On non-isometric extensions of some GR space-times: a branching perspective

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Outline

Aim: explore the interface of GR and a logic of real possibilities

Plan:

1. Preliminaries: manifest image, scientific image, GR, and real possibilities
2. Co-possibility
3. Hausdorffness and bifurcate curves
4. Non-isometric extensions of GR space-times: overview
5. Misner space-time
6. Whence bifurcate curves of II kind?
7. Interpreting Anaxagoras
8. Discussion: two options

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Preliminaries

Some areas of conflicts between manifest image and scientific image:

- are tenses objective?

- does time flow?

- are some possibilities real?

Preliminaries

Real possibilities: debates in GR inform (or are parallel to) the debates in logic and metaphysics of real possibilities.

On real possibilities (part of manifest image):

Dependent on time, location, and the present state of the world

Example:
It is still possible for Hans to take the 2.54 Berlin train from Keleti.

Co-possibility

Construction of a branching model for real possibilities:

As this particular event just occurred, what other events can occur as well?

Keep a particular event e fixed; ask what events are *co-possible* with it.

A *history* = a maximal set of events that are co-possible with a given one.

Co-possibility

Alternatively, consider the set of all possible events, carve from it maximal sets of events that can occur together

In a branching parlance, what is a criterion for being a (possible) history?

Similar question in GR: which manifolds represent space-times?

Common understanding: histories and GR space-times are modally flat, i.e., they do not tolerate a pair of events that are not co-possible.

An object larger than a history or a space-time: our world of all real possibilities? Modal representation of alternative space-times?

Co-possibility

Historical digression:

Co-possibility and maximal sets of co-possible events, i.e., possible histories or GR space-times - some proposal from metaphysics:

- Lewis: "spatiotemporal relations, or perhaps natural external relations generally—that unify a world".

- Prior: linear order, histories = maximal chains in a base set

- Belnap: having an upper bound, histories = maximal upward directed subsets of a base set

- Müller and TP : history = a maximal Hausdorff sub-manifold of an otherwise non-Hausdorff manifold.

Co-possibility

Is there a room for real possibilities (or non-trivial co-possibility) in a Block Universe view (part of scientific image)?

e' is co-possible with e iff e and e' inhabit same Block Universe.

Can there be alternative (not co-possible) events, e' and e'', but each co-possible with e? Only if e inhabits more than one Block Universe ... Qua one BU e is co-possible with e', qua the other BU e is co-possible with e''.

That's a highly non-standard concept of Block Universe.

Co-possibility

Standard BU:
Space and time are best described as a 4- dimensional space-time which represents all the places and all the times that ever exist as a single unchanging entity. There is no essential difference between the past and the future, because there is no present time defined to separate them; [...] The underlying dynamical idea is that given data at an arbitrary time, everything occurring at any later or earlier time can be uniquely determined from that initial data by time reversible Hamiltonian dynamics, which is assumed to be the basis of dynamics of physics in general and of gravitation in particular. (Ellis 2014)

Go non-standard: look at non-unique solutions to EFE (initial value problem, IVP for GR).

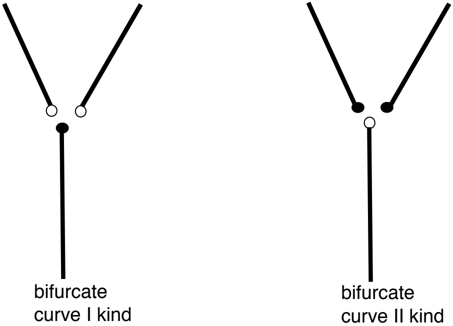
Co-possibility

Claim: IVP informs on how to conceive of co-possibility.

Hausdorffness and bifurcate curves

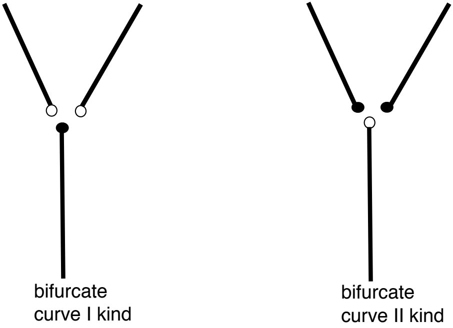
Topology $\langle X, \mathcal{T} \rangle$ has Hausdorff Property iff any two distinct elements of X have non-overlapping open neighborhoods.

Hausdorffness and bifurcate curves



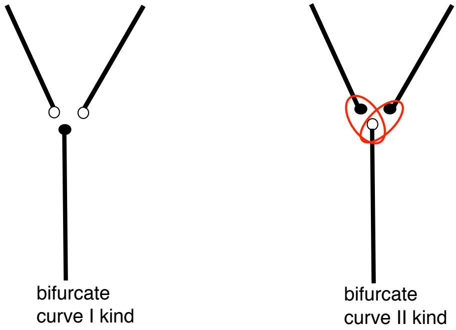
A bifurcate curve of kind II on a C^k manifold M is a triple $\langle C_1, C_2, g\rangle$, where $C_1 : I \to M$, $C_2 : I \to M$ are C^k-continuous curves, $g \in I$ and $\forall x \in I[x < g \Leftrightarrow C_1(x) = C_2(x)]$ (Hájíček 1971)

Hausdorffness and bifurcate curves



A bifurcate curve of kind I on a C^k manifold M is a triple $\langle C_1, C_2, g\rangle$, where $C_1 : I \to M$, $C_2 : I \to M$ are C^k-continuous curves, $g \in I$ and $\forall x \in I[x \leqslant g \Leftrightarrow C_1(x) = C_2(x)]$

A bifurcate curve of kind II implies non-Hausdorffness



but not the other way round.

Which bifurcate curves are really problematic? Those of II kind.

Hausdorffness and bifurcate curves

A happy discovery in GR:

Although non-Hausdorff manifolds naturally pop up in extensions of some GR space-times, they do not contain bifurcate curves.

Geroch (1968 Jour Math Phys 9, 450) on non-Hausdorff manifolds:

" It would seem that some restriction must be imposed on these non-Hausdorff manifolds which are to be deemed acceptable candidates for a space-time manifold. Two possible restrictions immediately come to mind:
1. Only those non-Hausdorff space-times are permitted in which every geodesic has a unique extension.
2. Only those non-Hausdorff space-times are permitted in which every curve has no more than one end point."

Hausdorffness and bifurcate curves

Non-Hausdorff manifolds without bifurcate curves

If a space-time (history) identified with a maximal Hausdorff sub-manifold, we have some weirdness:

alternative possible histories without any “small” object facing alternative developments

Hausdorffness and bifurcate curves

A common sentiment in GR and branching: a bifurcate curve II kind signals alternative possibilities, hence it cannot occur in a modally thin structure, like a GR space-time or a possible history of branching.

Non-isometric extensions of GR space-times

IVP in GR

A 3-dim spacelike surface Σ of a 4-dim space-time $\langle M, g \rangle$, with some data (fields) on Σ.

Is Σ with the data compatible with one space-time $\langle M, g \rangle$ only?

It depends on a kind of data assumed on Σ and on restrictions on sought-for extended space-times $\langle M, g \rangle$.

Non-isometric extensions of GR space-times

For a natural choice of initial data and by restricting attention to globally hyperbolic extended space-times: a uniqueness result (Choquet-Bruhat and Geroch 1969)

Moral from uniqueness results: non-unique developments of Σ cannot be globally hyperbolic.

Non-isometric extensions of GR space-times

How do multiple extensions arise?

Typical procedure:

(1) construction of a set of auxiliary manifolds, and

(2) pasting the auxiliary manifolds together (by a gluing map or taking a quotient wrt an equivalence relation), to obtain alternative extensions

(3) these extensions can be further glued together to produce (one) non-Hausdorff manifold.

The procedure illustrated by Misner space-time.

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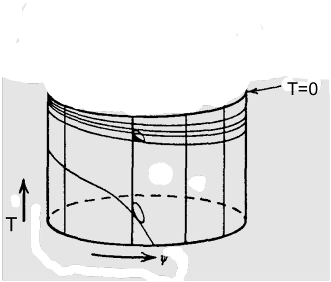
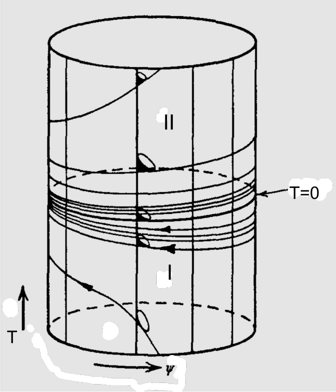
The procedure illustrated by Misner space-time.

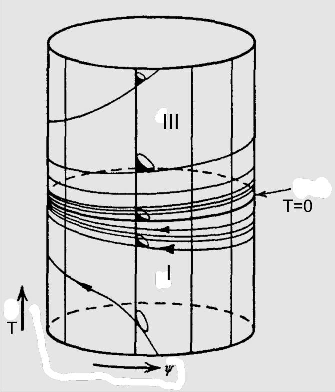
Misner space-time

$$ds^2 = \frac{dT^2}{T} - Td\varphi^2,$$



Misner space-time: its two extensions





$$ds'^2 = -T' d\varphi'^2 - 2sgn(T') d\varphi' dT',$$
$$ds''^2 = -T'' d\varphi''^2 + 2sgn(T'') d\varphi'' dT'',$$

Misner space-time: compactification

Topology: cylindrical. Identify φ, $\varphi + 2\pi, \ldots \varphi + k2\pi, \ldots$.

"Identifying" requires a gluing function (or an equivalence relation).

We glue together (regions of) countably many copies of Minkowski space-time, using a symmetry of Minkowski space-time, hyperbolic rotations through $k2\pi$ (k-integer); 0-copy and n-th copy are glued by:

$$t_n = t_0 \cosh(n\pi) + x_0 \sinh(n\pi)$$
$$x_n = t_0 \sinh(n\pi) + x_0 \cosh(n\pi).$$

This formula induces gluing between k-th and n-th copies.

Gluing preserves hyperbolas $t^2 - x^2 = const$

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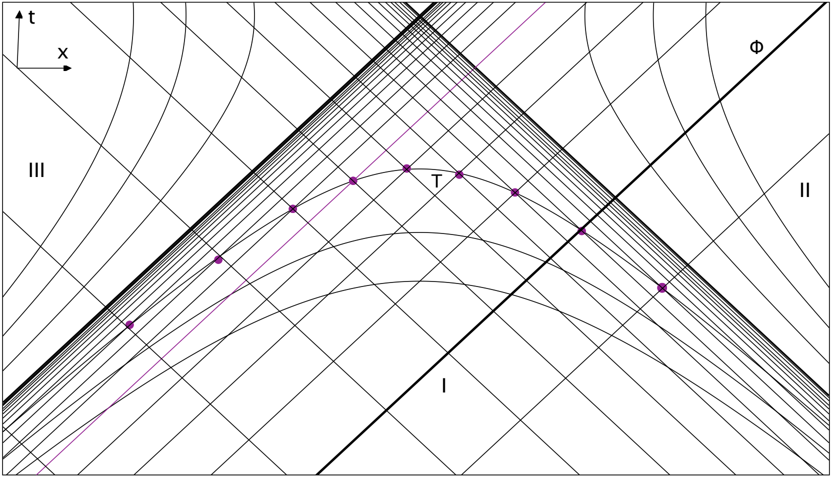
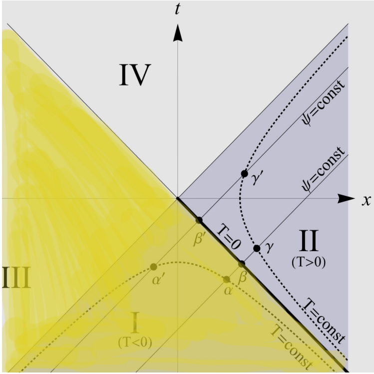
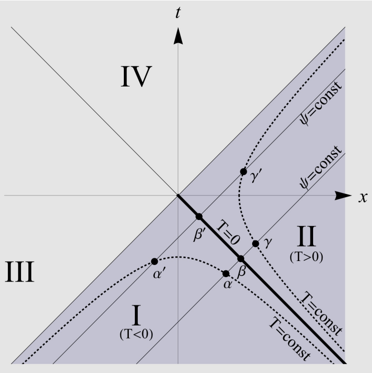
Misner space-time: points identified



Misner space-time - compactification

Quadrant I induces the initial Misner space-time
Quadrants I+II induces one extension of Misner space-time
Quadrants I + III induces the other extension of Misner space-time





Misner space-time

Gluing function A defines equivalence relation: $x \equiv_A y$ iff $x = y$ or $x = A(y)$ or $y = A(x)$.
This yields a quotient space (Munkres, p. 139)

Target space-times are quotient spaces $(I + II)/A$ and $(I + III)/A$.

Each extends the (initial) Misner space-time I/A.

Restricted to the Misner region, $(I + II)/A$ and $(I + III)/A$ are isometric.

But isometry of $(I + II)/A$ and $(I + III)/A$ is problematic.

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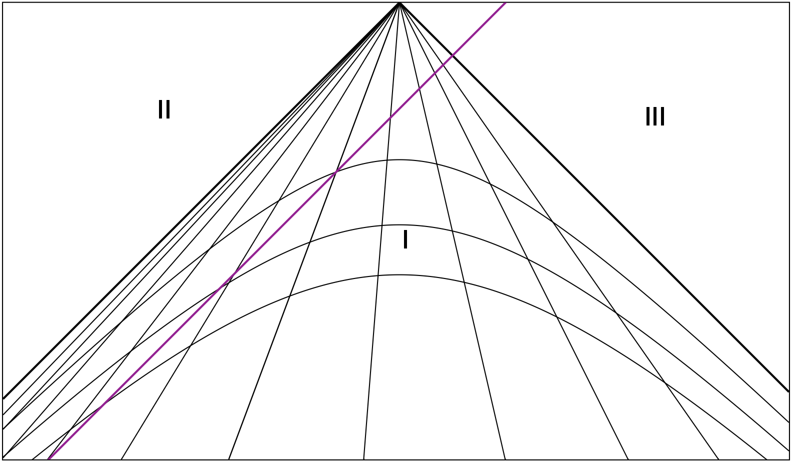
Misner space-time: : geodesics in $(I + III)/A$



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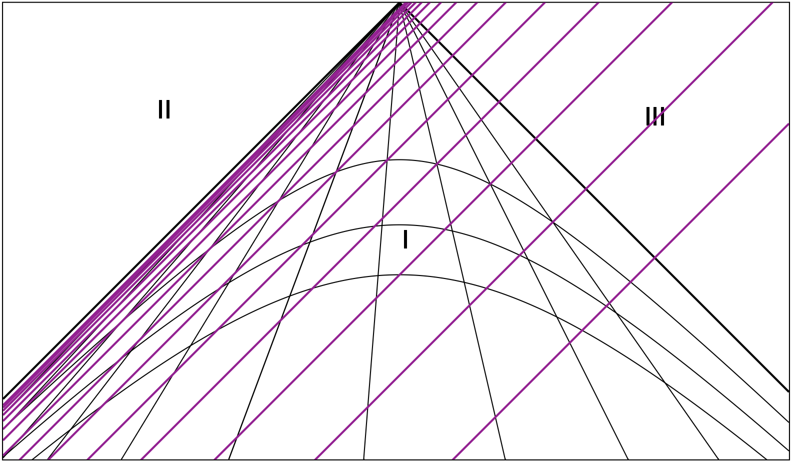

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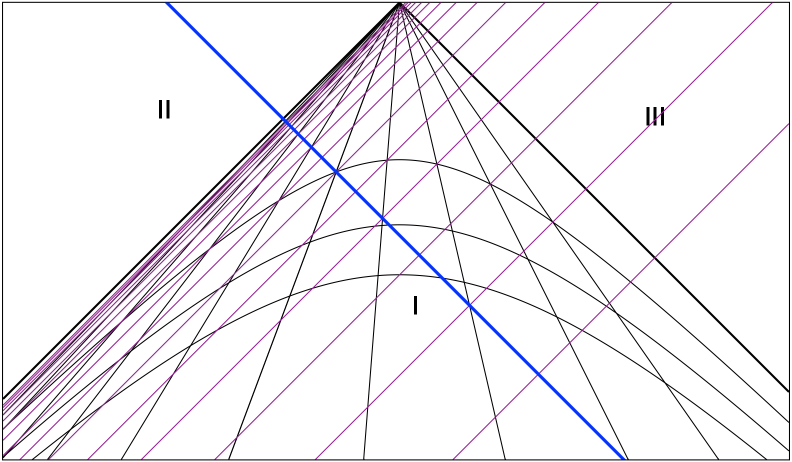

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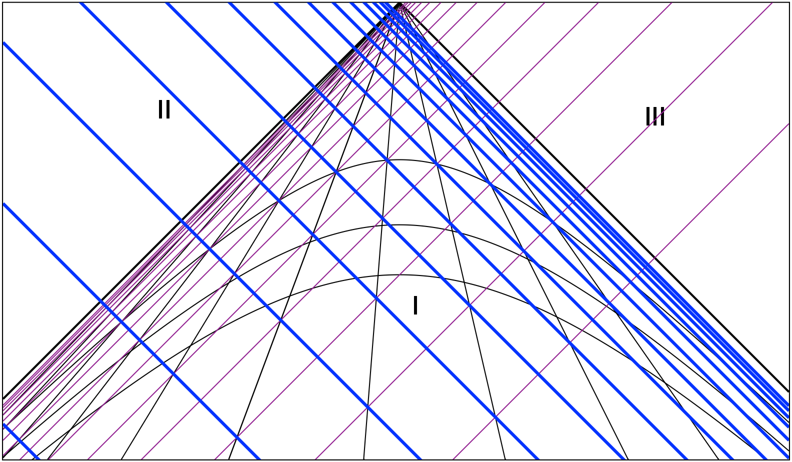

Misner space-time: geodesics

In $(I + II)/A$ (or $(I + III)/A$) for each point in the initial segment and a vector, there is exactly one geodesic passing through this point, whose tangent at this point coincides with the vector.

That's was to be expected, by the geodesics theorem, since these manifolds are Hausdorff.

Geodesics theorem for Hausdorff and non-Hausdorff space-times:

$(\star)$ if a metric g is appropriately continuous ($C^{1,1}_{loc}$ or smoother), given a point and a vector at this point, there is **locally** a unique geodesic that passes through the point and whose tangent at this point coincides with the vector (after Chruściel 2008).

This local result prohibits bifurcate geodesics of I kind, but does not exclude bifurcate geodesics of II kind.

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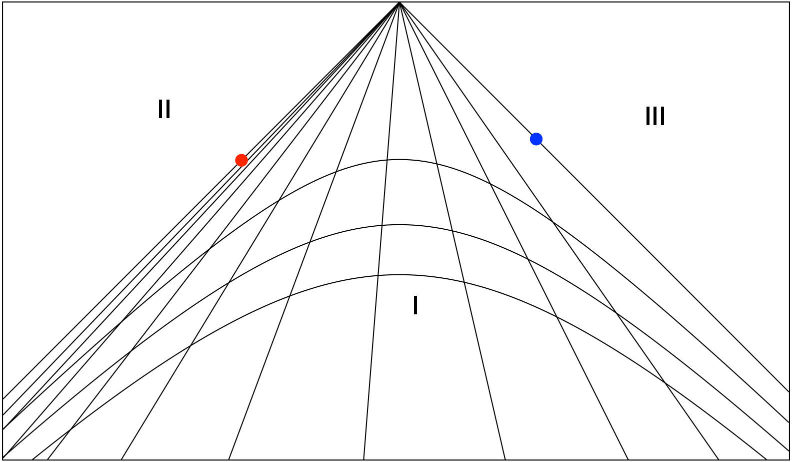
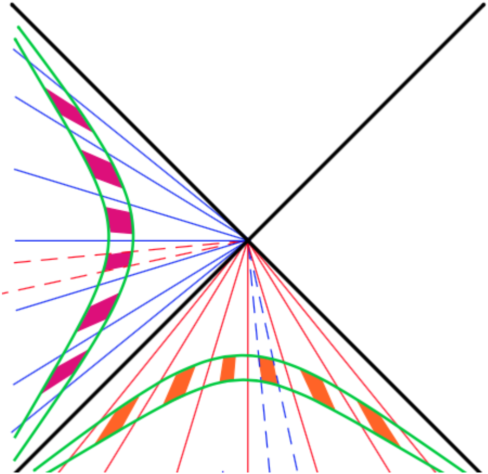
Misner space-times: $(I + II + III)/A$

Turn to $(I + II + III)/A$: it is non-Hausdorff.



Misner space-time: $(I + II + III)/A$

Base sets, for elements off diagonal (after Margalef–Bentabol, Villasenor 2015)



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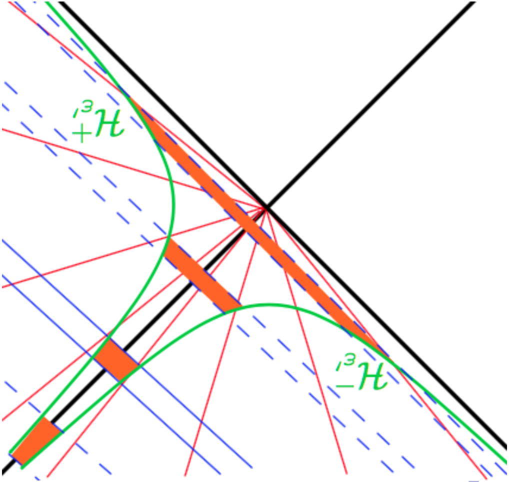
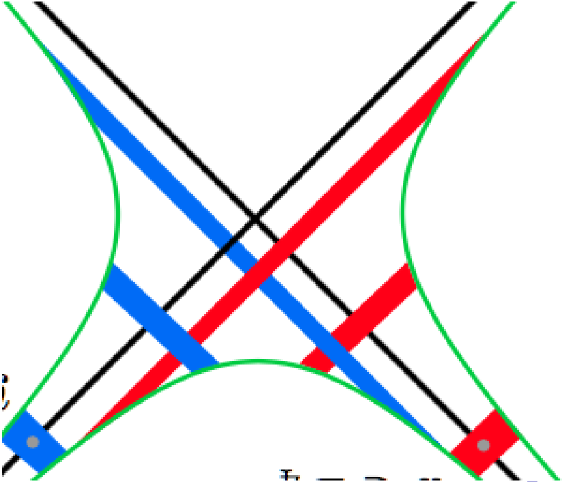


Misner space-time: $(I + II + III)/A$

Failure of Hausdorffness, in base sets (after Margalef–Bentabol, Villasenor 2015)



Misner space-time: $(I + II + III)/A$

Moral: a pair consisting of an (arbitrary) point on the left half-diagonal and a point on the right half-diagonal is a witness for non-Hausdorfness of $(I + II + III)/A$.

Each $(I + II)/A$ and $(I + III)/A$ is a maximal Hausdorff sub-manifold of $(I + II + III)/A$

What happens to a pair of geodesics, one going to the left extension, the other - to the right extension? They do not bifurcate.

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Misner space-time: $(I + II + III)/A$

Since $(I + II + III)/A$ is non-Hausdorff, it could tolerate bifurcating geodesics (of non-Hausdorff variety).

But $(I + II + III)/A$ does not tolerate them:

"No bifurcate geodesics" carries over, in this context, to "no bifurcate curves of II kind".

Why no bifurcate curves of II kind in $(I + II + III)/A$?

Misner space-time: $(I + II + III)/A$

Since $(I + II + III)/A$ is non-Hausdorff, it could tolerate bifurcating geodesics (of non-Hausdorff variety).

But $(I + II + III)/A$ does not tolerate them:

"No bifurcate geodesics" carries over, in this context, to "no bifurcate curves of II kind".

Why no bifurcate curves of II kind in $(I + II + III)/A$?

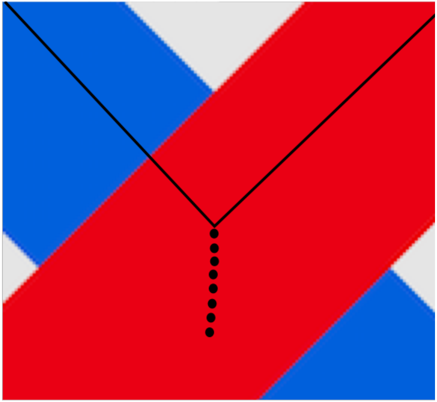
Why no bifurcate curves of II kind?

Recall a theorem:

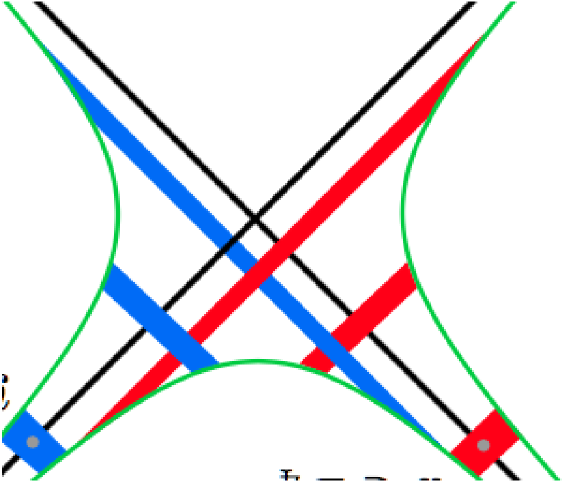
A topology (X, τ) is non-Hausdorff iff X contains a sequence (generally, a net) that has more than one point of convergence (in τ)

Where is a sequence with multiple points of convergence in $(I + II + III)/A$?

Here:



Recall:



Why no bifurcate curves of II kind?

The sequence $\{[x_i]\}_{i \in I}$ has three limits: two on the semi-diagonals, and $[x]$, where $x = \lim_{i \to \infty} x_i$ (lim in $\mathcal{R}^2$).

Can we use this sequence with three convergence pts to produce a bifurcate curve od II kind? No, problems with continuity (cf. literature on modular curves).

Observation: by gluing we produce a new topology in which an induced sequence have more convergence pts than the original sequence. But gluing alone seems incapable of producing new continuous curves.

Whence bifurcate curves of II kind?

Skepticism: Can bifurcate curves of II kind arise in gluing constructions? Yes.

Question: is a sequence with multiple points of convergence “better” than a bifurcate curve of II kind?

Whence bifurcate curves of II kind?

Motivating observation: $(I + II + III + \{o\})/A$ admits bifurcate curves of II kind (here o is the origin point, $(0, 0)$).

More precisely, take two copies of $(I + II + III + \{o\})$ glued by hyperbolic rotation in $(I + II + III)$ only. There will be a bifurcate curve of II kind, (γ_1, γ_2) s.t. $\gamma_1(x) = \gamma_2(x)$ for $x \in (0, 1)$ but $o_1 = \gamma_1(1) \neq \gamma_2(1) = o_2$.

Observation: gluing wasn't total, it was extendable (moreover, continuously e.).

Consequence of Hájíček's criterion for the existence of bifurcate curves of II kind.

Interpreting Anaxagoras

Back to co-possibility

"For it is no more fitting for what is established at the center and equally related to the extremes to move up rather than down or sideways. And it is impossible for it to make a move simultaneously in opposite directions. Therefore it is at rest of necessity."
Anaxagoras, as reported by Aristotle in *On Heavens*.

Majority interpretation: Anaxagoras's argument for the Earth being at rests.

Minority interpretation: A's discovered a notion of co-possibility, like "alternative trajectories are not co-possible".

Discussion: two options

Recall the objection to bifurcate curves, that they indicate alternative developments

Does this objection carry over to sequences with multiple convergence points?

If yes, go Hausdorff.

Discussion: two options

But if one opts for non-Hausdorffness without bifurcate curves, they need to answer why the objection to bifurcate curves do not carry over to sequences with multiple convergence points.

Discussion: two options

I. Co-possibility defined in terms of Hausdorffness:

A history is a maximal Hausdorff sub-manifold of a (possibly non-Hausdorff) manifold.

A GR space-time is a Hausdorff manifold (satisfying a few other conditions)

- Pluses: GR mathematical tradition, no bifurcate curves, no sequences with multiple convergence points.

- Minuses: GR comes out indeterministic (by e.g. J.N. Butterfield's DM2 analysis, 1989) and it is a weird kind of indeterminism

- What are those non-Hausdorff manifolds met in IVP? Suggestion: modal representation of alternative space-times.

Discussion: two options

II. Co-possibility defined in terms of no bifurcate curves:

A history is a maximal sub-manifold (Hausdorff or not) that does not admit bifurcate curves of II kind.

A GR space-time is a manifold (possibly non-Hausdorff) without bifurcate curves of II kind (satisfying a few other conditions)

- Pluses: non-isometric extensions need not witness indeterminism—if they can be further extended to manifolds without bifurcate curves of II kind.
- No conflict between global indeterminism and local determinism

- Minuses: (1) history /spacetime contains a sequence with multiple convergence points, (2) A role of Hausdorffness in basic constructions of GR. See Joanna Luc's talk.

Thank you for your attention

Thanks to Joanna Luc for many discussions.

Butterfield's (1989) analysis of determinism

A theory with models $\langle M, O_i \rangle$ is **S**-deterministic, where **S** is a kind of region that occurs in manifolds of the kind occurring in the models, iff: given any two models $\langle M, O_i \rangle$ and $\langle M', O_i' \rangle$ containing regions S, S' of kind **S** respectively, and any diffeomorphism α from S onto S':
if $\alpha^*(O_i) = O_i'$ on $\alpha(S) = S'$, then:
there is an isomorphism β from M onto M' that sends S to S', *i.e.* $\beta^*(O_i) = O_i'$ throughout M' and $\beta(S) = S'$.

By taking for S and S' the Misner region in $(I + II)/A$ and $(I + III)/A$, resp., and observing that isomorphism = isometry in this case, we get it that the two extensions witness indeterminism.