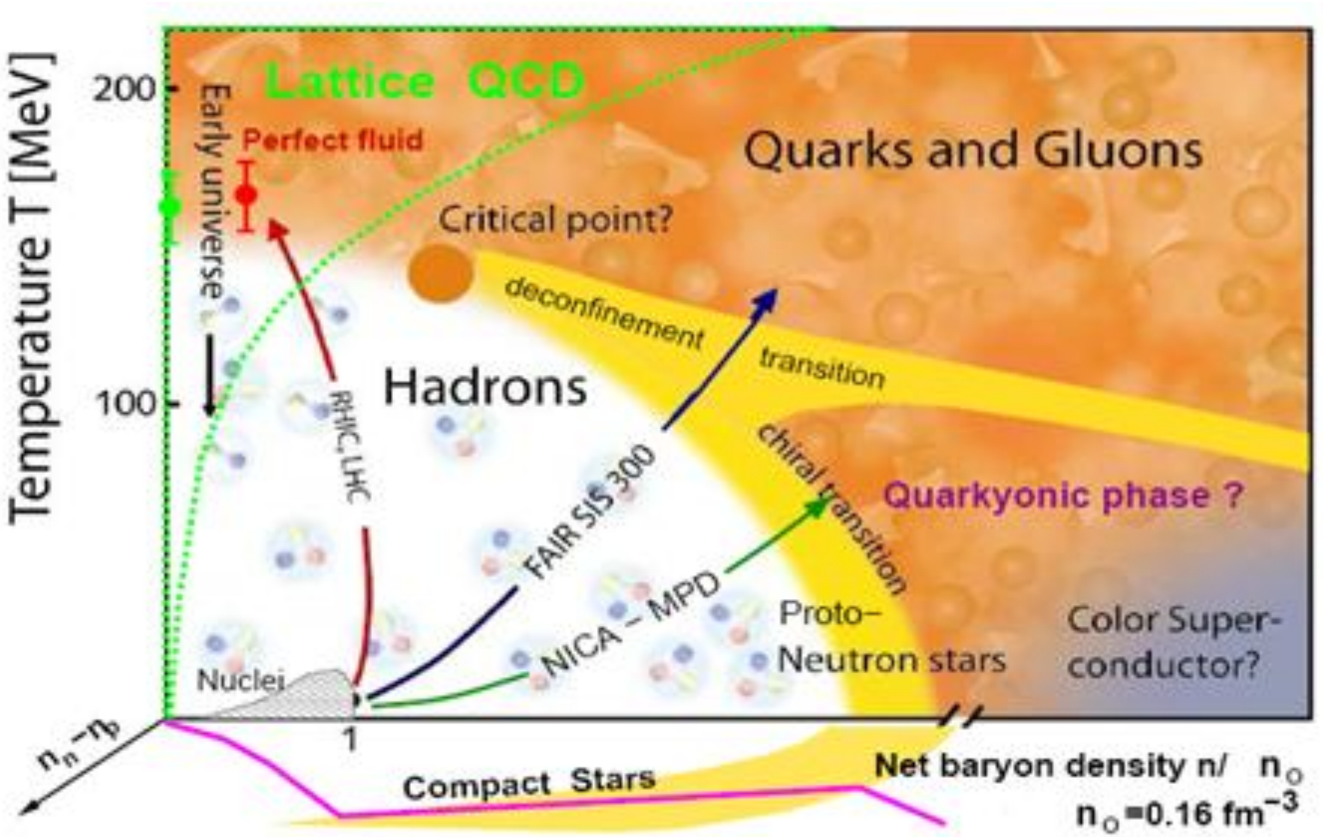
Péter Pósfay

Supervisors : Antal Jakovác, Gergely G. Barnaföldi

Outline

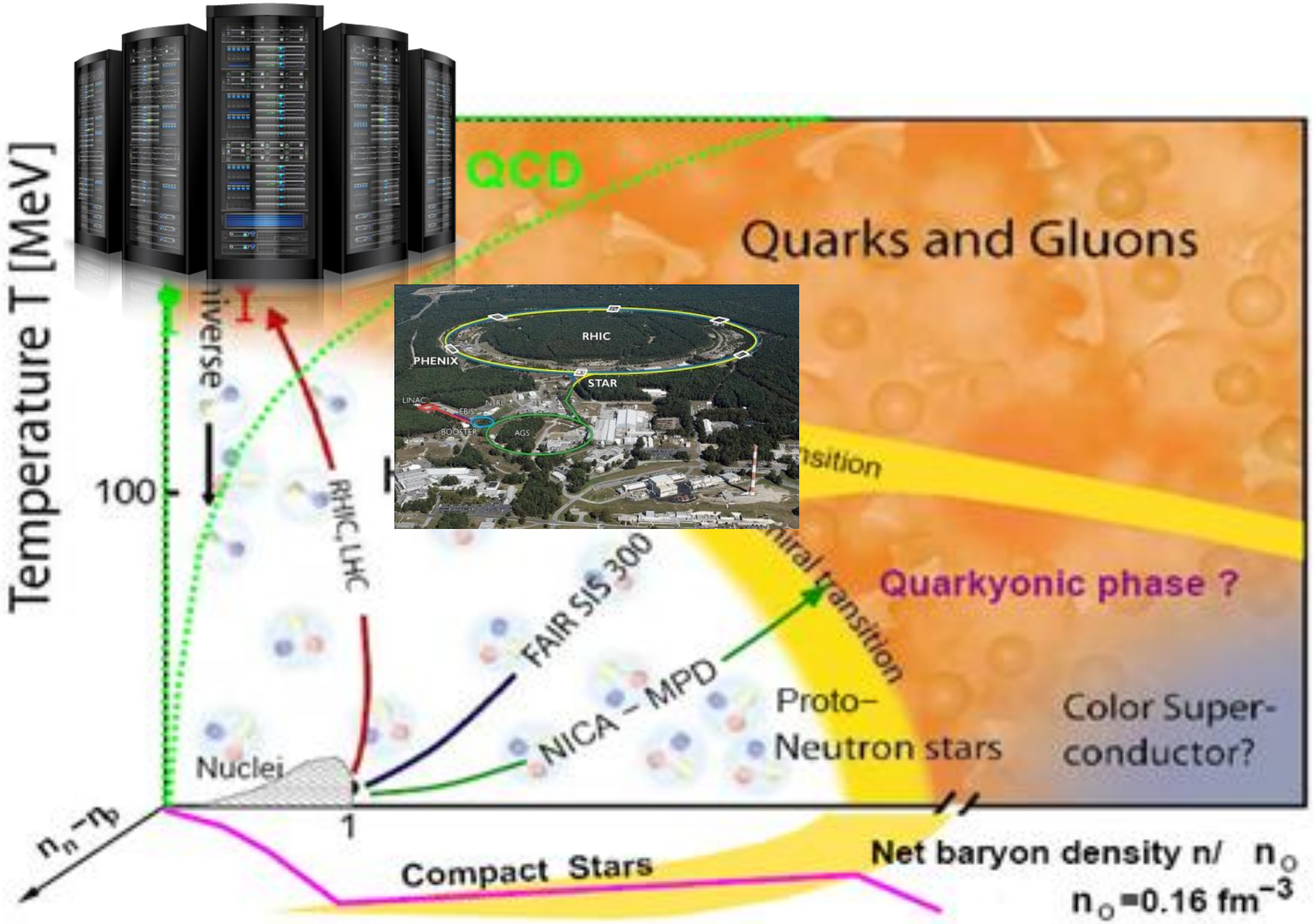
1. Motivation for using FRG in nuclear physics
2. Introduction to FRG
3. Solving the Wetterich equation at finite chemical potential
4. Proof of concept: application for compact stars

Motivation



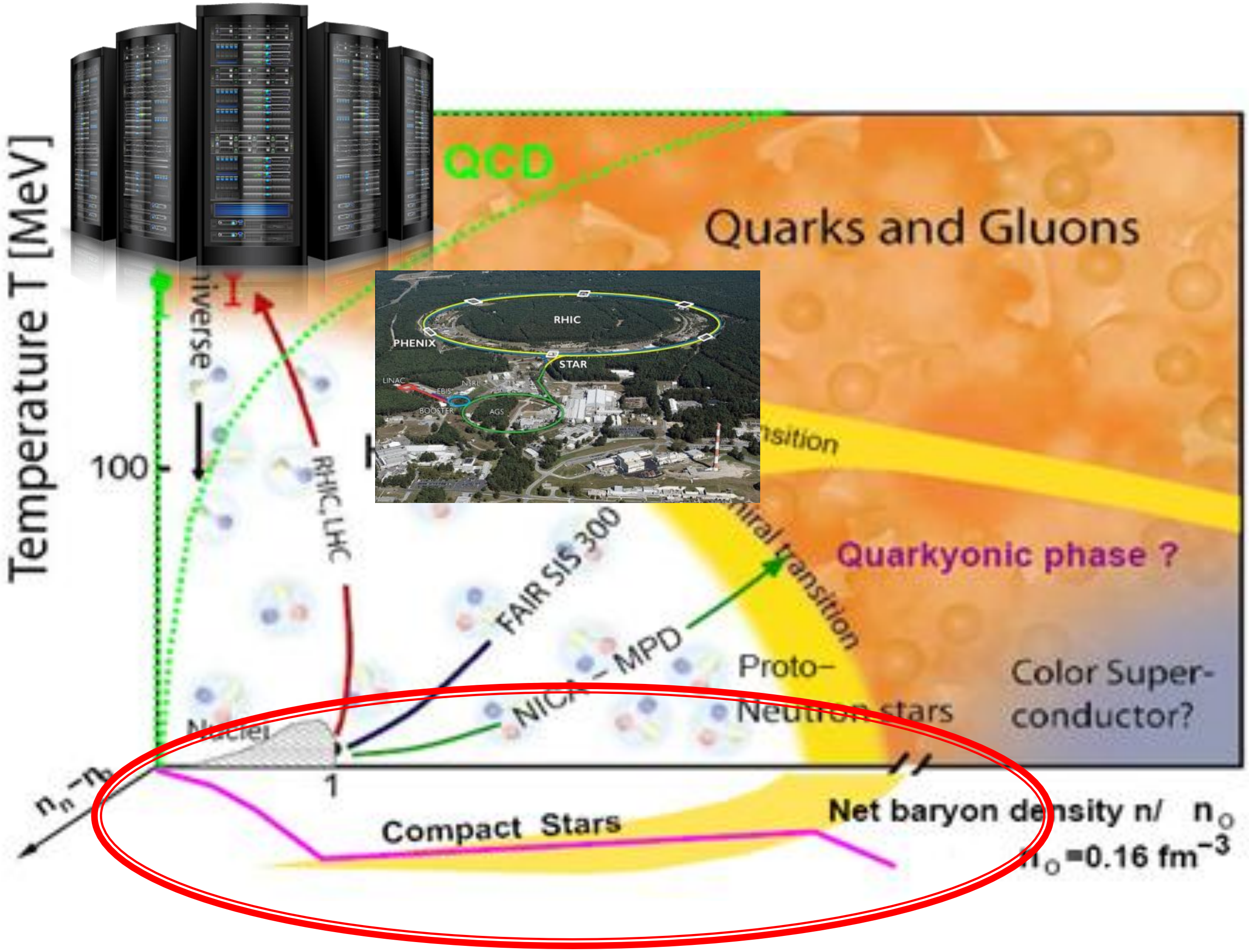
Temperature T [MeV]
200
100
Lattice QCD
Early universe
Perfect fluid
Quarks and Gluons
Critical point?
deconfinement
transition
Hadrons
RHIC,LHC
FAIR SIS 300
chiral transition
Quarkyonic phase ?
NICA – MPD
Proto-
Neutron stars
Color Super-
conductor?
Nuclei
1
n_n-n_p
Compact Stars
Net baryon density n/ n_0
n_0=0.16 fm^{-3}

Motivation



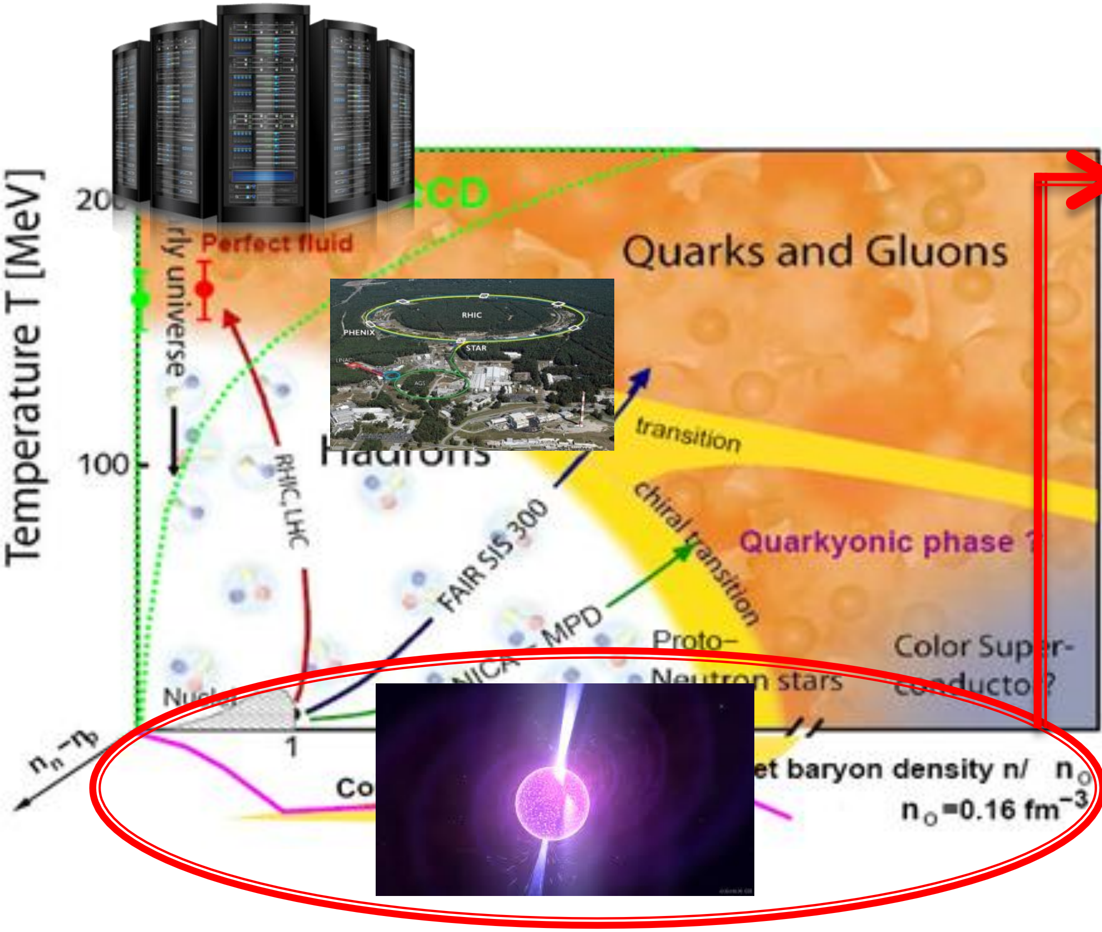
QCD
Quarks and Gluons
Temperature T [MeV]
100
RHIC
PHENIX
STAR
RHIC,LHC
FAIR SIS 300
NICA – MPD
Quarkyonic phase ?
Proto-
Neutron stars
Color Super-
conductor?
Nuclei
1
$n_n - n_p$
Compact Stars
Net baryon density n/ n_o
n_o=0.16 fm^{-3}

Motivation



Temperature T [MeV]
100
QCD
Quarks and Gluons
RHIC
PHENIX
STAR
RHIC,LHC
FAIR SIS 300
NICA - MPD
Quarkyonic phase ?
Proto-
Neutron stars
Color Super-
conductor?
Nuclei
1
Compact Stars
Net baryon density n/ n_0
n_0=0.16 fm^{-3}

Motivation

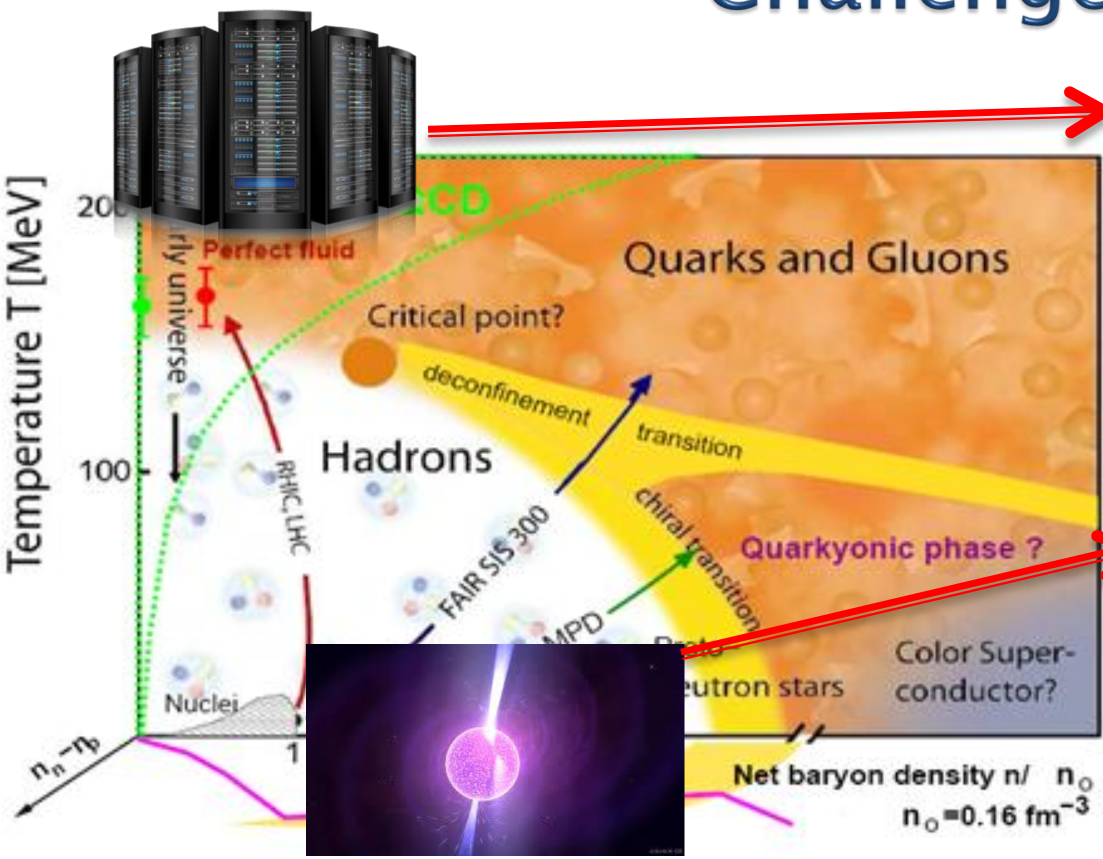


Hard to study area:

- ❑ no numerical results from lattice
- ❑ hard to reach with experiments

- Effective models
- indirect methods
- Astrophysics
- Compact stars

Challenges



This region known:
Interaction, particles, degrees of freedom

Using this knowledge, how can we understand the behavior of matter in a different state?

Analogy: to describe fluid flow, the knowledge of the quantum mechanics of fluid molecules is not needed. This separation makes hard to predict fluid flow based on laws describing the elementary particles of water.

Main questions

- **How can complex behaviour of many particles can be understood and explained from the simple laws gowerning one particle?**
- **Quantum measurement: one particle is quantum mechanical, but in the description of enormous number of particles it is not important.**
- **Why fine tuning effects arise in many physical theories?**
- **How can we reconcile the fact that seemingly different theories work exceptionally well in different regions?**

The framework to address
these is **FRG**

What is FRG ?

- Recently very succesful **renormalization method**
- FRG is used in the formulation of **Wetterich** (1993)

- Main ingredients
  - **Functional** (path integral) methods of QFT
  - Wilsonian renormalization idea: Renormalization Group

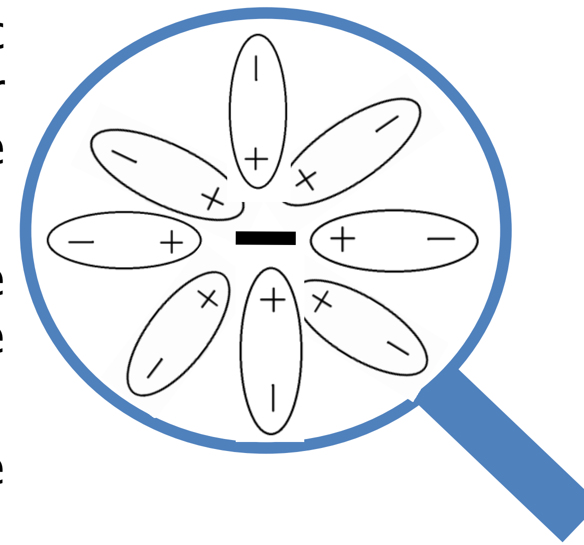
→ **FRG**

- **Interpolates** smoothly between microscopic laws and complicated macroscopic phenomena, by introducing a scale
  - Description of phase transitions, quantum measurements?

- Why are we interested ?
  - There is a hint that FRG maybe solves the **triviality problem** and the **hierarchy problem (fine tuning)** of the Higgs sector in the SM
  - **Quantum Gravity**: Many calculations show that gravity has an UV fixed point.

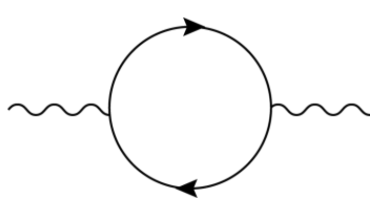
Renormalization for skilled pedestrians

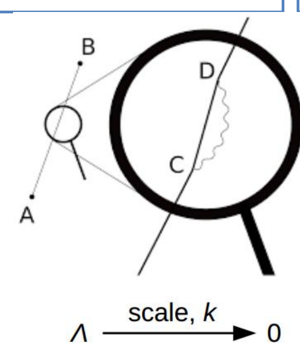
- **Picture**: The point charge polarizes the dielectric material. The point charge appears to be smaller at large distance, because of the screening of the medium.
- **Basic Idea**: Thank to the interaction the measurable (effective) quantities differ from the original (bare) quantites
- **Renormalization:** Taking into account the active medium.
- **Origin of quantum corrections**

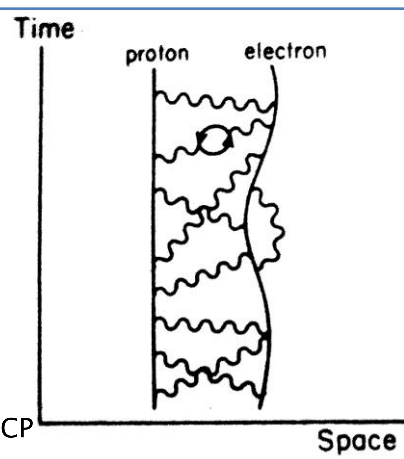


The **uncertainty principle** allows the particle to gain high energy for a short time: it can participate in a high energy process.	Example: The photon creates an e^- e^+ pair which annihilates into a photon. This process changes the **propagator** of the electron.	The interaction between the electron and the photon is a result of lots of small scattering processes.

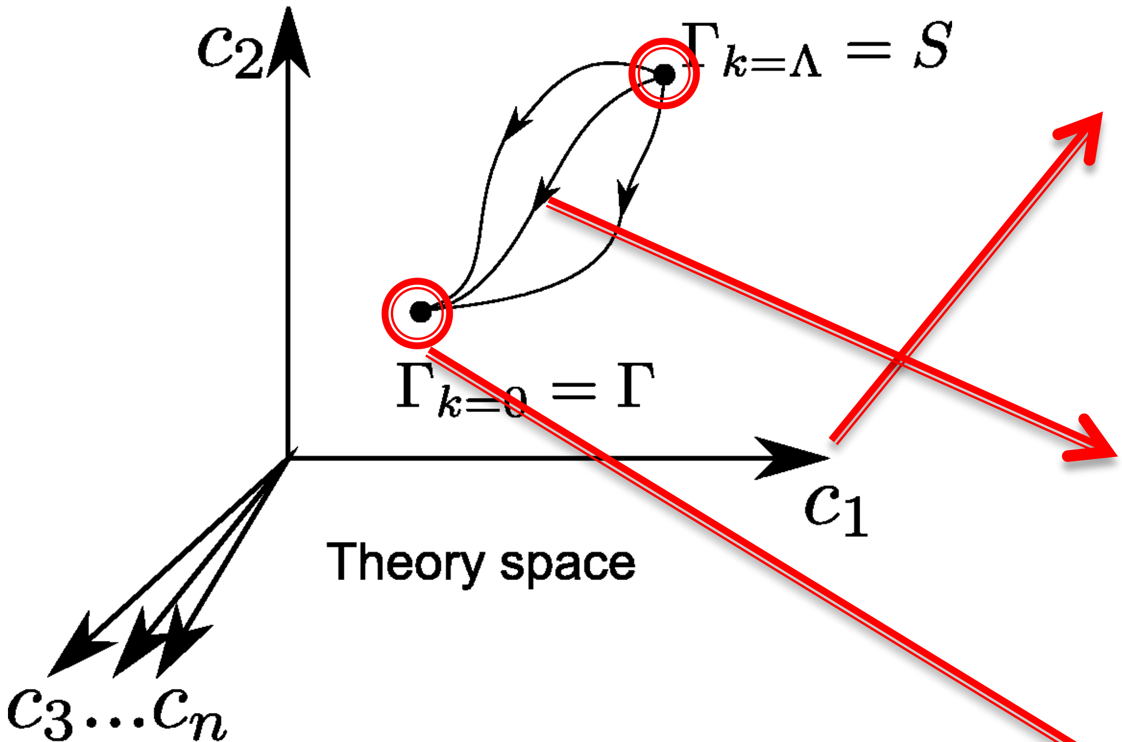
$$\Delta E \, \Delta t \geq \frac{\hbar}{2}$$







Theory space



Operators, relevant physical interactions particles etc…

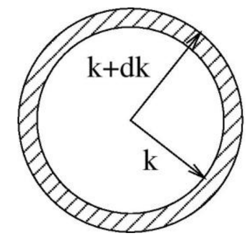
Operators change between fixed points, some become less important some appear. A physical ” theory” is valid until it’s operators are intact.

Fixed Points: Physical theories live near these points. They have given set of operators: physical quantities, particles etc.

Physical theories ”**born**” near fixed points: Near these points the operators can be **experimentally** determined, because they do not change by the scale

The recipe for FRG

- **Generating Functional+ Regulator**
  - The regualtos acts as a **mass term** and suppresses fluctuations below scale *k*
  - gradual momentum integration



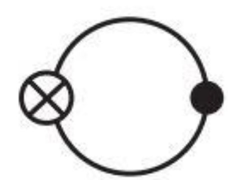
$$Z_k[J] = \int \left(\prod_a d\Psi_a\right) e^{-S[\Psi]-\frac{1}{2}R_{k,ab}\Psi_a\Psi_b+\Psi_a J_a}$$

- The **effective action** is the Legenrdre-transform of the Schwinger functional:

$$\Gamma_k[\psi] = \sup_J (\psi_a J_a - W[J]) - \frac{1}{2}R_{k,ab}\psi_a\psi_b$$

- The scale-dependece of the effective action is given by the **Wetterich-equation:**

$$\partial_k \Gamma_k = \frac{1}{2} Str\left[(\partial_k R_k)\left(\Gamma_k^{(1,1)} + R_k\right)^{-1}\right]$$



The Wetterich equation

- Exact equation for the effective action, but it is very hard to solve directly
  - **Scale dependent effective action (k scale parameter)**

$$\partial_k \Gamma_k = \frac{1}{2} \int dp^D \; STr \left[\frac{\partial_k R_k}{\Gamma_k^{(2)} + R_k} \right]$$

Wetterich equation

k=Λ

Classical action

Integration

k=0

Quantum fluctuations included

The Wetterich equation

- Exact equation for the effective action, but it is very hard to solve directly
  - **Scale dependent effective action (k scale parameter)**

$$\partial_k \Gamma_k = \frac{1}{2} \int dp^D \, STr \left[\frac{\partial_k R_k}{\Gamma_k^{(2)} + R_k} \right]$$

We need an ansatz for the integration

Not necessarly perturbative ansatz!

$$\Gamma_k = \sum_{l=1}^{l=N} \frac{g_l(k)}{l!} \hat{O}_l$$

Scale dependent coupling

The Wetterich equation

- Exact equation for the effective action, but it is very hard to solve directly
  - **Scale dependent effective action (k scale parameter)**

$$\partial_k \Gamma_k = \frac{1}{2} \int dp^D \; STr \left[\frac{\partial_k R_k}{\Gamma_k^{(2)} + R_k} \right]$$

Regulator:
- determines the modes present on scale *k*
- physics is regulator independent

Interacting Fermi-gas model

Ansatz for the effective action:

$$\Gamma_k[\varphi,\psi] = \int d^4x \left[\bar{\psi}\left(i\partial\!\!\!/ - g\varphi\right)\psi + \frac{1}{2}\left(\partial_\mu\varphi\right)^2 - U_k(\varphi) \right]$$

Fermions : *m=0,* **Yukawa-coupling** generates mass

Bosons: the **potential** contains self interaction terms

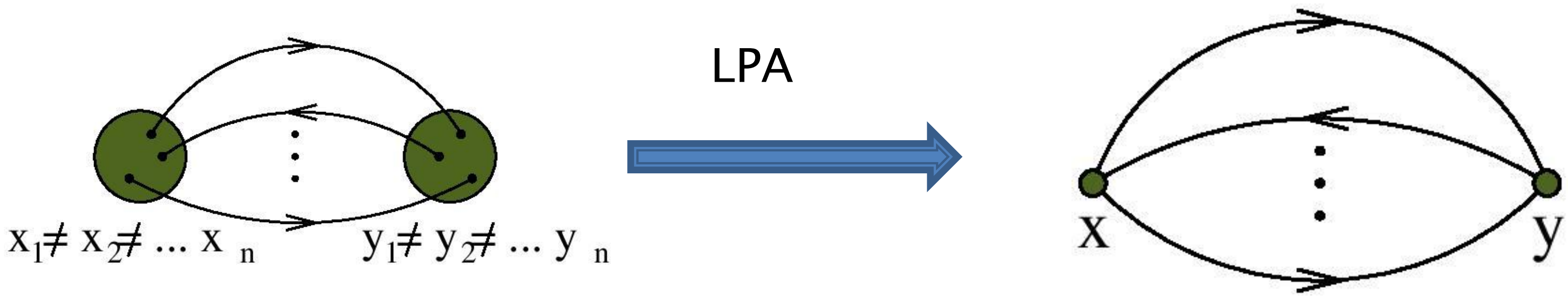
We study the scale dependence of the potential only!!

Local Potential Approximation (LPA)

What does the ansatz exactly mean ?

LPA is based on the assumption that the contribution of these two diagrams are close.

(momentum dependence of the vertices is suppressed)



This implies the following ansatz for the effective action:

$$\Gamma_k[\psi] = \int d^4x \left[\frac{1}{2}\psi_i K_{k,ij}\psi_j + U_k(\psi)\right]$$

Interacting Fermi-gas at finite temperature

Ansatz for the effective action:

$$\Gamma_k[\varphi,\psi] = \int d^4x \left[\bar{\psi}\left(i\partial\!\!\!/ - g\varphi\right)\psi + \frac{1}{2}\left(\partial_\mu\varphi\right)^2 - U_k(\varphi)\right]$$

Wetterich -equation

$$\partial_k U_k = \frac{k^4}{12\pi^2}\left[\underbrace{\frac{1+2n_B(\omega_B)}{\omega_B}}_{\text{Bosonic part}} + 4\underbrace{\frac{-1+n_F(\omega_F-\mu)+n_F(\omega_F+\mu)}{\omega_F}}_{\text{Fermionic part}}\right]$$

Bosonic part

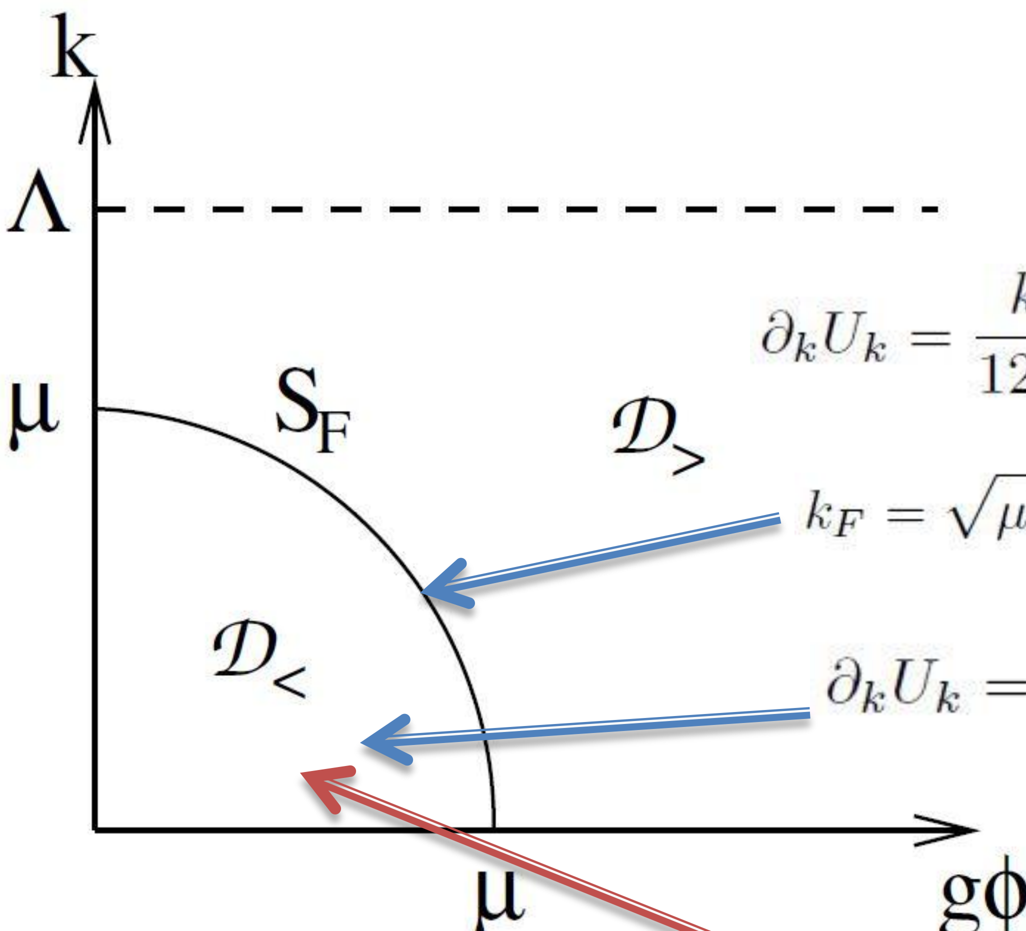
Fermionic part

$$U_\Lambda(\varphi) = \frac{m_0^2}{2}\varphi^2 + \frac{\lambda_0}{24}\varphi^4 \qquad \omega_F^2 = k^2 + g^2\varphi^2 \qquad \omega_B^2 = k^2 + \partial_\varphi^2 U \qquad n_{B/F}(\omega) = \frac{1}{1 \mp e^{-\beta\omega}}$$

Interacting Fermi-gas at zero temperature

T=0 , μ≠0 ⟹ $n_F(\omega) \to \Theta(-\omega)$

We have two equations for the two values of the step function each valid on different domain



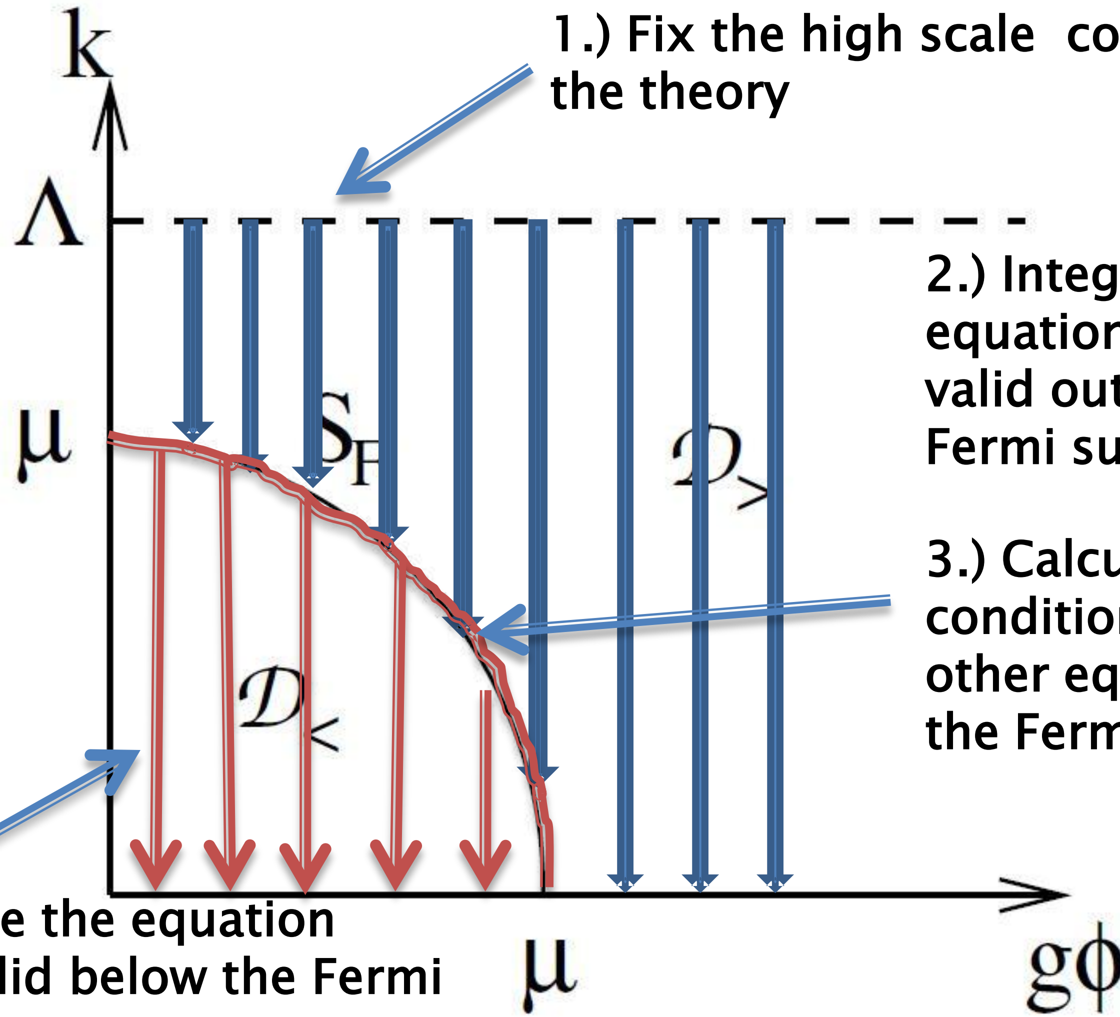
$$\partial_k U_k = \frac{k^4}{12\pi^2}\left[\frac{1}{\omega_B} - \frac{4}{\omega_F}\right]$$

$k_F = \sqrt{\mu^2 - g^2\varphi^2}$; Fermi-surface

$$\partial_k U_k = \frac{k^4}{12\pi^2}\frac{1}{\omega_B}$$

Fermionic vacuum fluctuations and thermodynamic fluctuations cancel

Integration of the Wetterich-equaiton



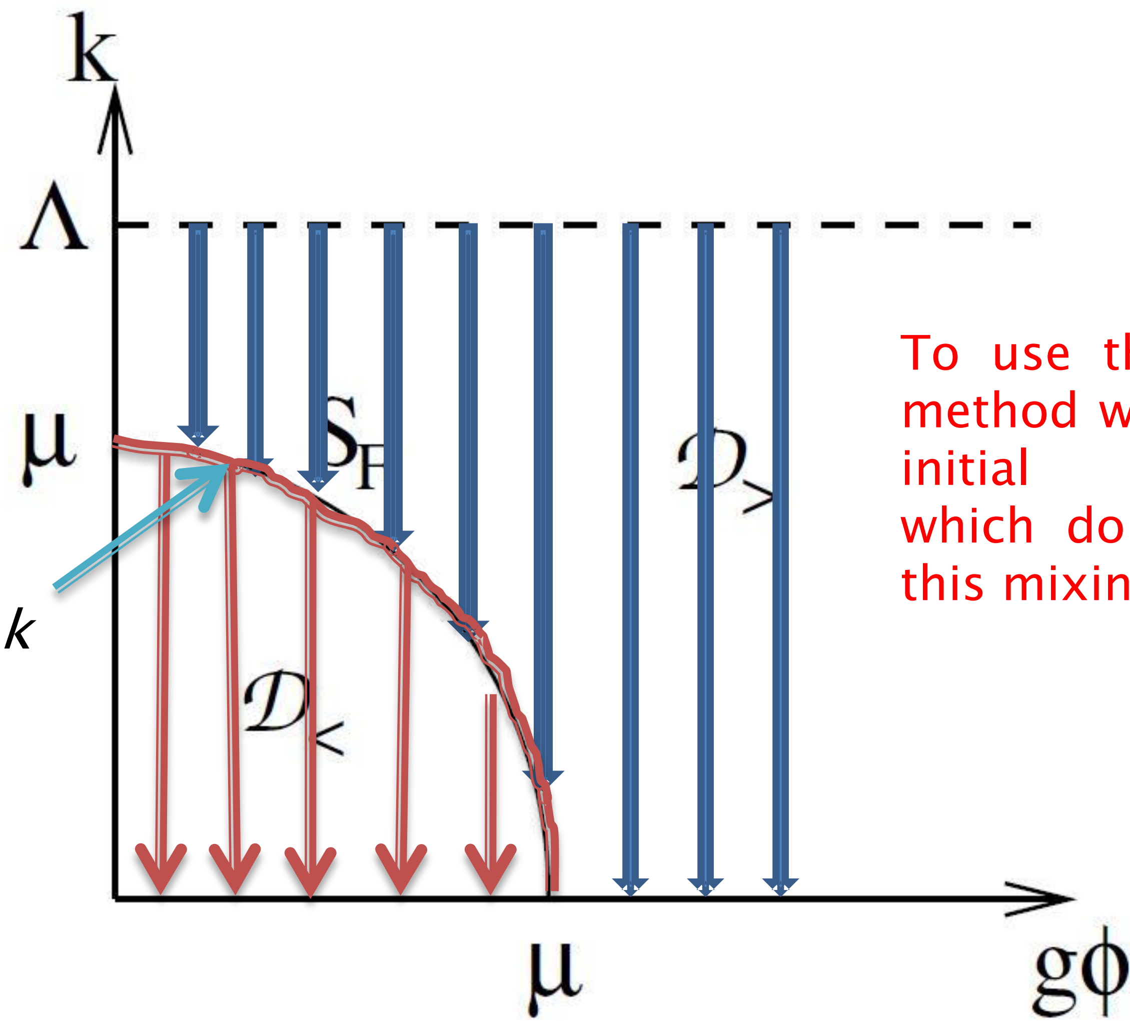
1.) Fix the high scale couplings in the theory

2.) Integrate the equation which is valid outside of the Fermi surface

3.) Calculate the initial conditions for the other equation inside the Fermi surface

4.) Integrate the equation which is valid below the Fermi surface

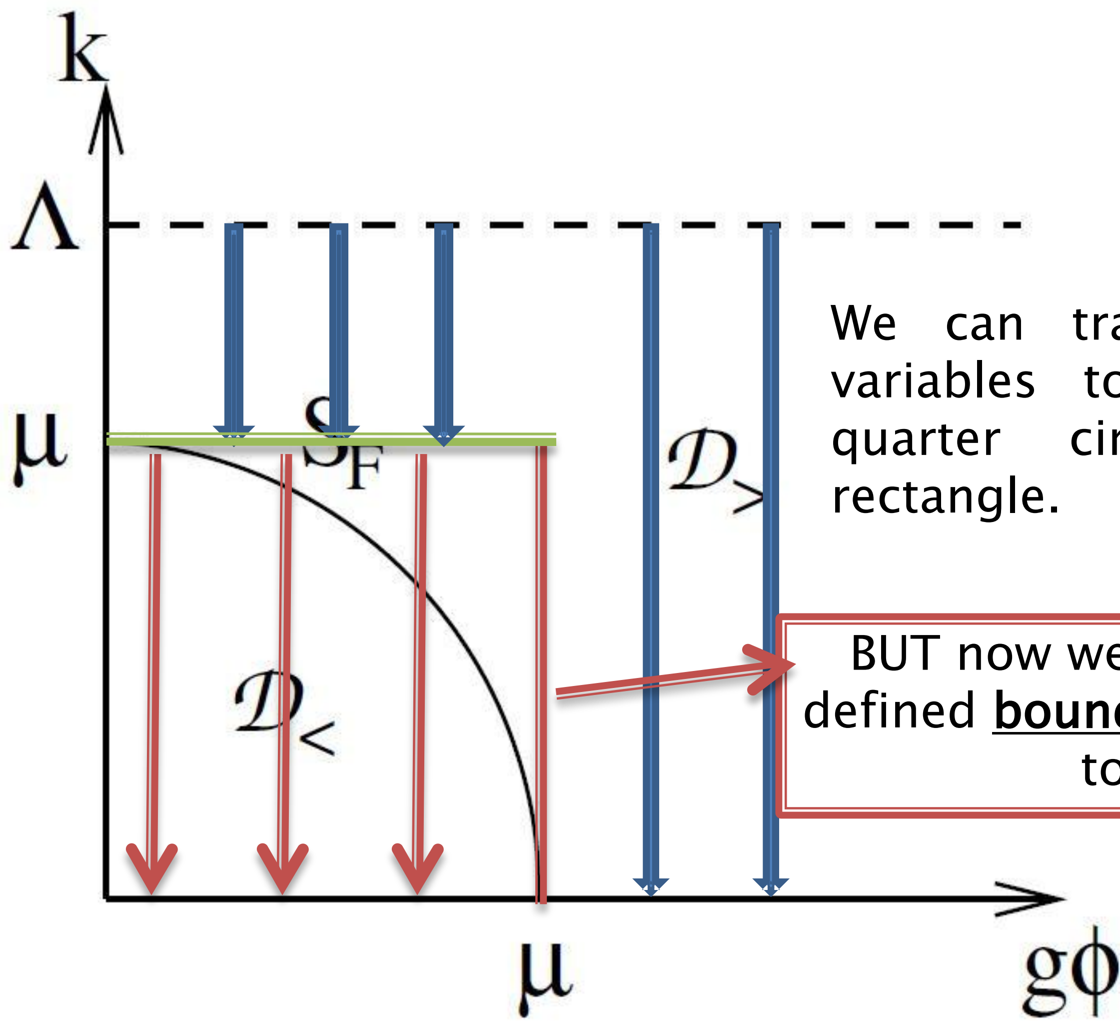
BUT…



The boundary condition mix *k* and *gϕ*

To use the orginal method we need an initial condition which do not have this mixing

Transform the variables



We can transform the variables to make the quarter cirle into a rectangle.

BUT now we have a well defined **boundary condition** too!

Solution by orthogonal system

- Solution is expanded in an **orthogonal basis** to accomodate the strict boundary conditoin in the trasformed area

$$\tilde{u}(x,y) = \sum_{n=0}^{\infty} c_n(x) h_n(y) \qquad h_n(1) = 0 \qquad \int_0^1 dy\, h_n(y) h_m(y) = \delta_{nm}$$

- The **square root** in the Wetterich-equation is also expanded:

$$x c_n'(x) = \int_0^1 dy\, h_n(y) \left[-x V_0' + y \partial_y \tilde{u} - \frac{g^2 (kx)^3}{12\pi^2} \sum_{p=0}^{\infty} \binom{-1/2}{p} \frac{(\partial_y^2 \tilde{u} - M^2)^p}{\omega^{2p+1}} \right]$$

Where: $\omega^2 = (kx)^2 + M^2$

Expanded square root

We use harmonic base

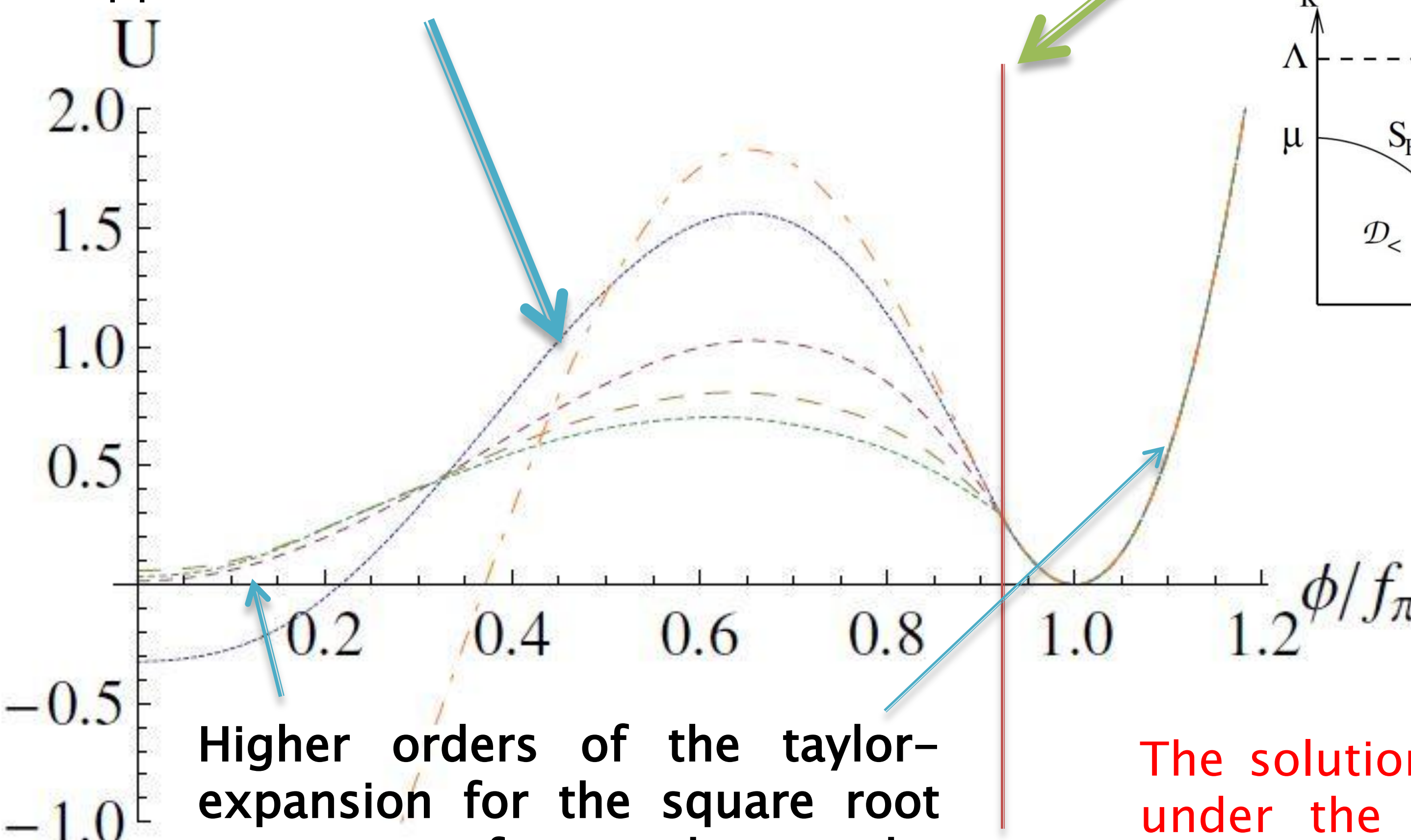
$$h_n(y) = \sqrt{2} \cos q_n y, \qquad q_n = (2n+1)\frac{\pi}{2}$$

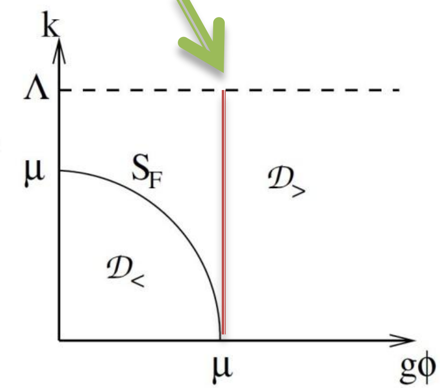
The solution of the Wetterich equation for the interacting Fermi gas

Results-I

Potential in one-loop approximation

Fermi surface in the field variable





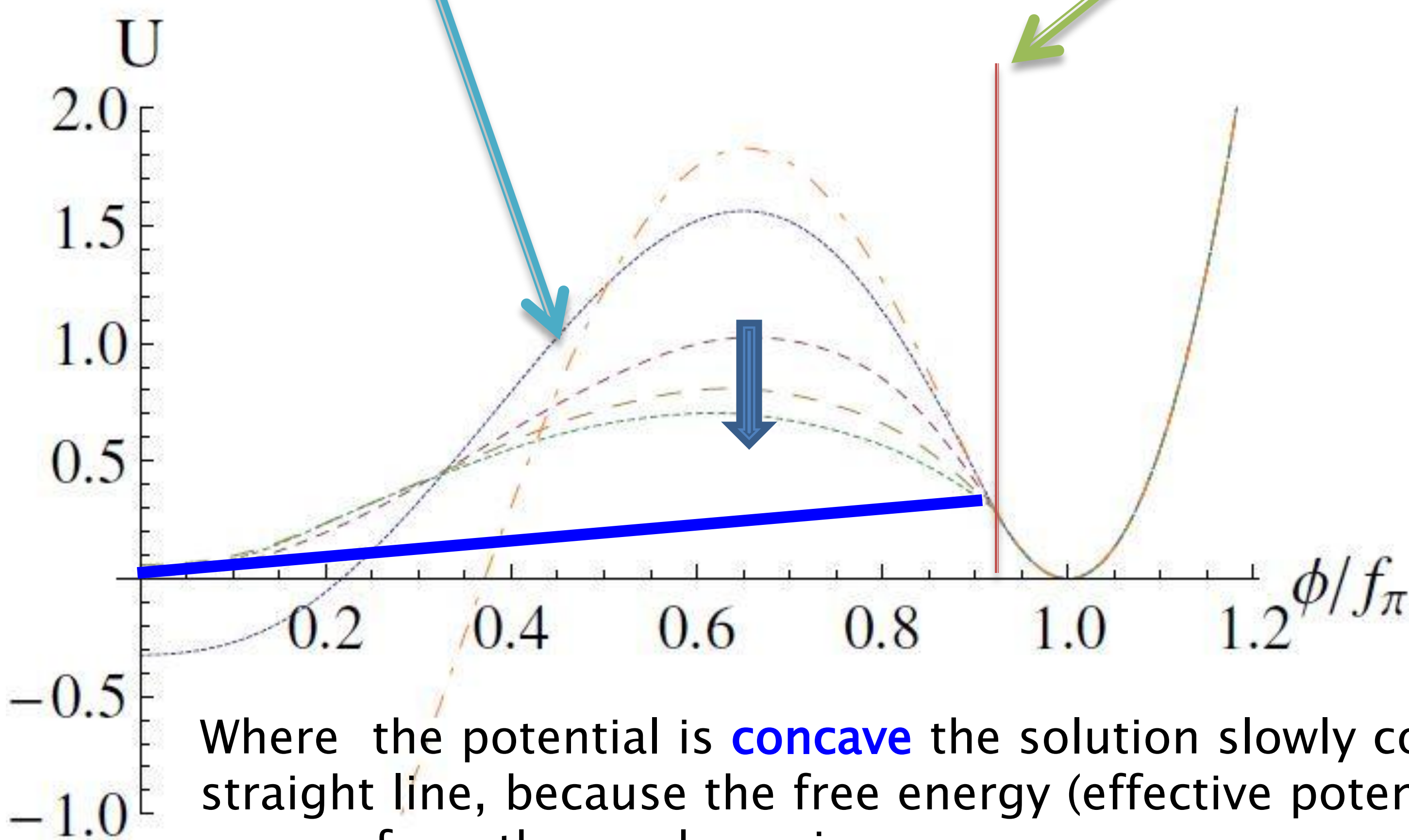
Higher orders of the taylor-expansion for the square root converge fast where the potential is convex

The solution changes only under the Fermi surface, because here we switch to the other equation

Results-I

Potential in one-loop approximation

Fermi-surface in the field variable

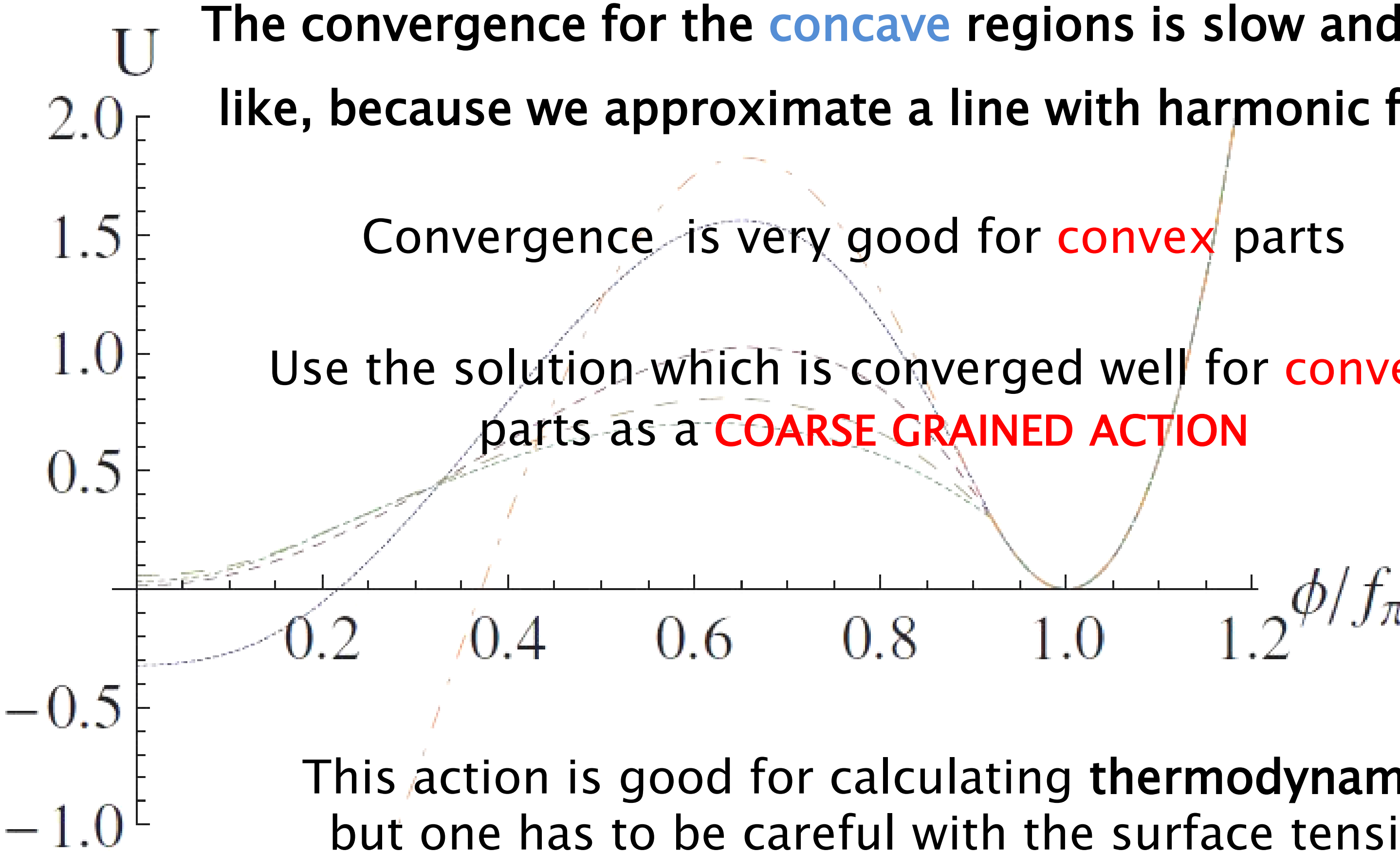


Where the potential is **concave** the solution slowly converges to a straight line, because the free energy (effective potential) must be convex from thermodynamics reasons.

This is the **Maxwell construction.**

Results-I

The convergence for the concave regions is slow and not point like, because we approximate a line with harmonic functions.

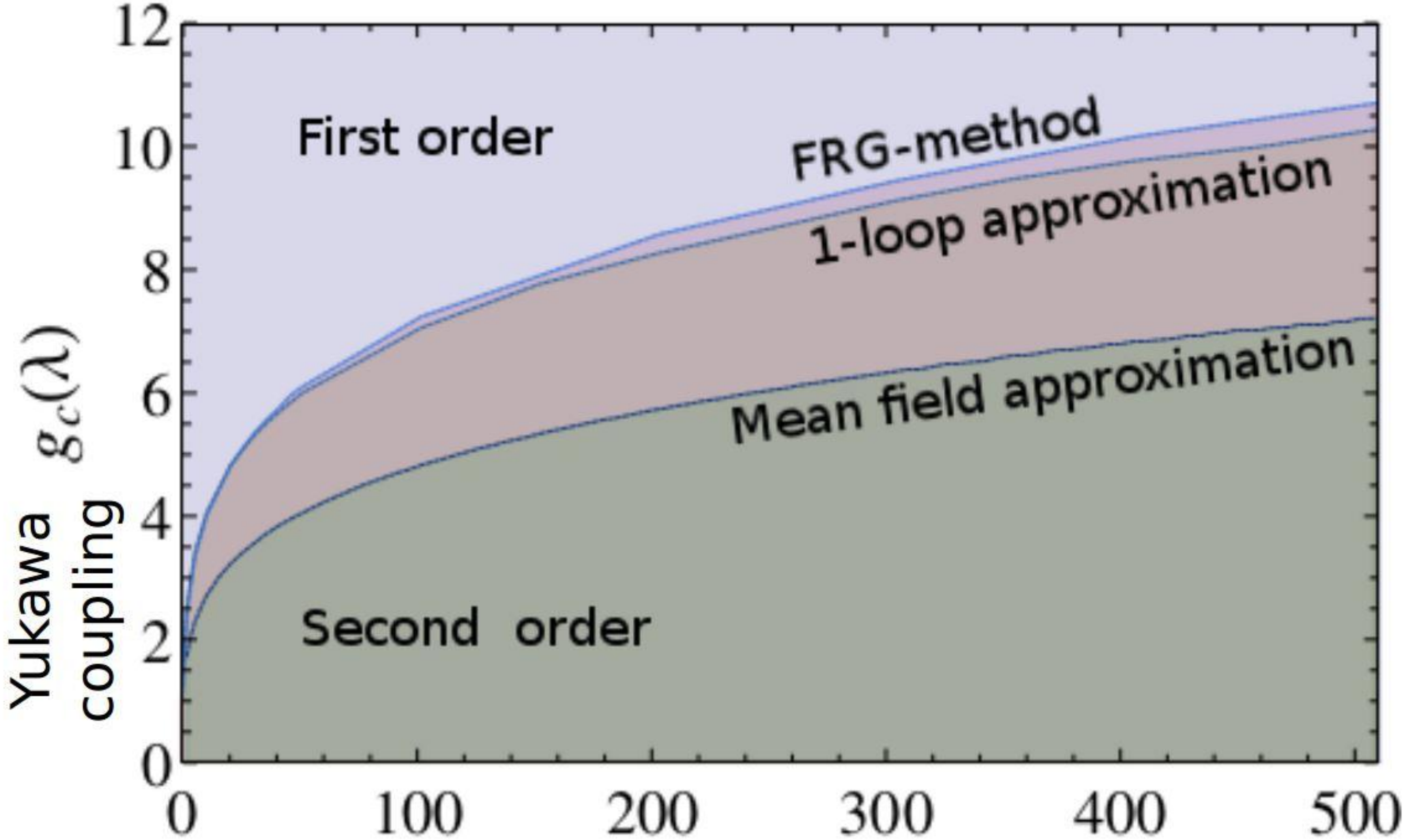


Convergence is very good for convex parts

Use the solution which is converged well for convex parts as a COARSE GRAINED ACTION

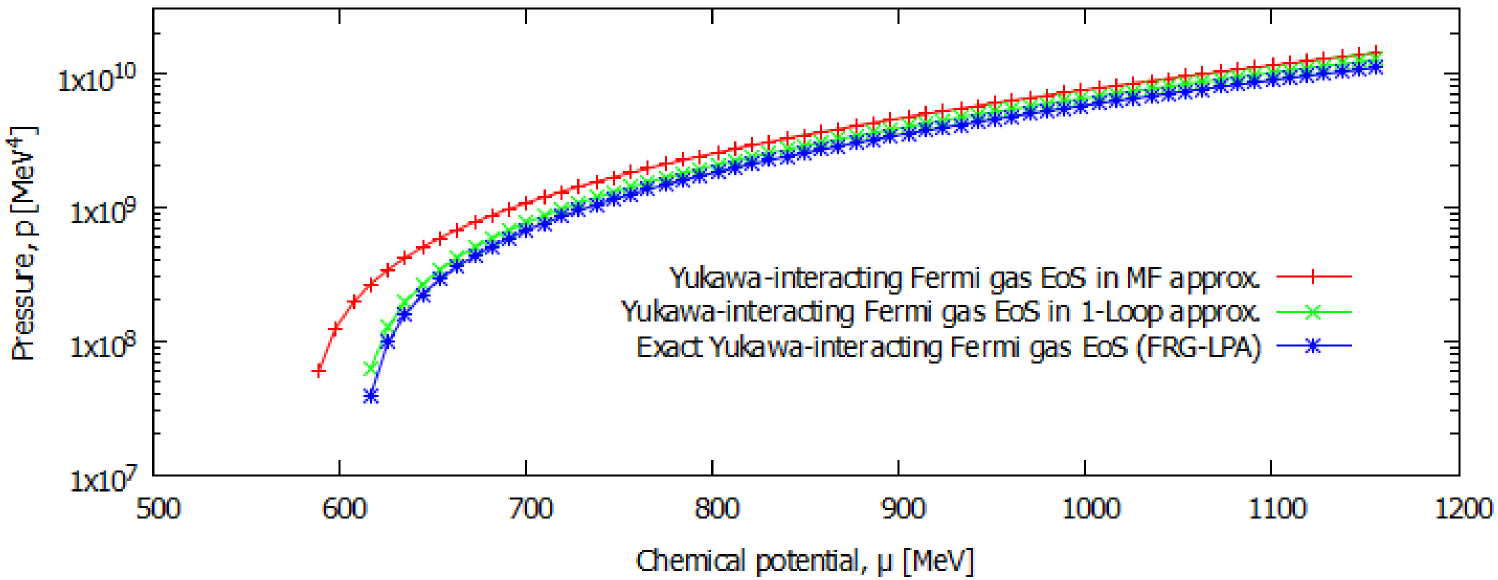
This action is good for calculating **thermodynamics**, but one has to be careful with the surface tension

Results-II



Scalar self interaction

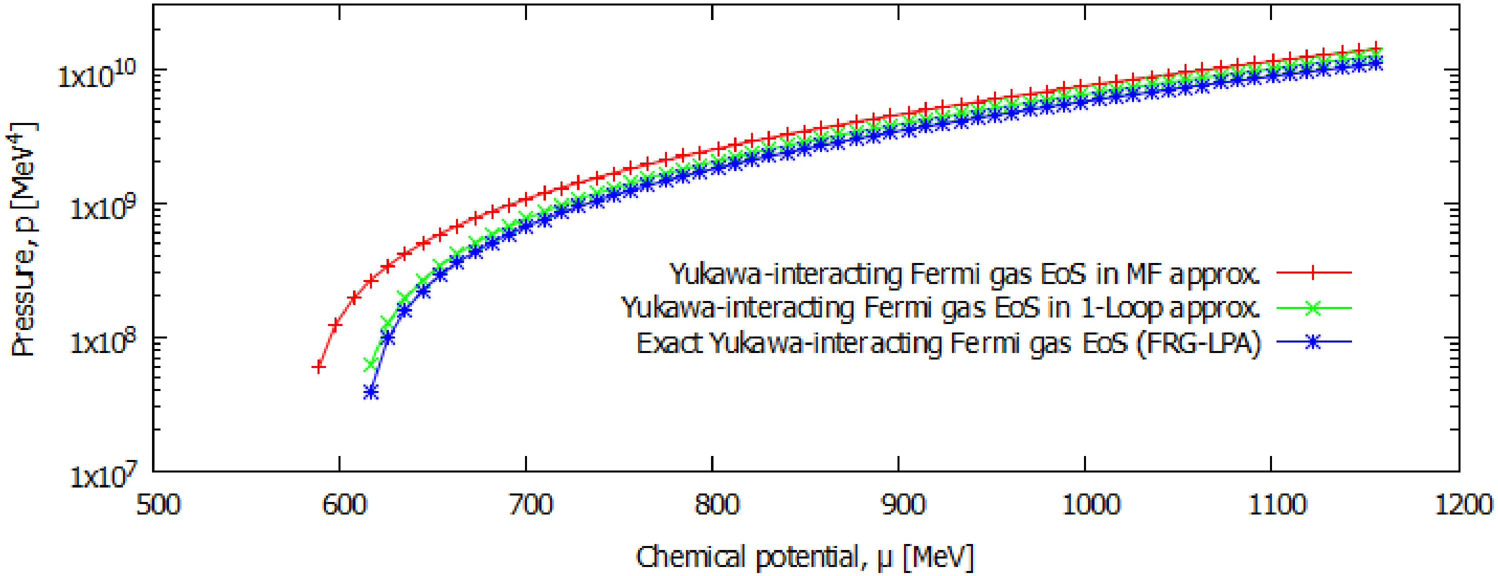
The equation of state

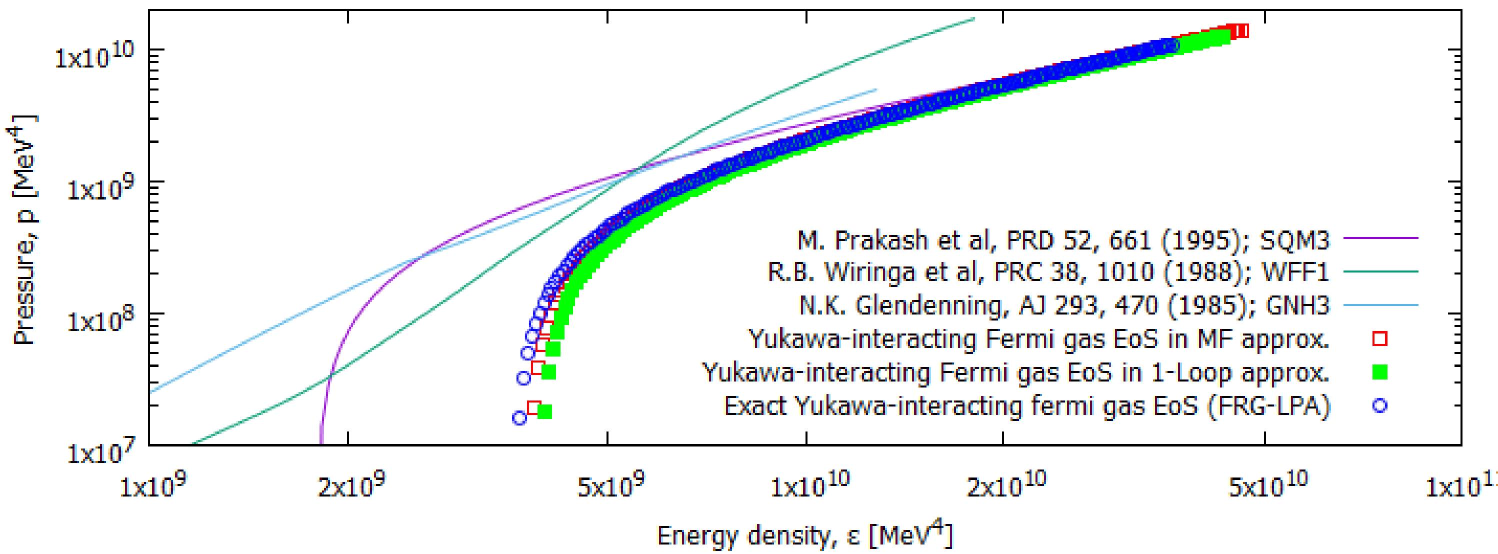


PRESSURE

Mean Field > 1-LOOP > FRG

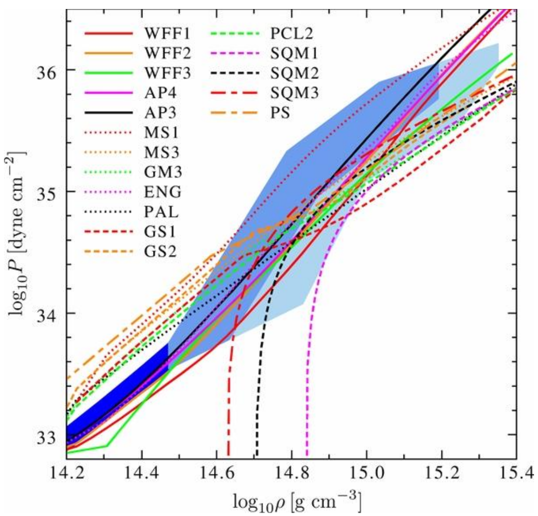
The equation of state



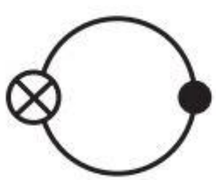


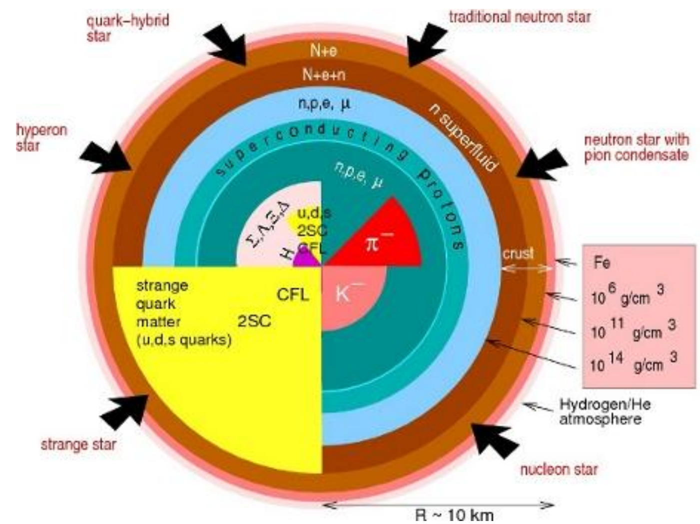
From EoS to Compact Stars

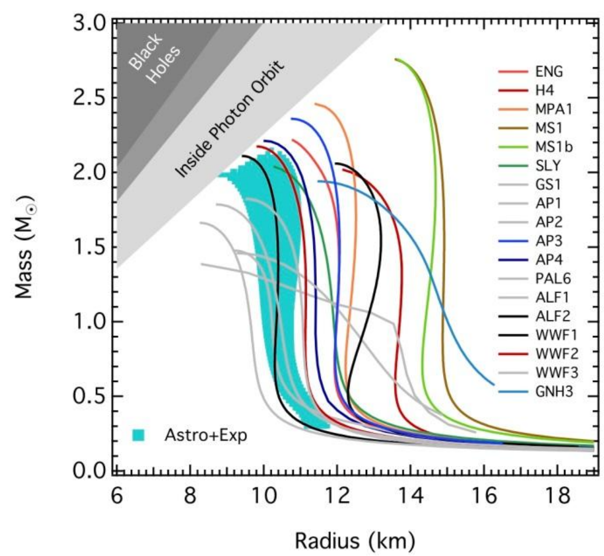
EOS



FRG







Phase structure

Quantum fluctuations included

M-R diagram

Application for compact stars

The Fermi-gas is not a realistic model of a neutron star, this is just demonstrates that an apparently small change in the EoS, due to quantum fluctuations means a noticeable change in the solution of the TOV equations.

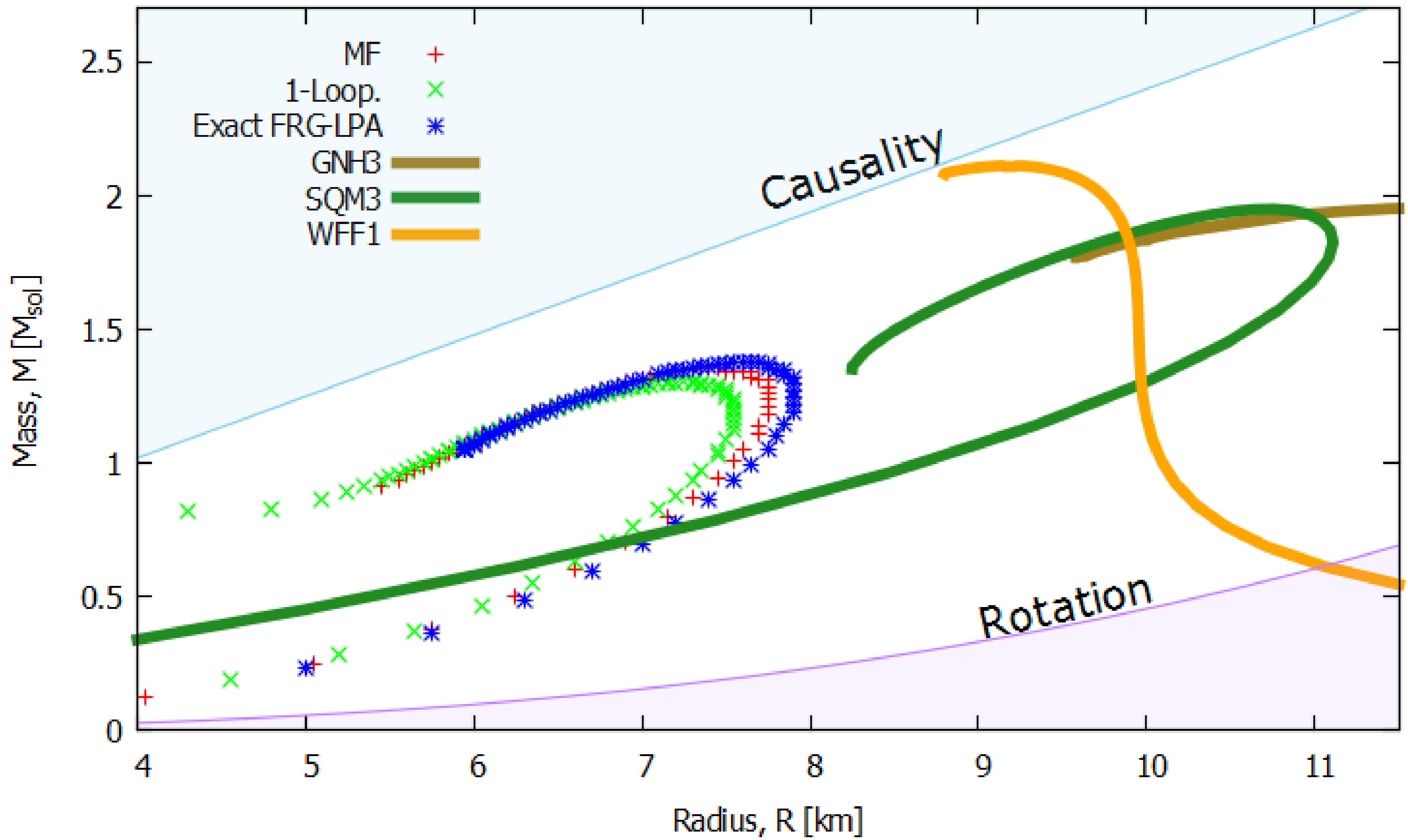
$M_{FRG}=1.377$

+1.5 %

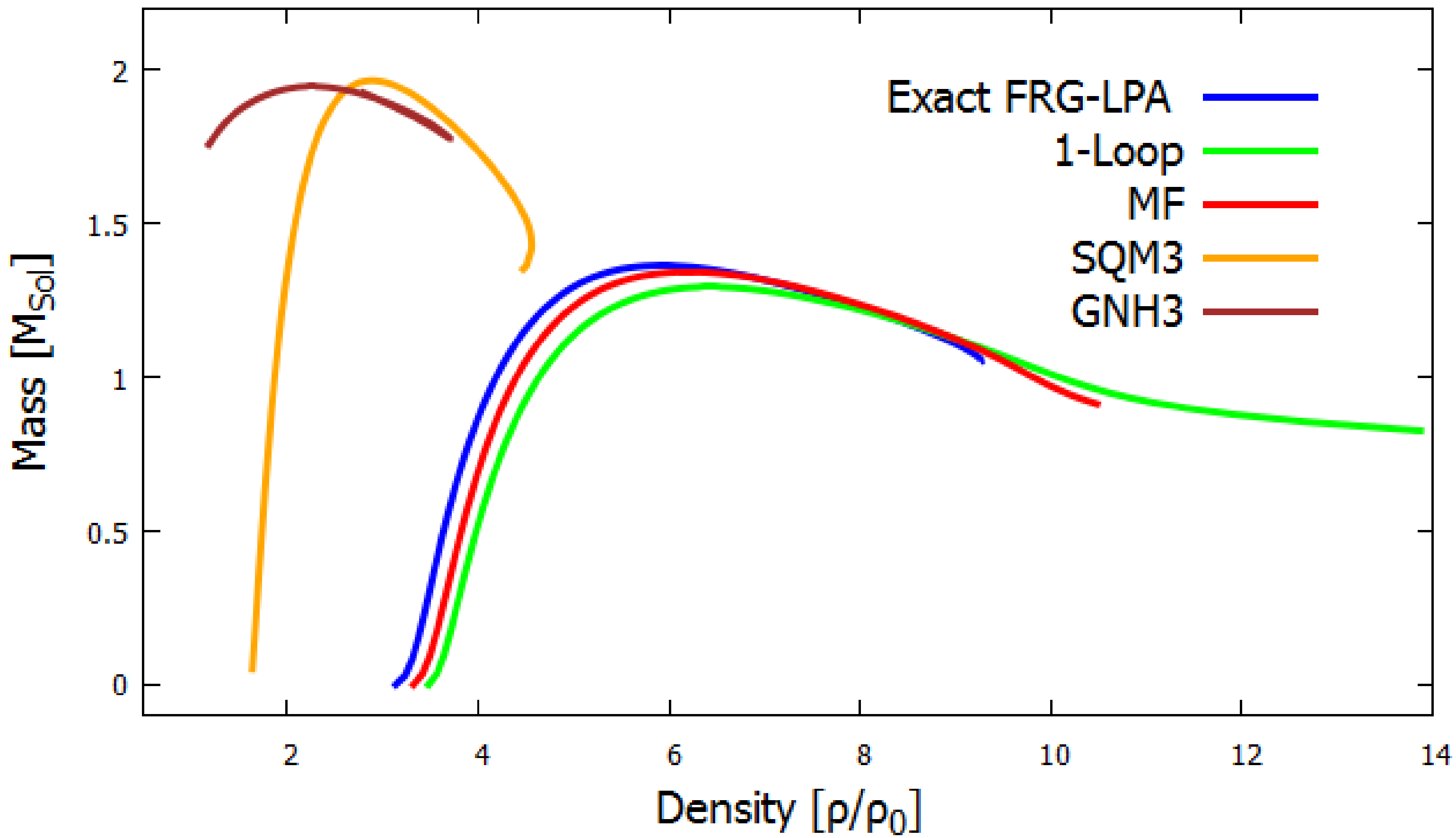
$M_{MF}=1.358$

-3.5 %

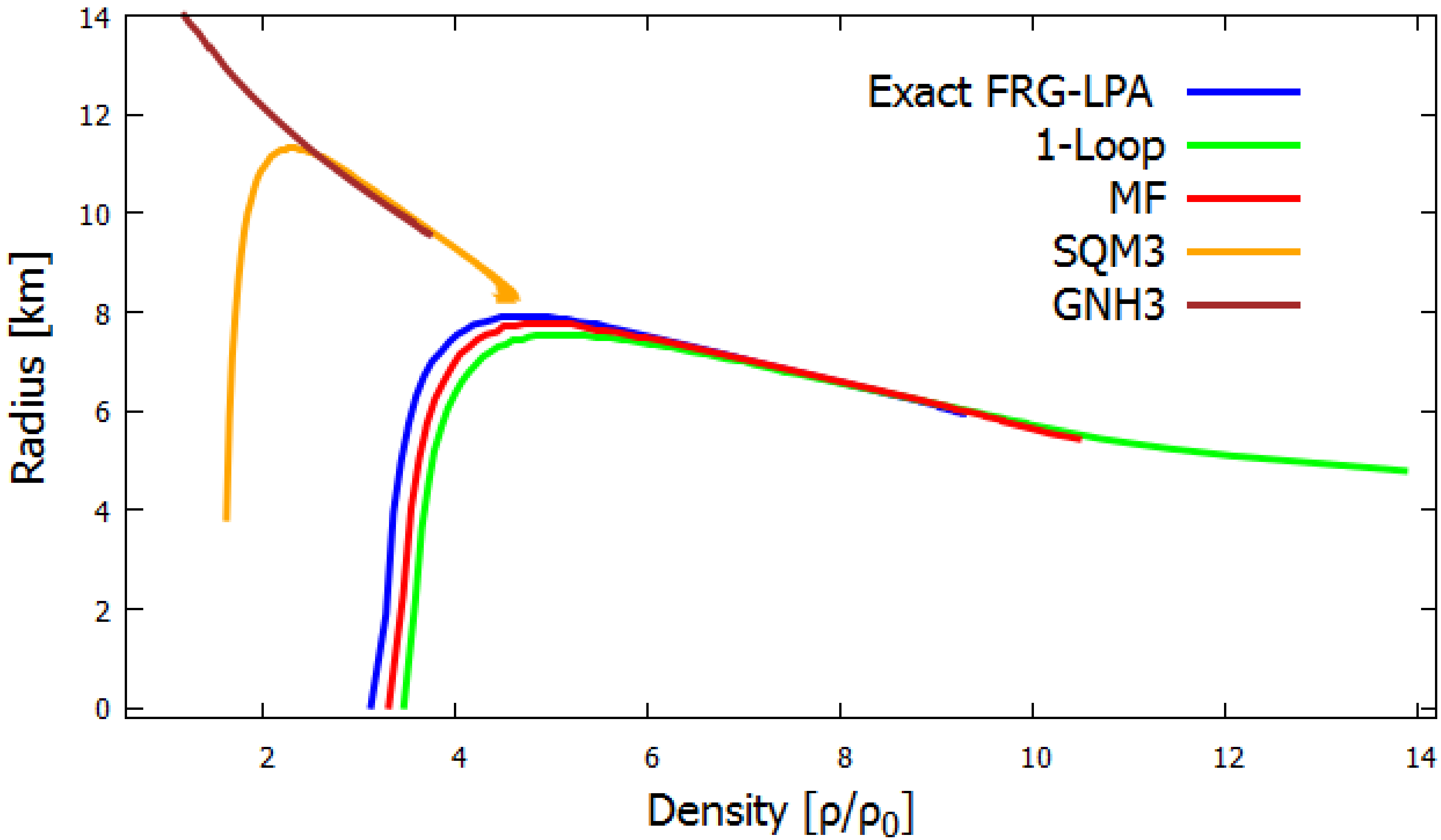
$M_{1L}=1.309$



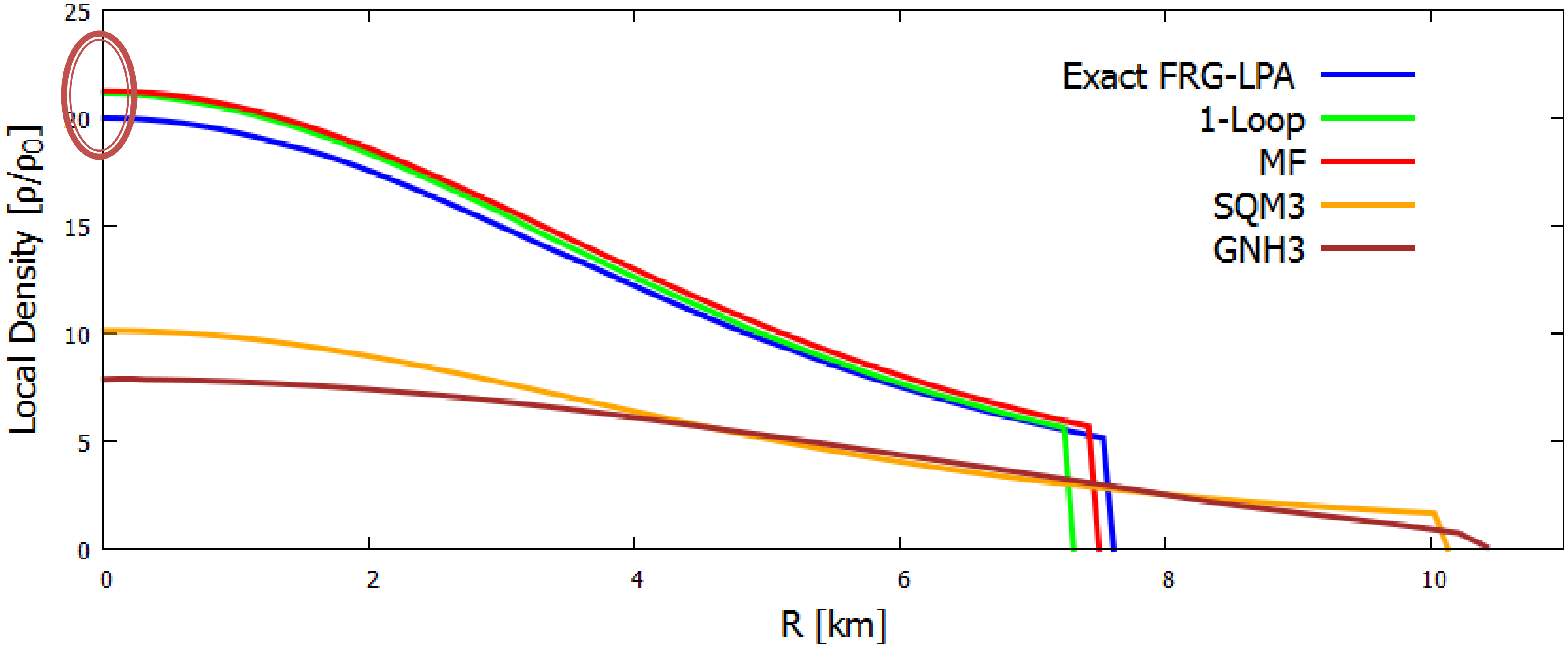
Difference between approximations as function of density



Difference between approximations as function of density



Density profile of highest mass stars

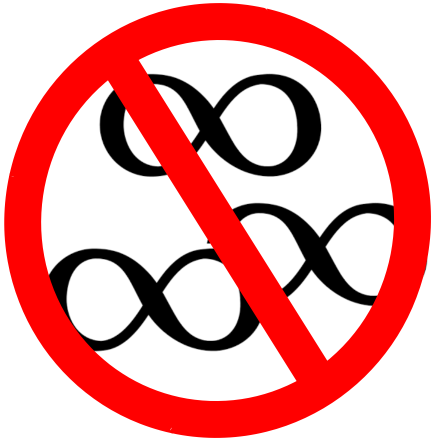


Thank you for the attention !

If you have an FRG Problem

http://pospet.web.elte.hu/

(Contact and related materials)



Acknowledgements:

This work was supported by Hungarian OTKA grants, NK106119, K104260, K104292, K120660, *TET 12 CN-1-2012-0016* , NewCompStar COST action 1304. *Author G.G.B. also thanks the János Bolyai research Scholarship of the Hungarian Academy of Sciences. Author P. P. acknowledges the support by the Wigner RCP of the H.A.S.*

What is FRG good for? Asymptotic freedom

- How can we control the UV behavior of a QFT?
  - **Asymptotic freedom**: theory becomes free at large energies (QCD)

- **Asymptotic safety**: running of the couplings stops at high energies, so increasing the energy does not change the value of couplings
  - Couplings can be fixed and valid up to arbitrary high energies. The theory is renormalizable.
  - Perhaps the SM is asymptotically safe?
  - An UV fixed point is found for Gravity-> Quantum gravity

What is FRG good for? Mass of the Higgs boson

- Using **gravity** in the running of the Higgs potential Wetterich et.al. predicted the **mass of the Higgs** boson (2009 – 126 GeV !)
  - Mikhail Shaposhnikov, Christof Wetterich :Asymptotic safety of gravity and the Higgs boson mass Phys.Lett.B683:196-200,2010, arXiv:0912.0208 [hep-th]
- **Message**: Irrelevant operators (example: gravity in SM) has important role in the running of the couplings.
  - **Irrelevant operators**: they influence the running of the couplings, but they have negligible effect in the IR theory
  - If this is true **FRG is the most convenient** method to study this behavior.