Quantum theory and local causality

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Local causality

- **Bell, 1988:** Local causality "is the idea that what you do has consequences only nearby, and that any consequences at a distant place will be weaker and will arrive there only after the time permitted by the velocity of light. Locality is the idea that consequences propagate continuously, that they don't leap over distances."

Local causality

- **Rédei, 2014:** Local causality “is not a single property a physical theory can inprinciple have but an intricately interconnected web of features.”

Project

I. What is a local physical theory?

II. Bell's local causality in a local physical theory

III. Other locality and causality concepts

  a. Local primitive causality

  b. Common Cause Principle

  c. Causal Markov Condition

IV. Bell's inequalities and "noncommutative beables"

I. What is a local physical theory?

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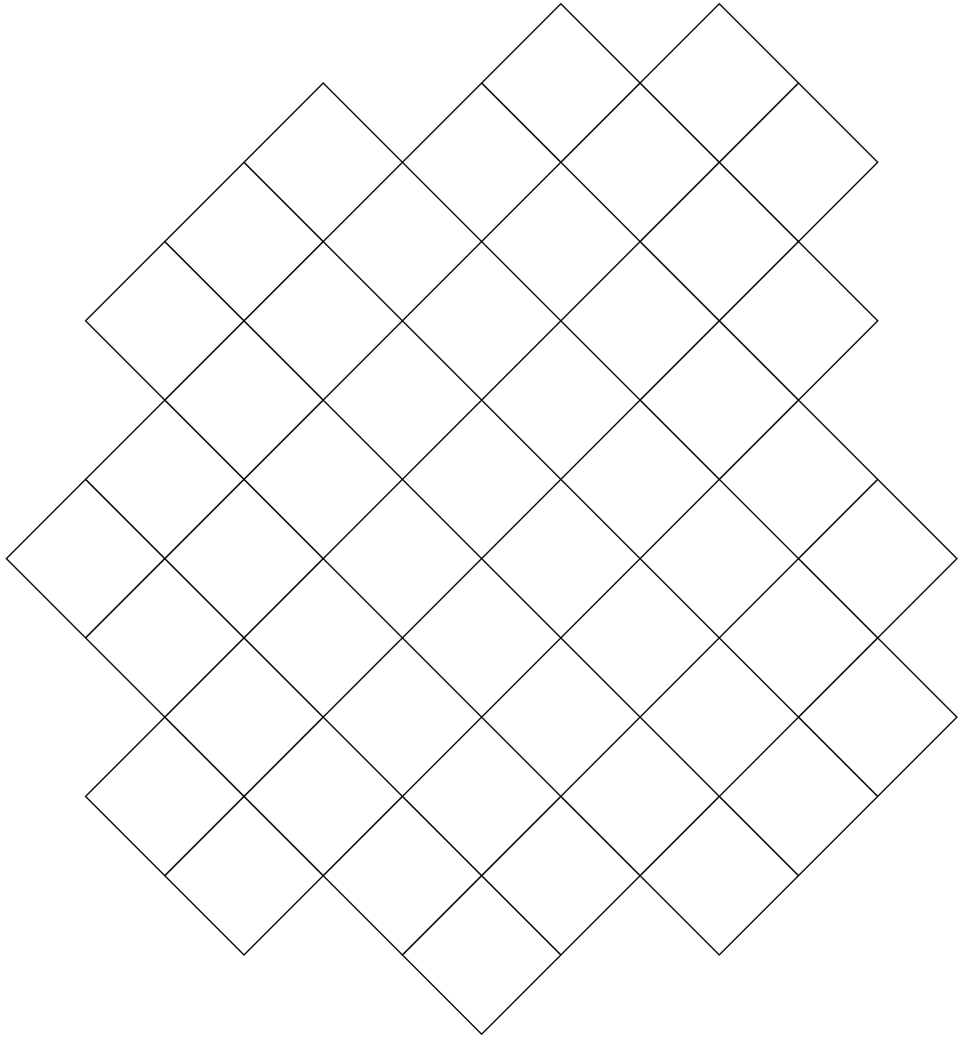
- $\mathcal{M}$: **globally hyperbolic spacetime**
- $\mathcal{K}$: **covering collection** of bounded, globally hyperbolic regions of $\mathcal{M}$
- $(\mathcal{K}, \subseteq)$: **directed poset**

I. What is a local physical theory?

- **Discretized two dimensional Minkowski spacetime:**

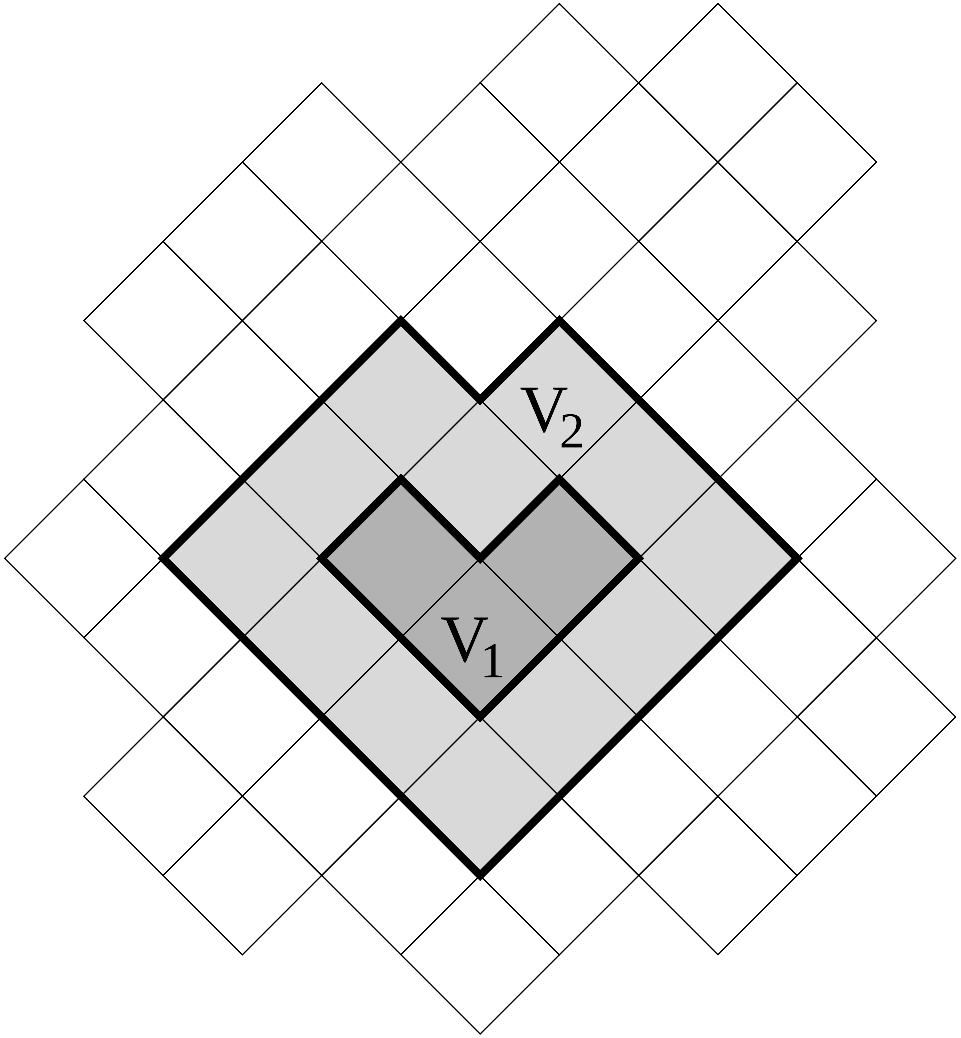


I. What is a local physical theory?

- A **local physical theory (LPT)** is an association of algebras to spacetime regions satisfying **isotony** and **microcausality**.

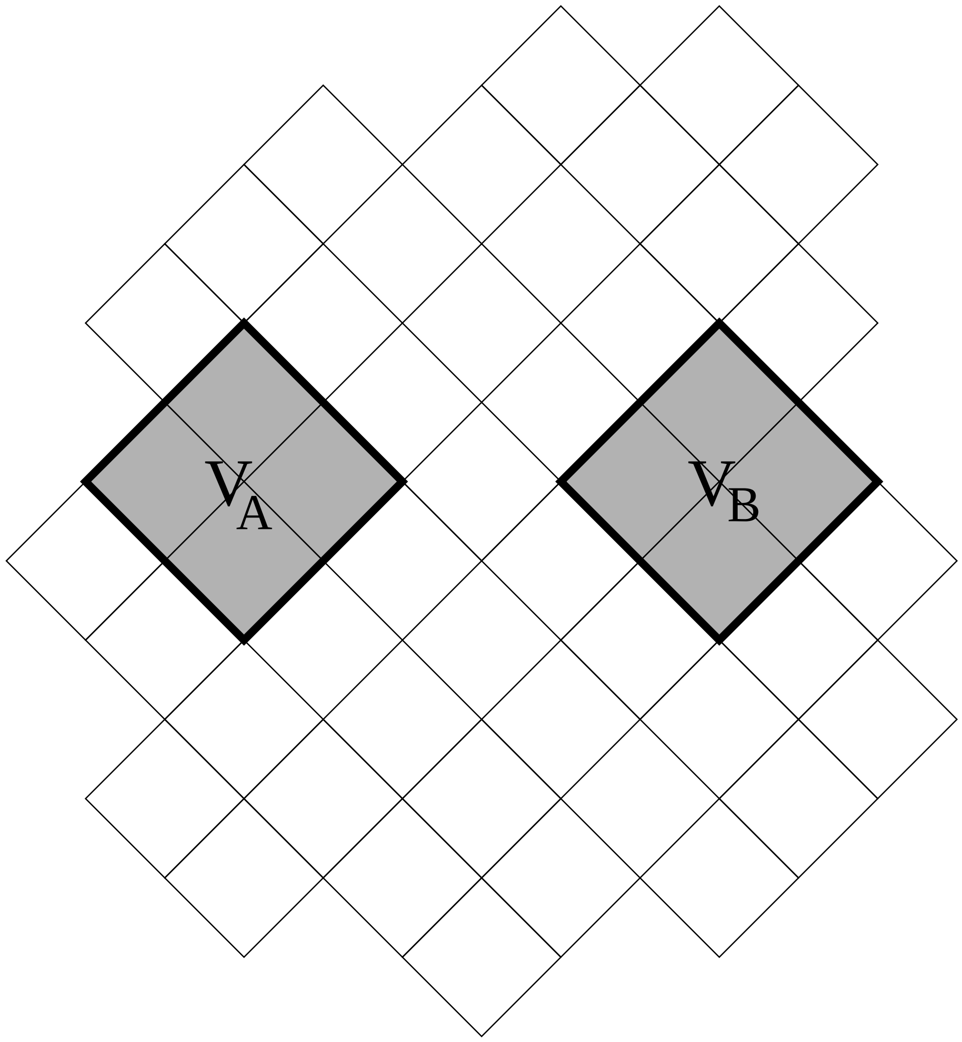
I. What is a local physical theory?

- **Isotony:** if $V_1 \subset V_2$, then $\mathcal{N}(V_1)$ is a unital subalgebra of $\mathcal{N}(V_2)$



I. What is a local physical theory?

- **Microcausality:** if $V_A \subset V_B'$, then $[\mathcal{N}(V_A), \mathcal{N}(V_B)] = 0$

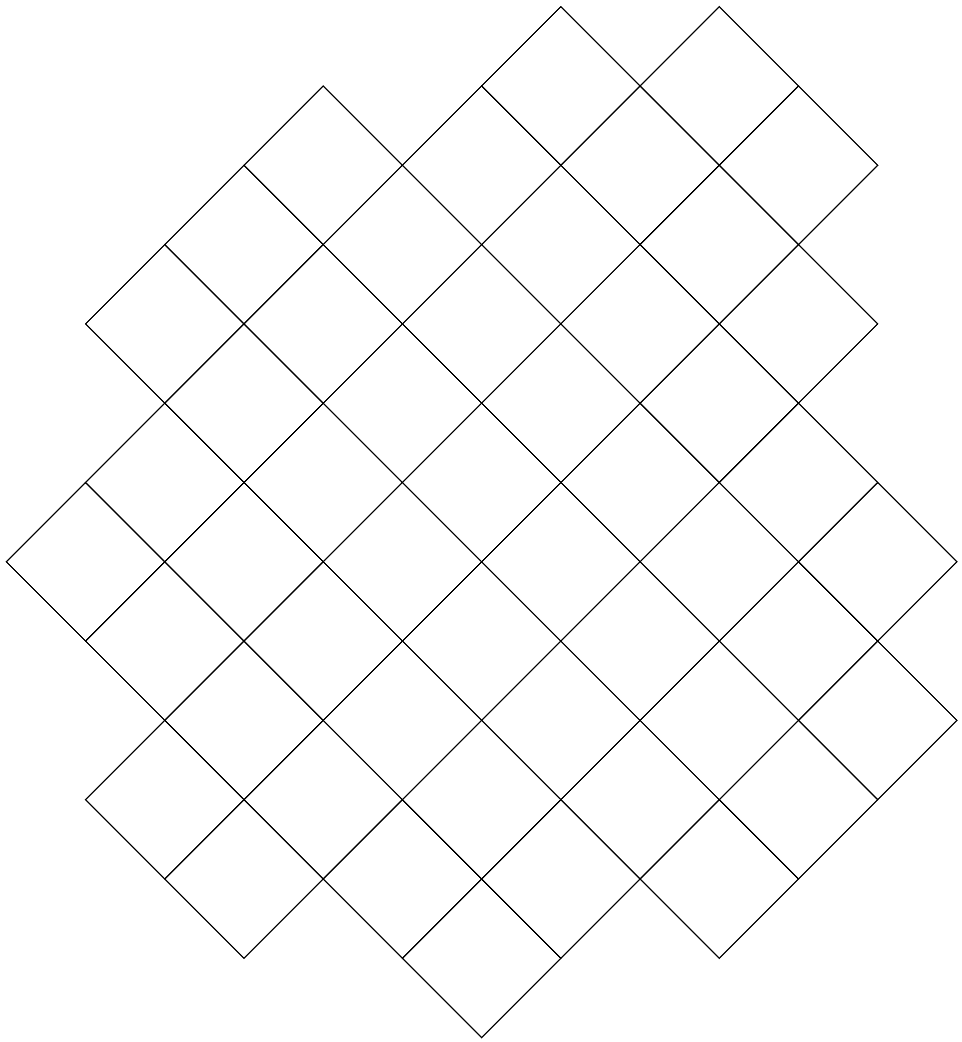


I. What is a local physical theory?

- A local physical theory is **classical** (LCT) if the local algebras are commutative and **quantum** (LQT) if they are noncommutative.

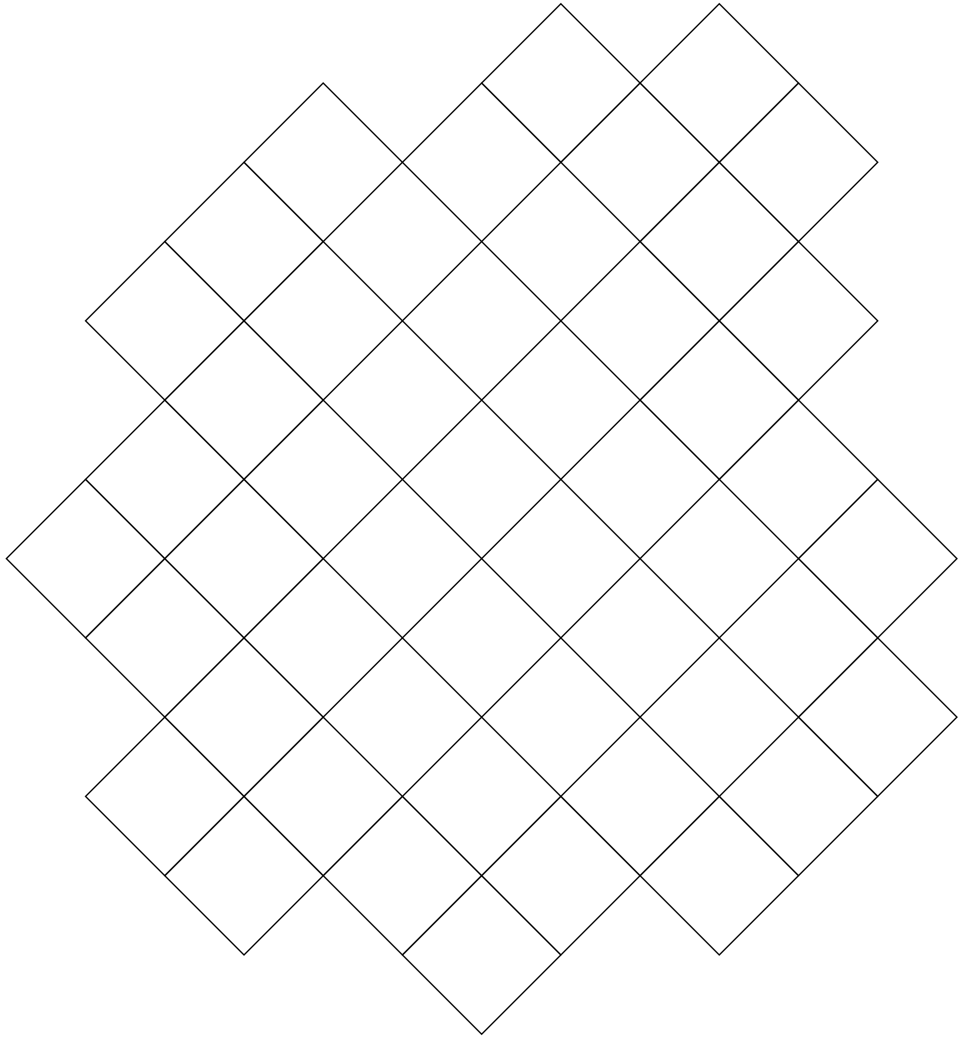
I. What is a local physical theory?

- **Examples:**



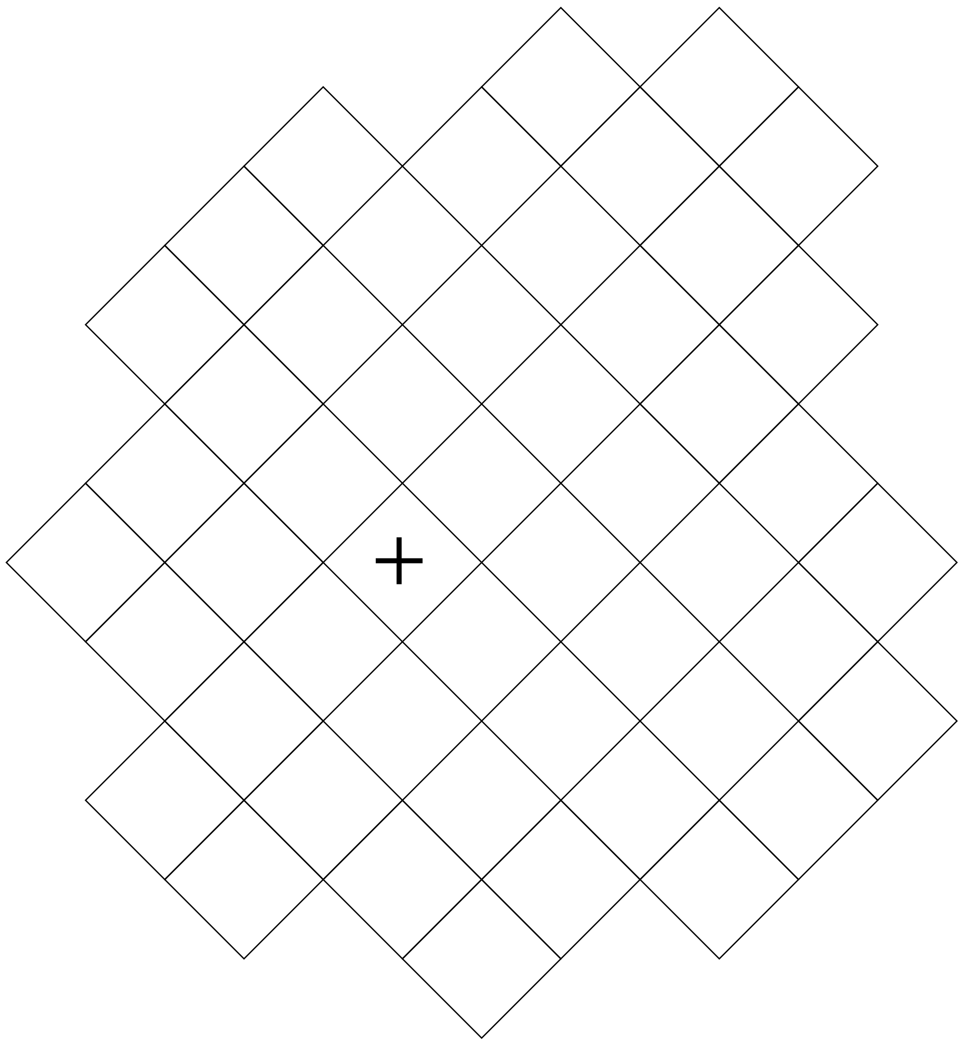
I. What is a local physical theory?

- **Example 1. Deterministic LCT**



I. What is a local physical theory?

- **Local algebras:**



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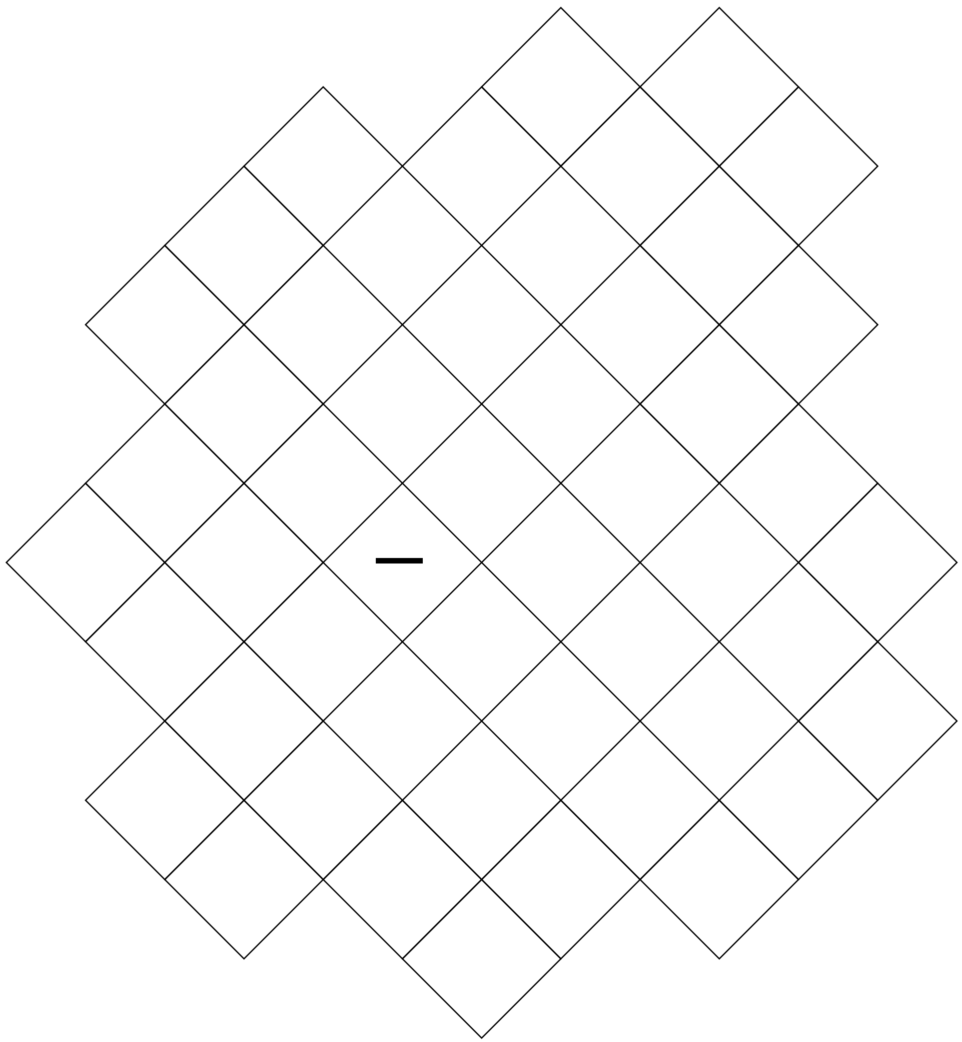
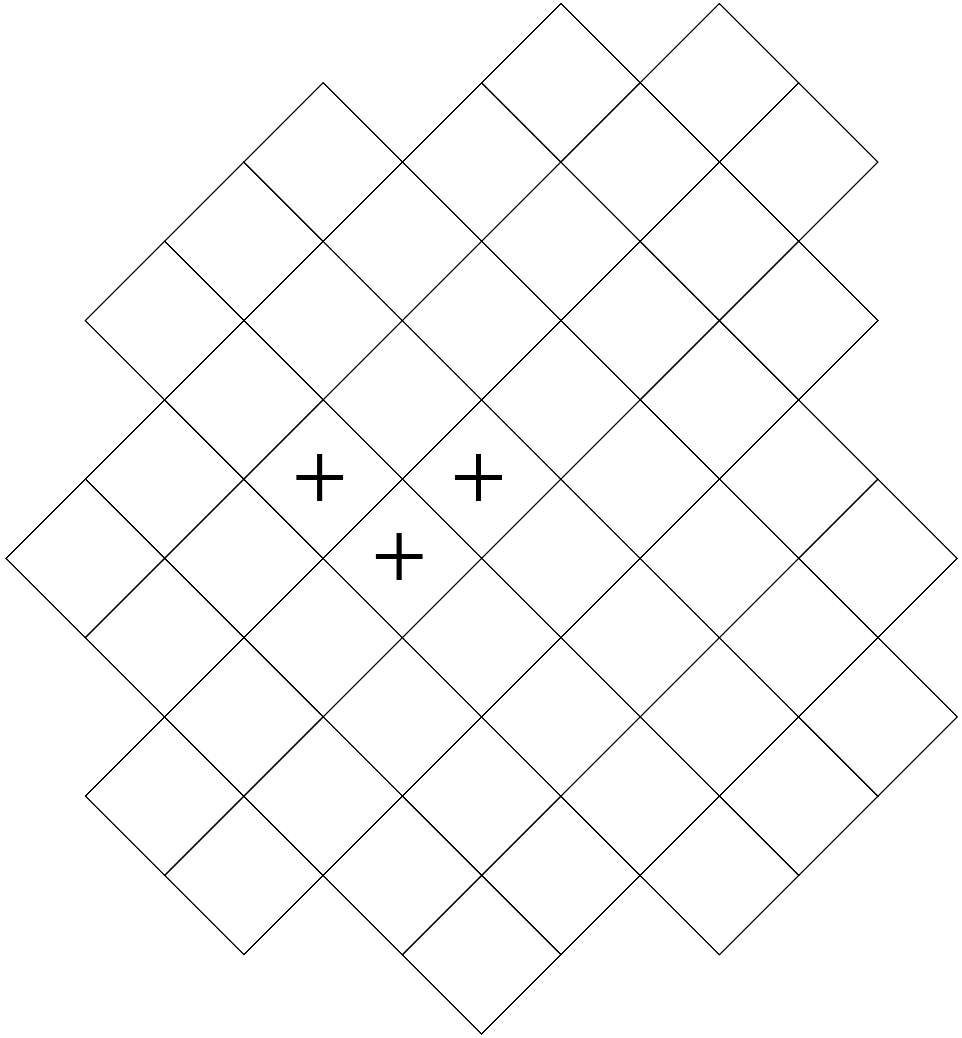
- **Local algebras:**



- **Deterministic dynamics:**



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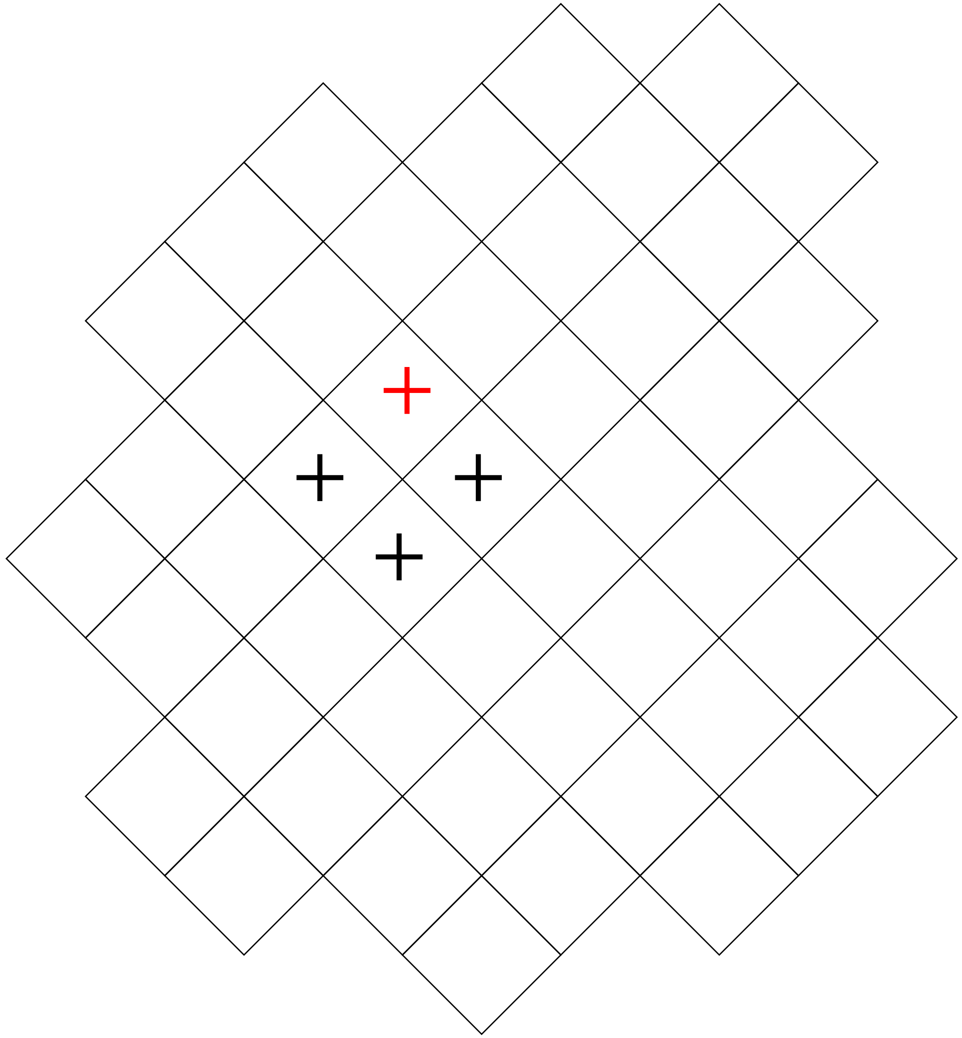

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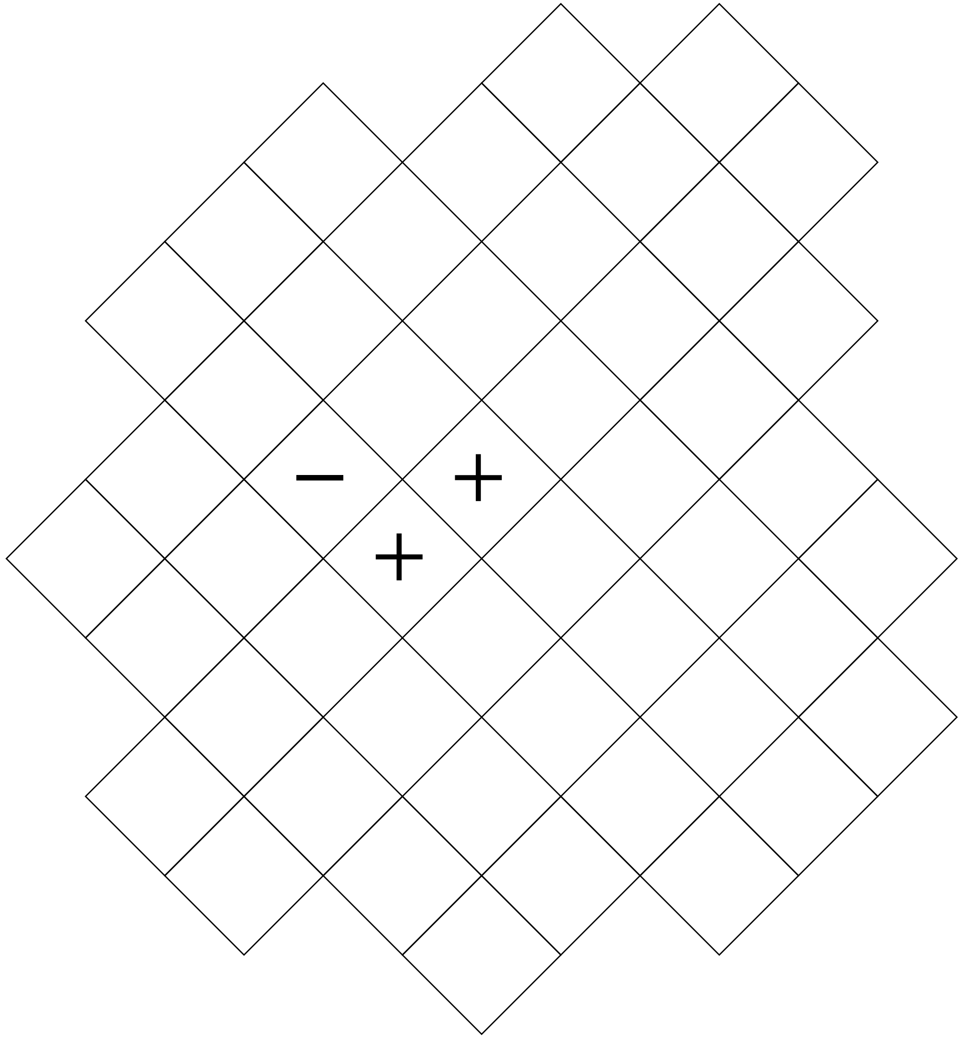

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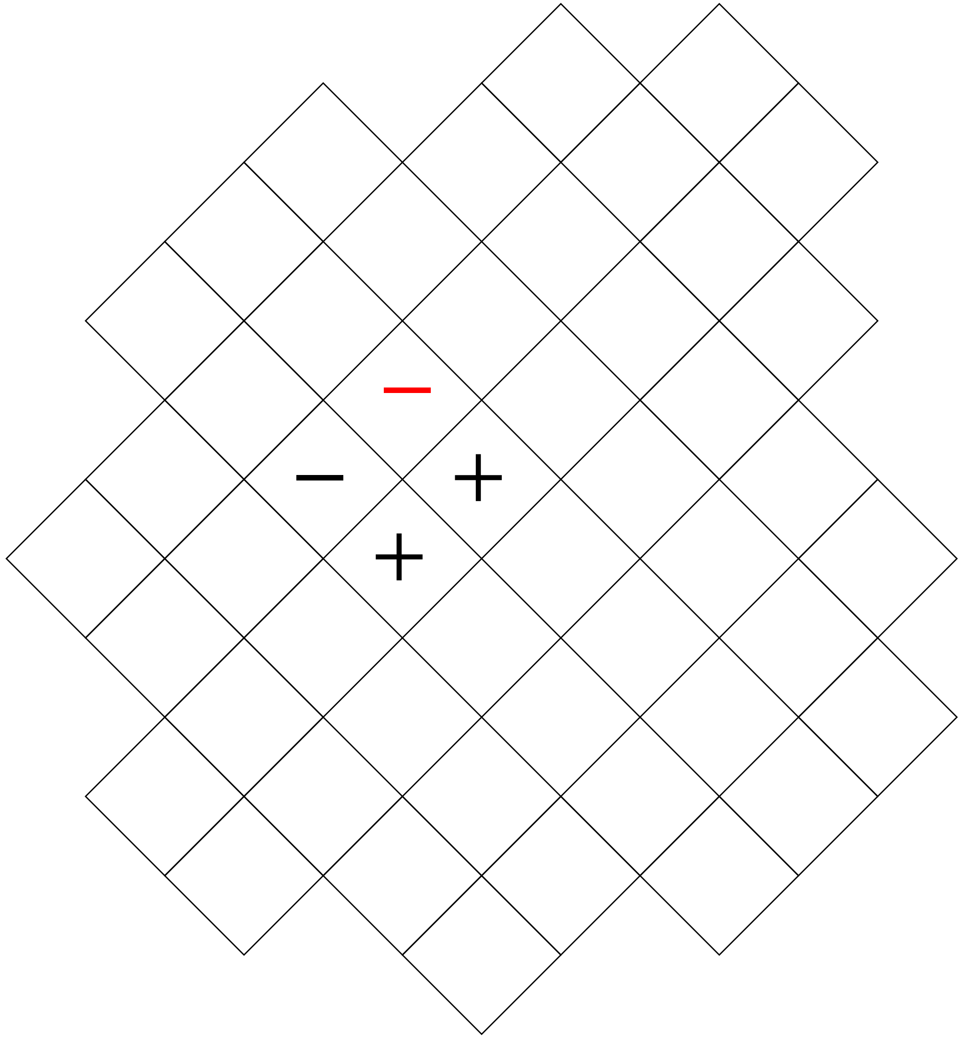
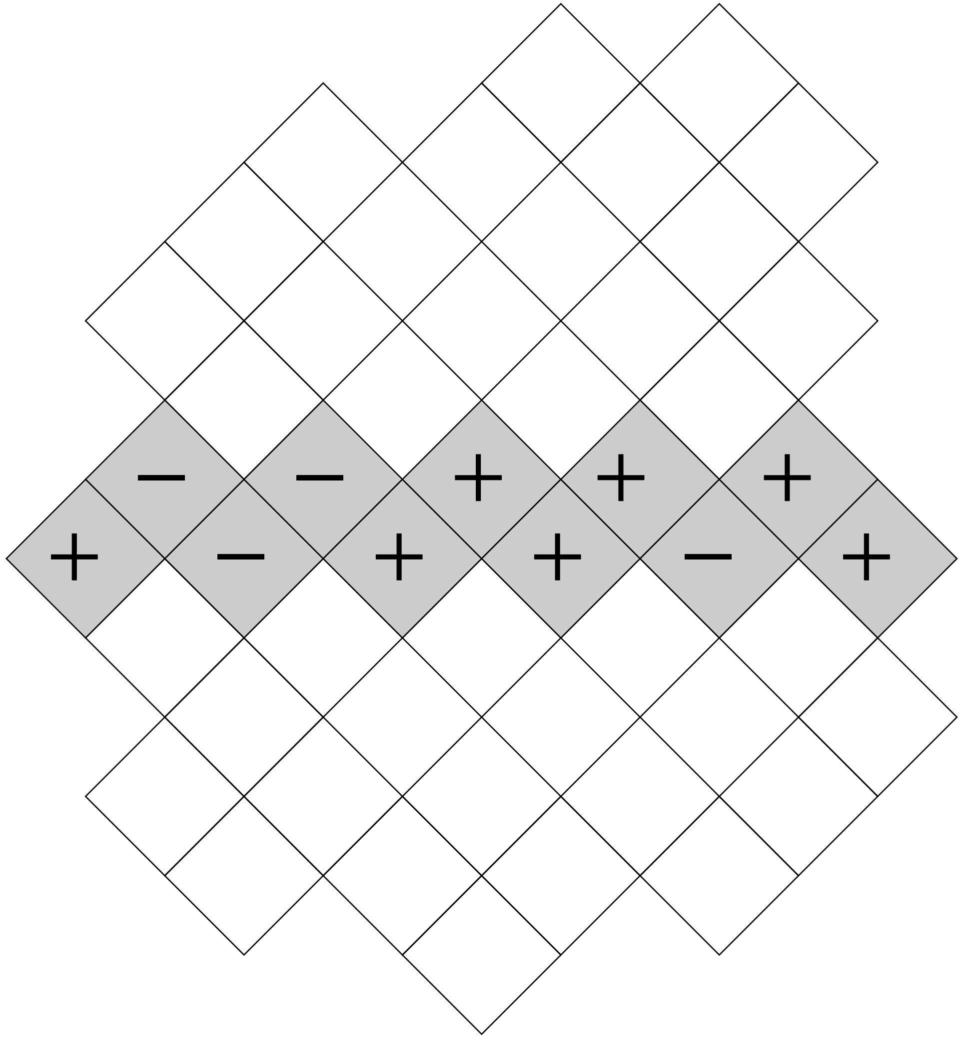


I. What is a local physical theory?

- **Dynamics in action:**



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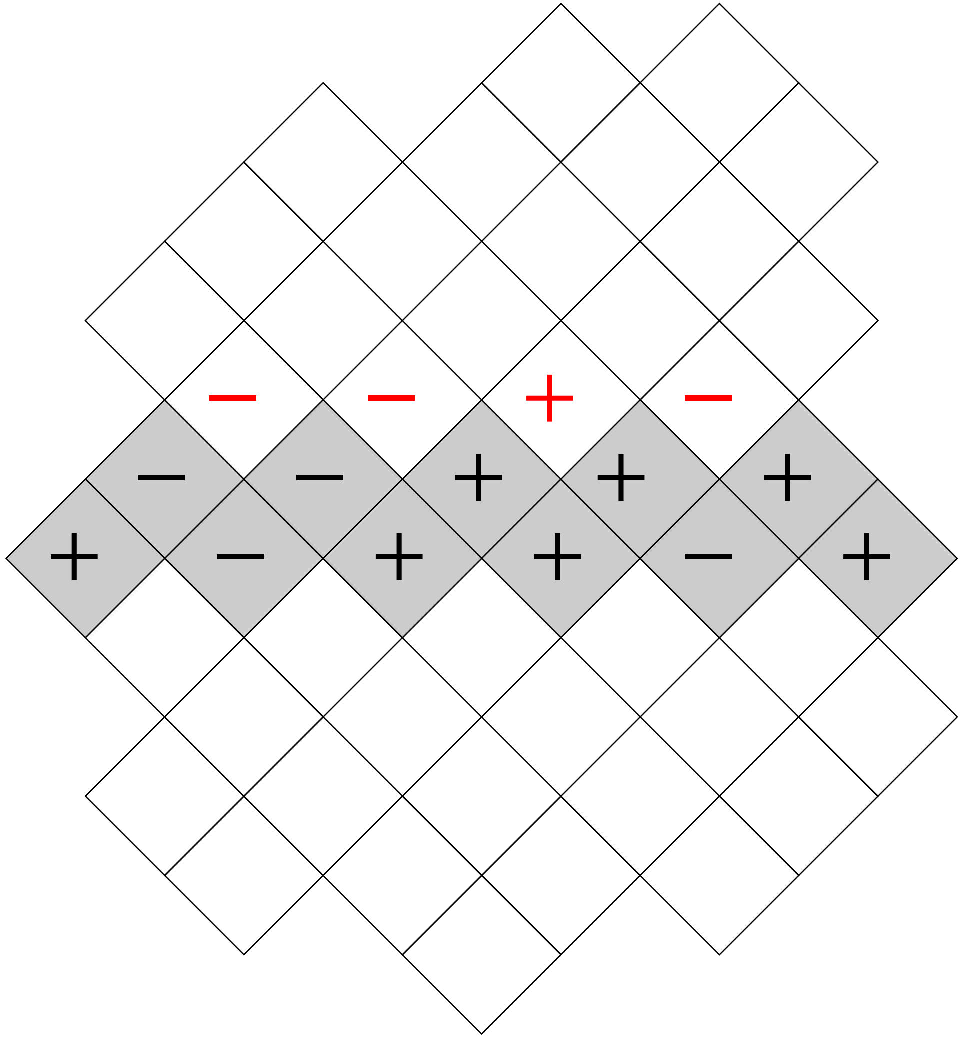

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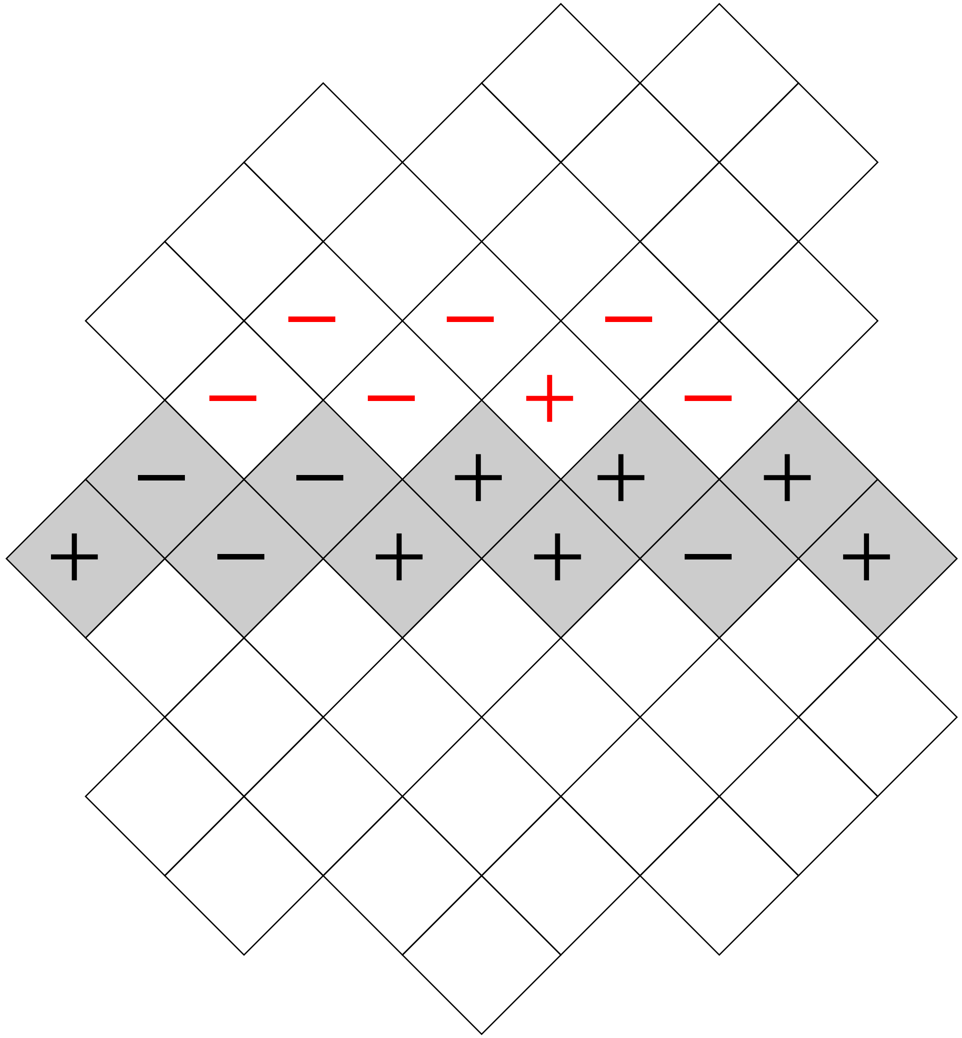

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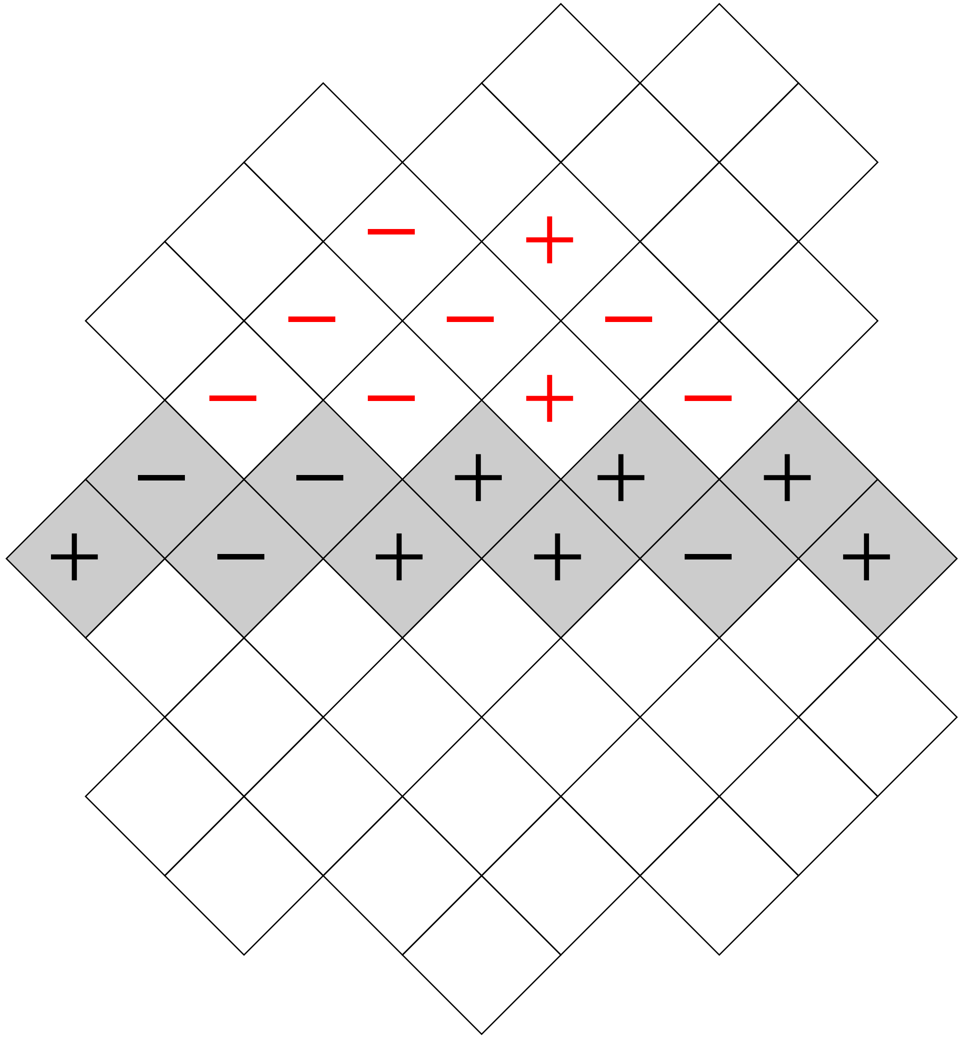

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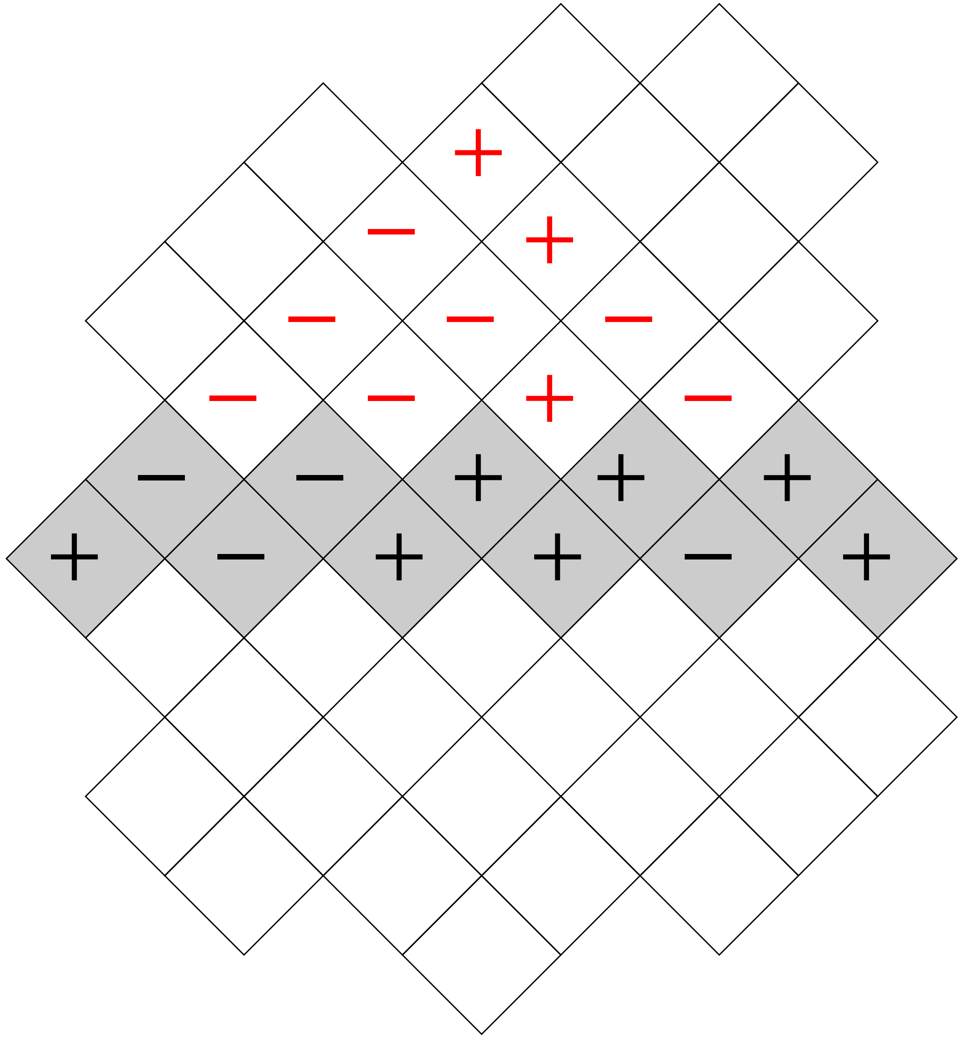
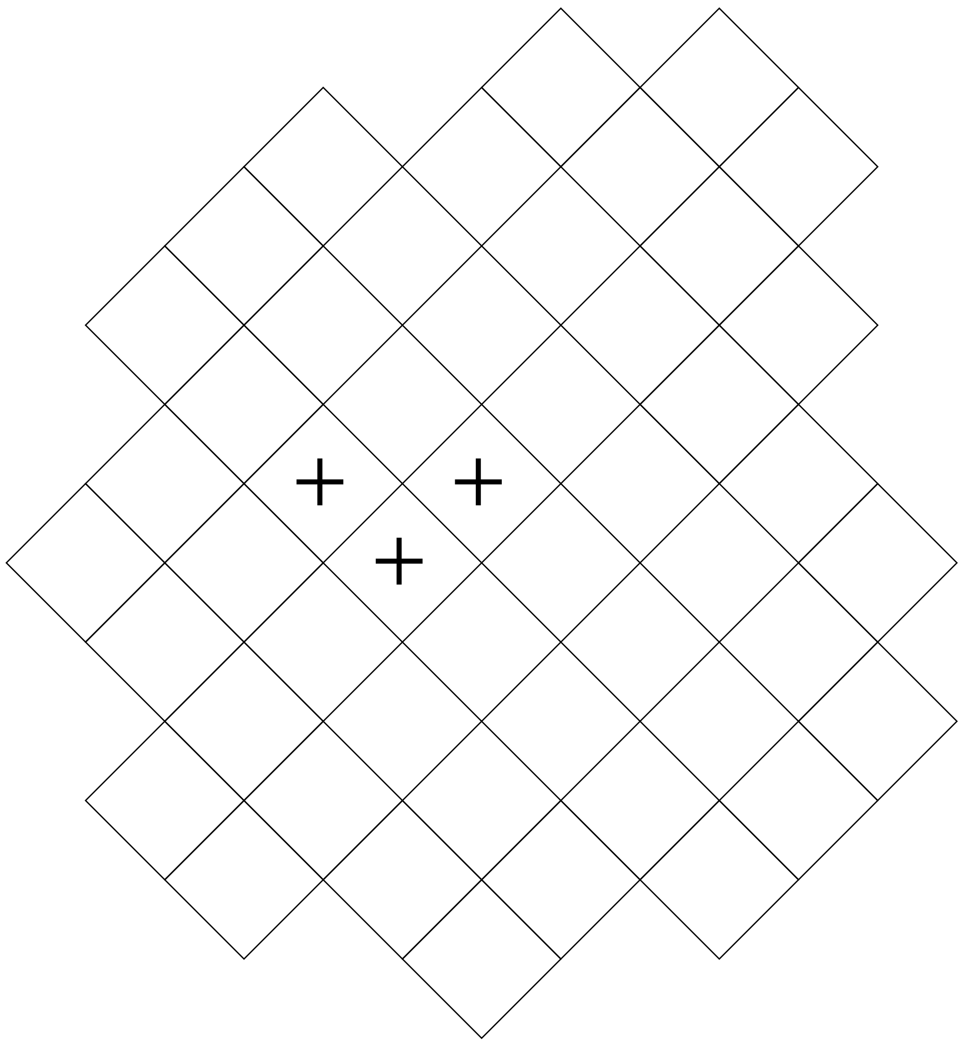


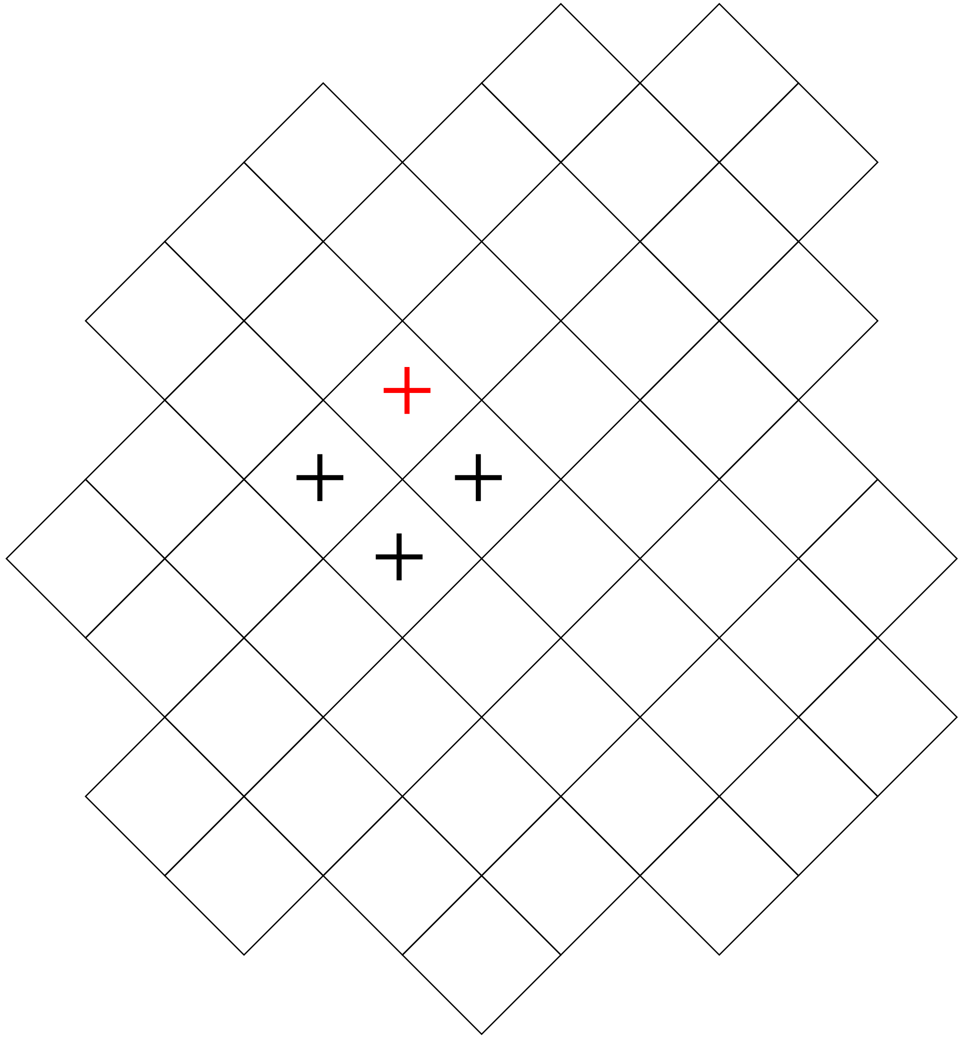
I. What is a local physical theory?

- **Example 2.** **Stochastic LCT**



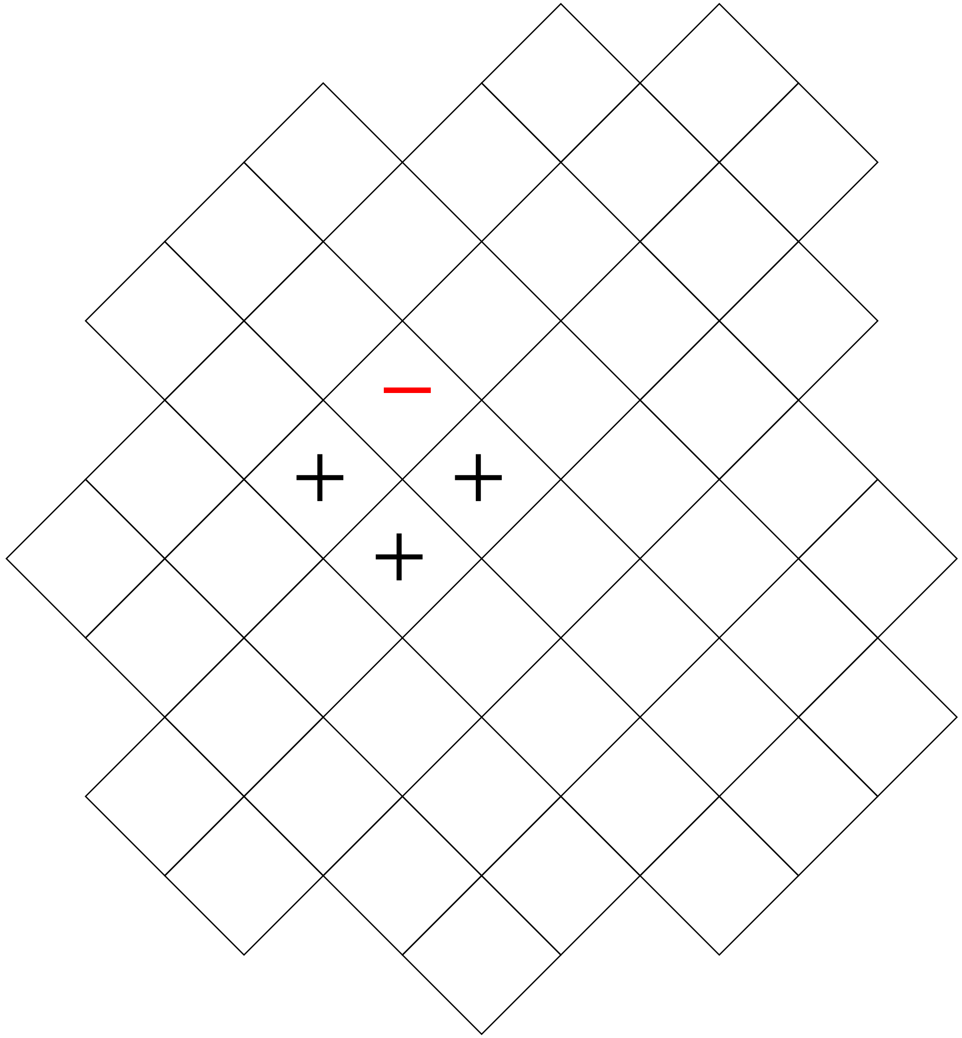
I. What is a local physical theory?

- **Stochastic dynamics:** with probability p



I. What is a local physical theory?

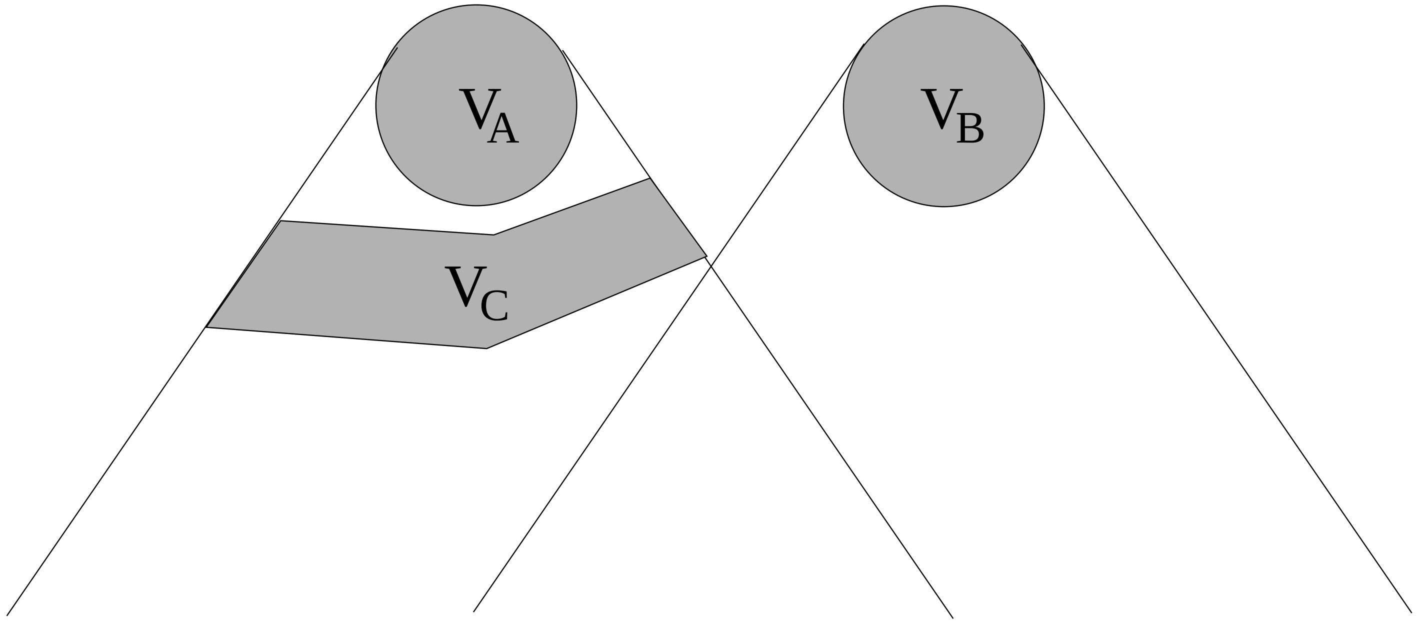
- **Stochastic dynamics:** with probability $1 - p$



II. Bell's local causality in a LPT

II. Bell's local causality

- **Bell, 1990:** "A theory will be said to be **locally causal** if the probabilities attached to values of local beables in a space-time region V_A are unaltered by specification of values of local beables in a space-like separated region V_B, when what happens in the backward light cone of V_A is already sufficiently specified, for example by a full specification of local beables in a space-time region V_C."



II. Bell’s local causality in a LPT

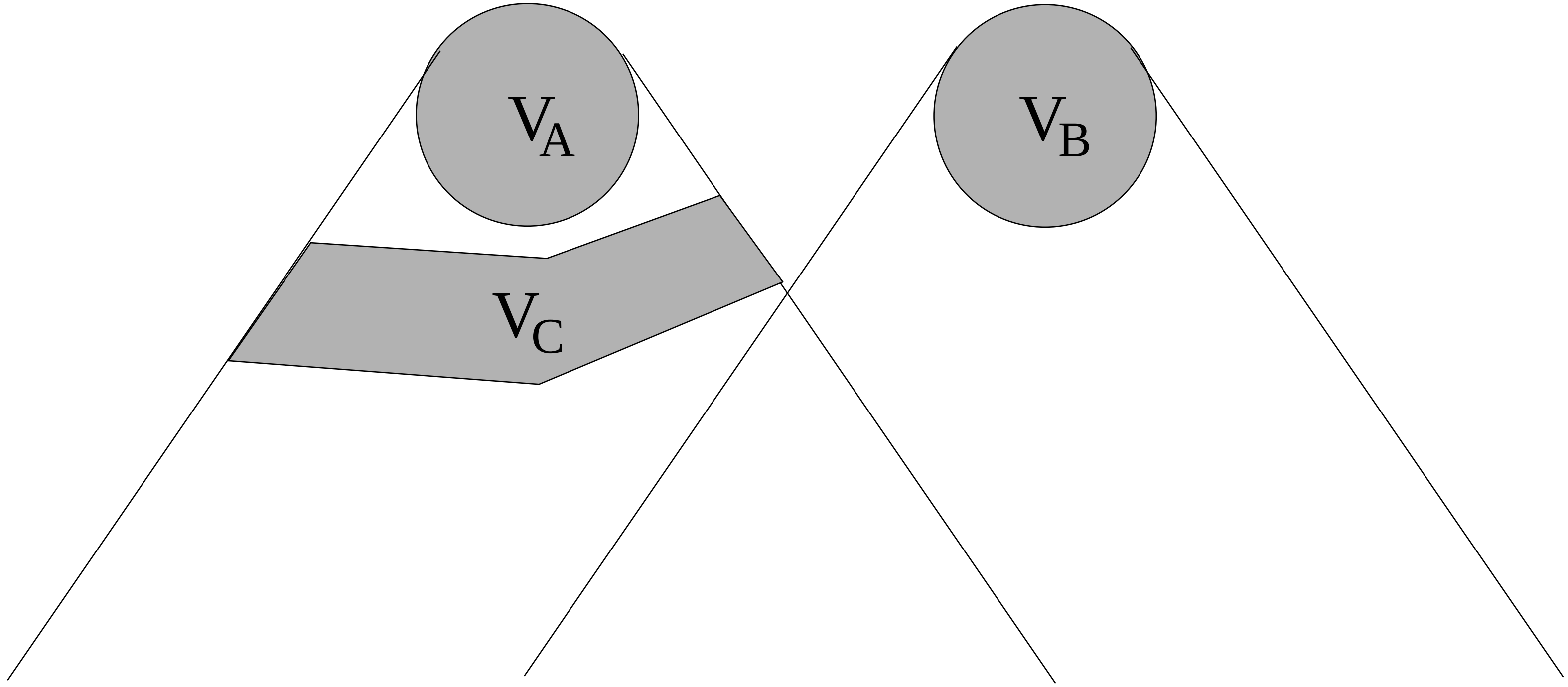
Basic terms:

1. “The ***beables*** of the theory are those entities in it which are, at least tentatively, to be taken seriously, as corresponding to something real.”
2. “there *are* things which **do go faster than light**. British sovereignty is the classical example. When the Queen dies in London (long may it be delayed) the Prince of Wales, lecturing on modern architecture in Australia, becomes instantaneously King.”
3. “***Local* beables** are those which are definitely associated with particular space-time regions. The electric and magnetic fields of classical electro-magnetism, $\mathbf{E}(t,x)$ and $\mathbf{B}(t,x)$ are again examples.”

II. Bell's local causality in a LPT

Basic terms:

4. "It is important that region V_C **completely shields off** from V_A the overlap of the backward light cones of V_A and V_B."



II. Bell’s local causality in a LPT

Basic terms:

5. “And it is important that events in V_C be **specified completely**. Otherwise the traces in region V_B of causes of events in V_A could well supplement whatever else was being used for calculating probabilities about V_A.”

Translation:

- “local beable” $\longrightarrow$ element of a local algebra
- “complete specification” $\longrightarrow$ an atomic element of a local algebra

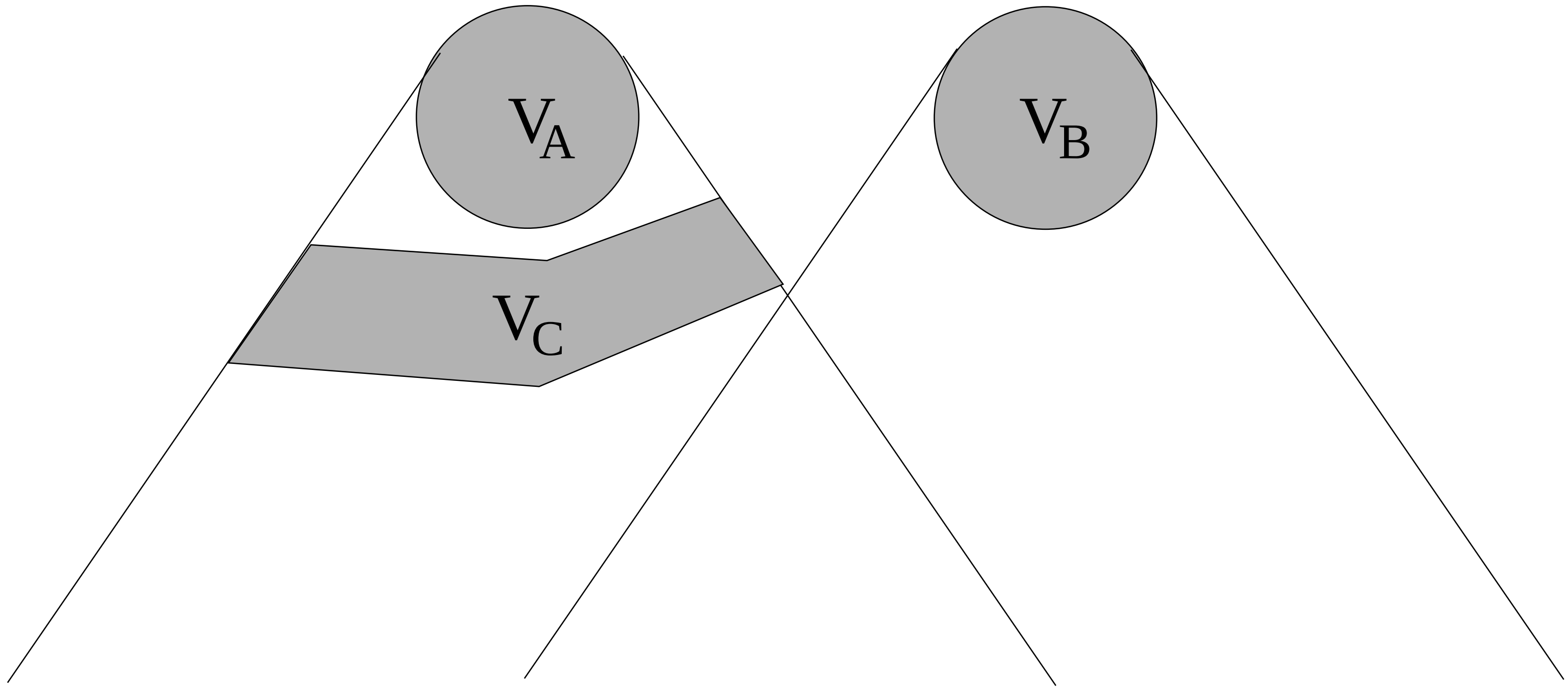
II. Bell's local causality in a LPT

- "shielder-off region" $\longrightarrow$

  L$_1$: $V_C \subset J_-(V_A)$

  L$_2$: $V_A \subset V_C''$

  L$_3$: $V_C \subset V_B'$



II. Bell's local causality in a LPT

- **Definition.** A LPT is called **(Bell) locally causal**, if
  - for any *pair of projections* $A \in \mathcal{N}(V_A)$ and $B \in \mathcal{N}(V_B)$ supported in spacelike separated regions, and
  - for every locally normal and faithful *state* ϕ establishing a correlation between A and B, $\phi(AB) \neq \phi(A)\phi(B)$, and
  - for any *spacetime region* V_C satisfying **L**$_1$- **L**$_3$, and
  - for any *atomic event* C_k in $\mathcal{N}(V_C)$:

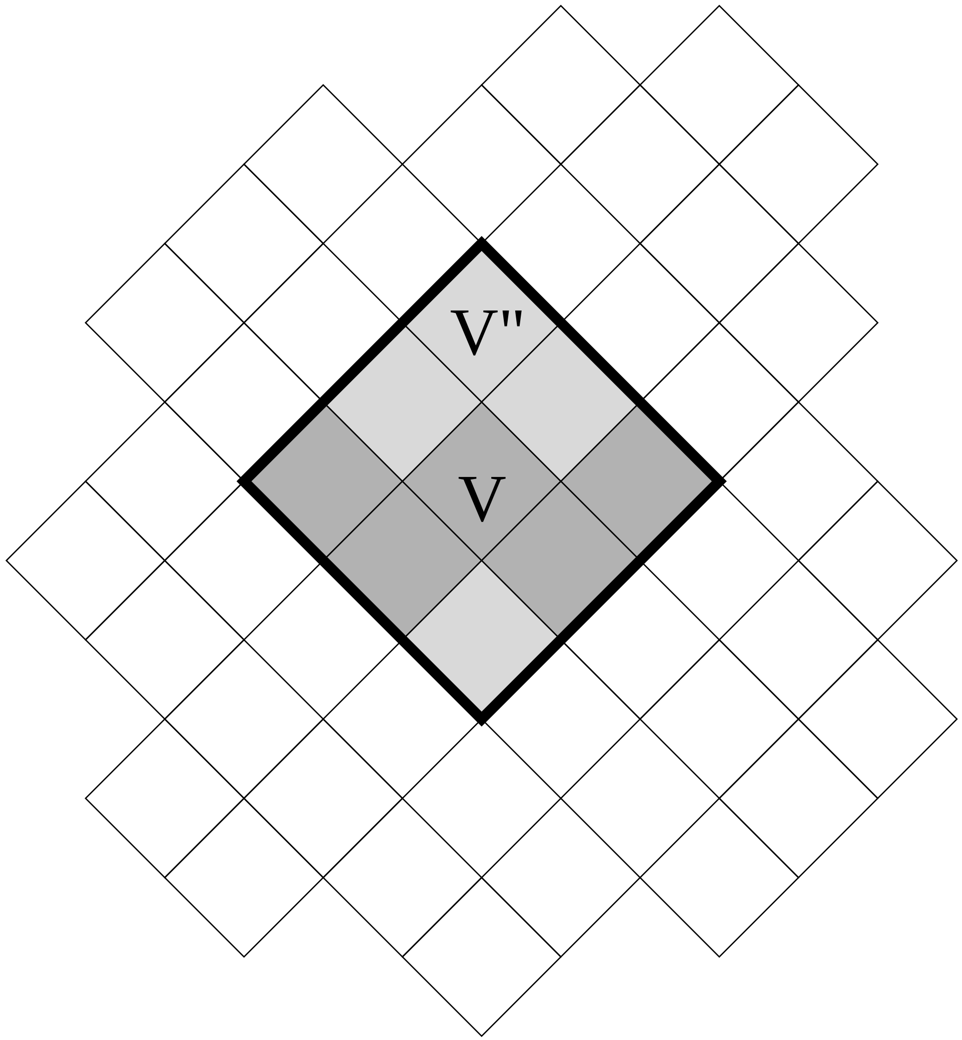
$$\frac{\phi(C_kABC_k)}{\phi(C_k)} = \frac{\phi(C_kAC_k)}{\phi(C_k)}\frac{\phi(C_kBC_k)}{\phi(C_k)}$$

III. Other locality and causality concepts

a. Local primitive causality
b. Common Cause Principle
c. Causal Markov Condition

III. Other locality and causality concepts

- **Local primitive causality:** $\mathcal{N}(V) = \mathcal{N}(V'')$ for any $V \in \mathcal{K}$



III. Other locality and causality concepts

Proposition:

- Any **atomic** LPT satisfying **local primitive causality** is locally causal.

III. Other locality and causality concepts

- **Reichenbach's Common Cause Principle:** If there is a correlation between two events and there is no direct causal (or logical) connection between them, then there always exists a common cause of the correlation.

III. Other locality and causality concepts

- **Correlation:** $\phi(AB) \neq \phi(A)\phi(B)$
- **Common cause:** partition $\{C_k\}_{k \in K}$ of the unit

$$\frac{\phi(C_k ABC_k)}{\phi(C_k)} = \frac{\phi(C_k AC_k)}{\phi(C_k)} \frac{\phi(C_k BC_k)}{\phi(C_k)}$$

III. Other locality and causality concepts

- **Correlation:** $\phi(AB) \neq \phi(A)\phi(B)$
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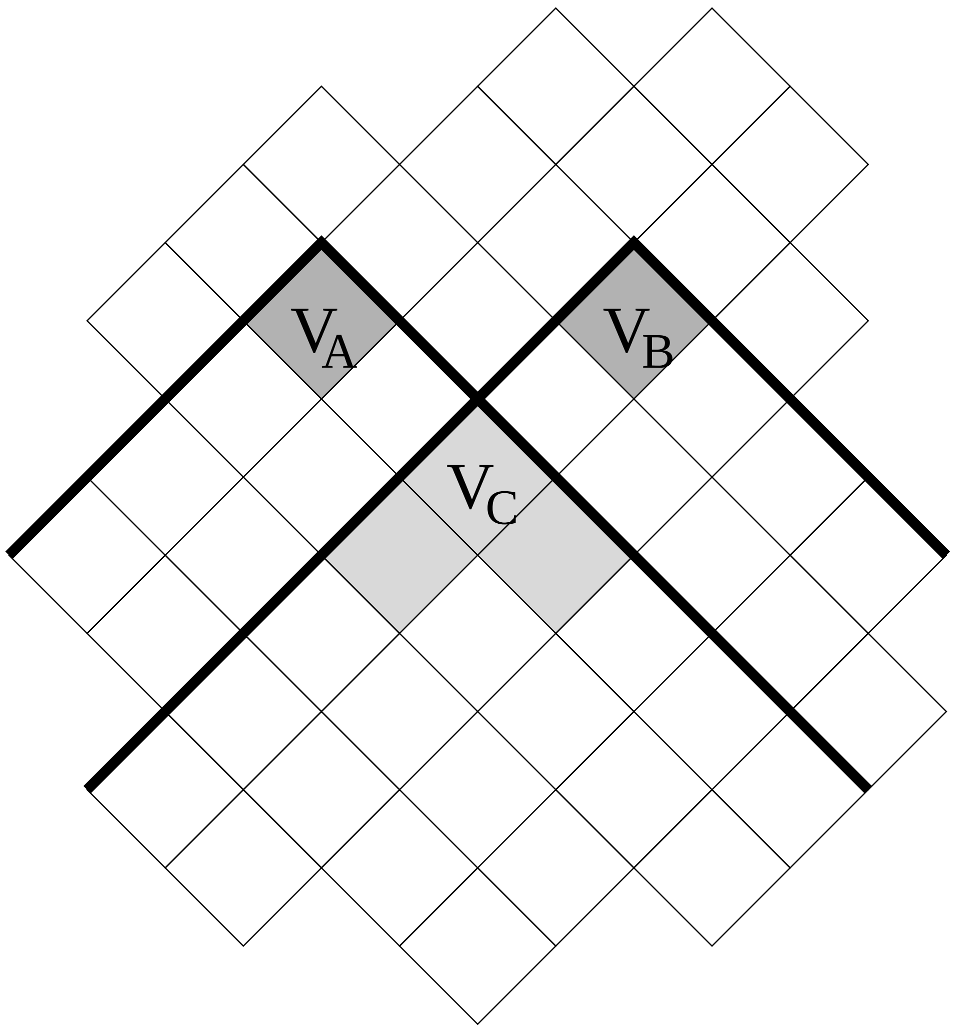
$$\frac{\phi(C_k ABC_k)}{\phi(C_k)} = \frac{\phi(C_k AC_k)}{\phi(C_k)} \frac{\phi(C_k BC_k)}{\phi(C_k)}$$

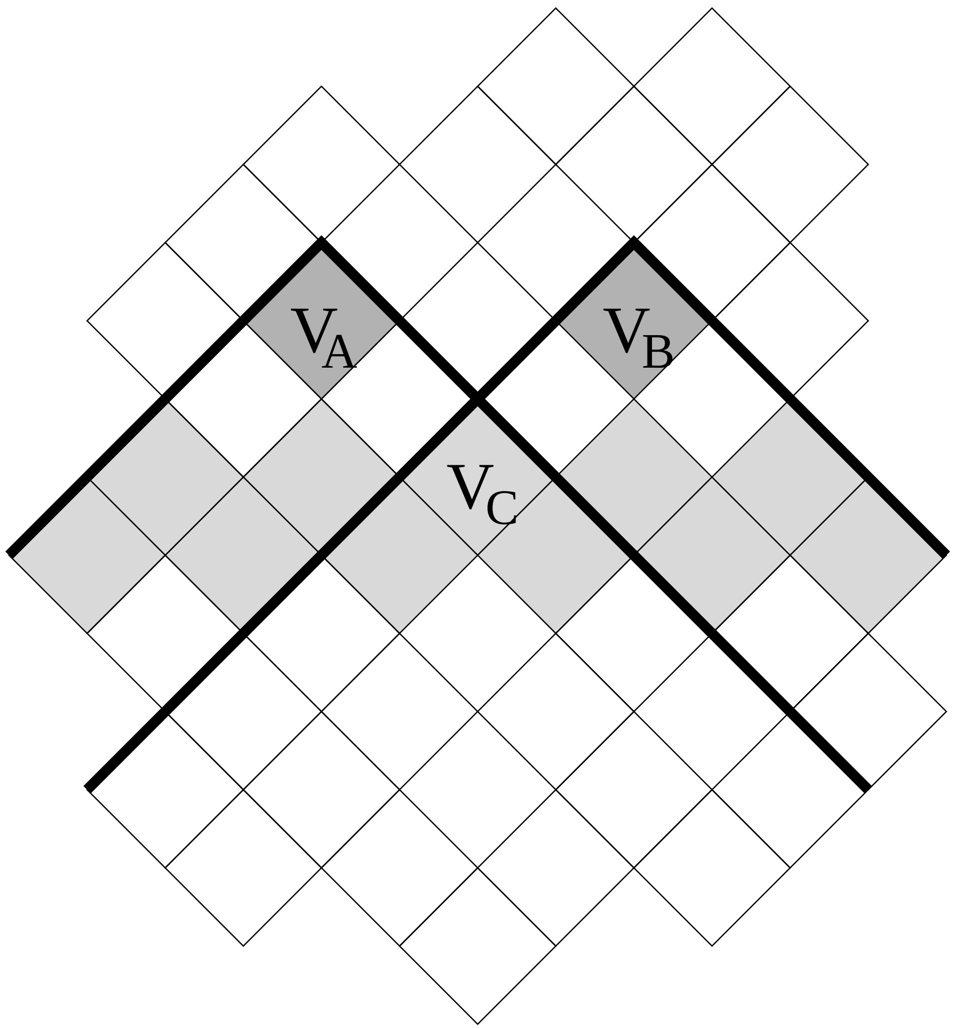
  - **Commuting / Noncommuting common cause:** $\{C_k\}_{k\in K}$ is commuting / not commuting with A and B

III. Other locality and causality concepts

- **Common Cause Principle:**

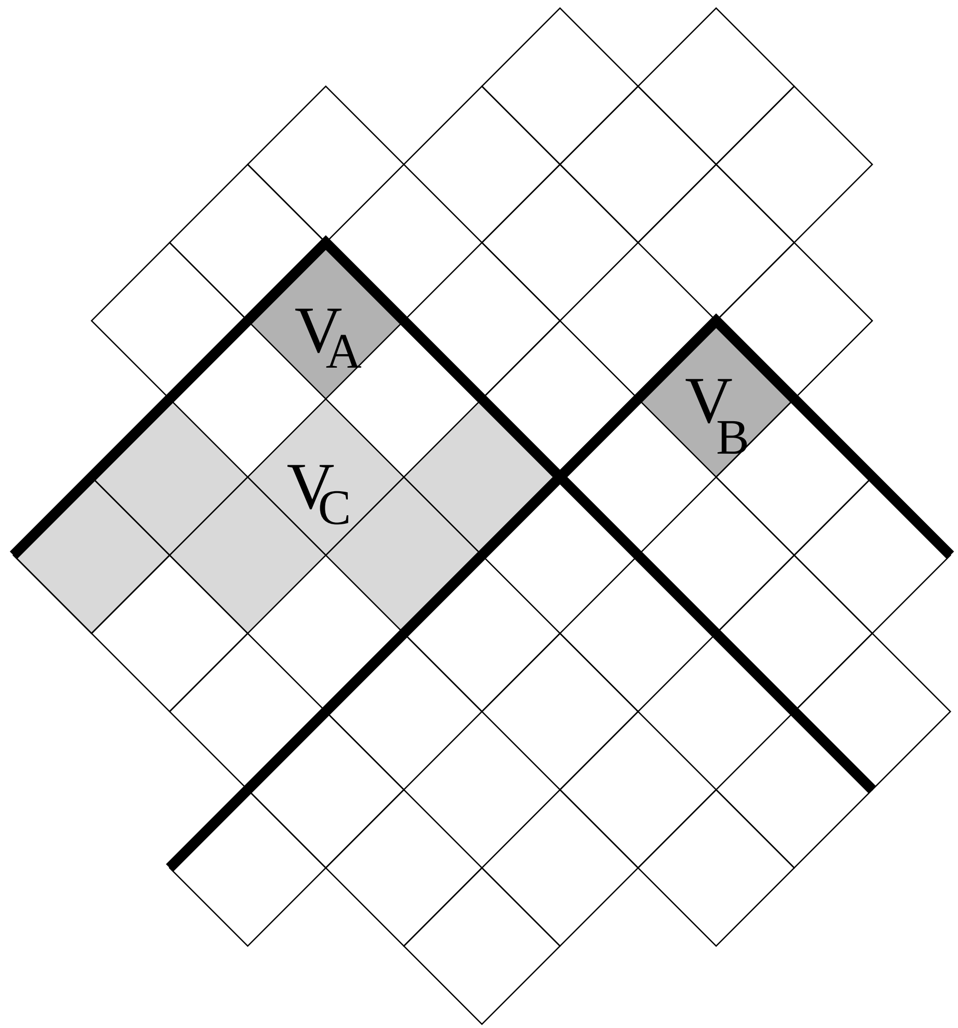


- **Weak Common Cause Principle:**

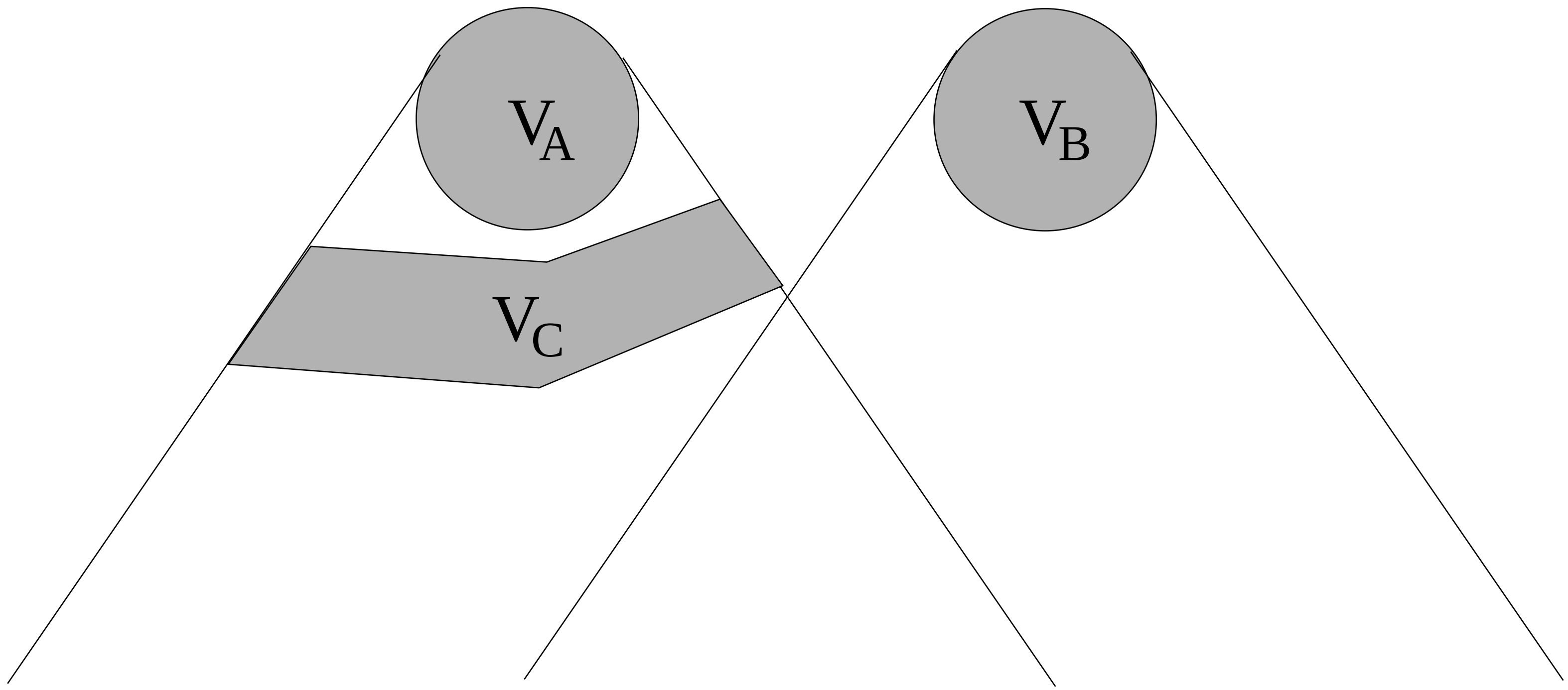


III. Other locality and causality concepts

- **Local causality:**

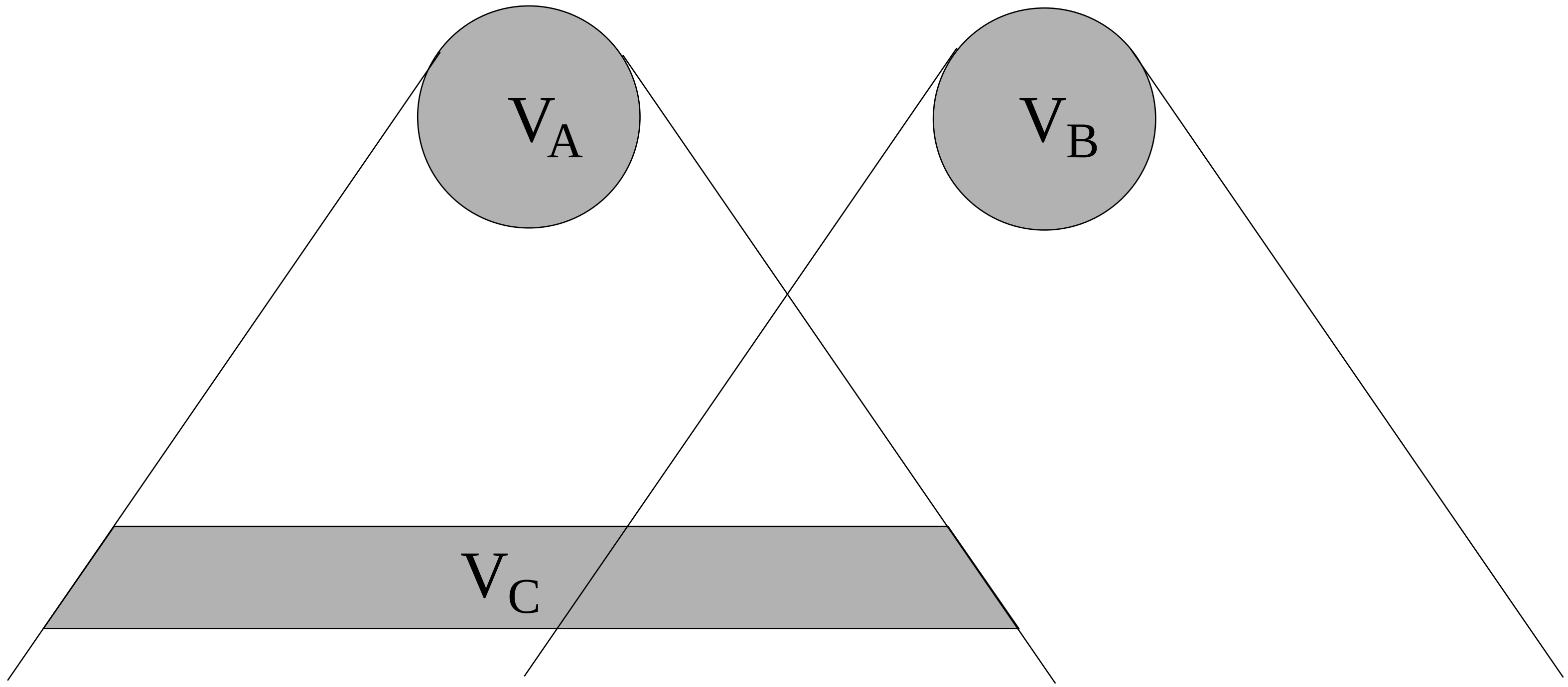


- **Localization of V_C**



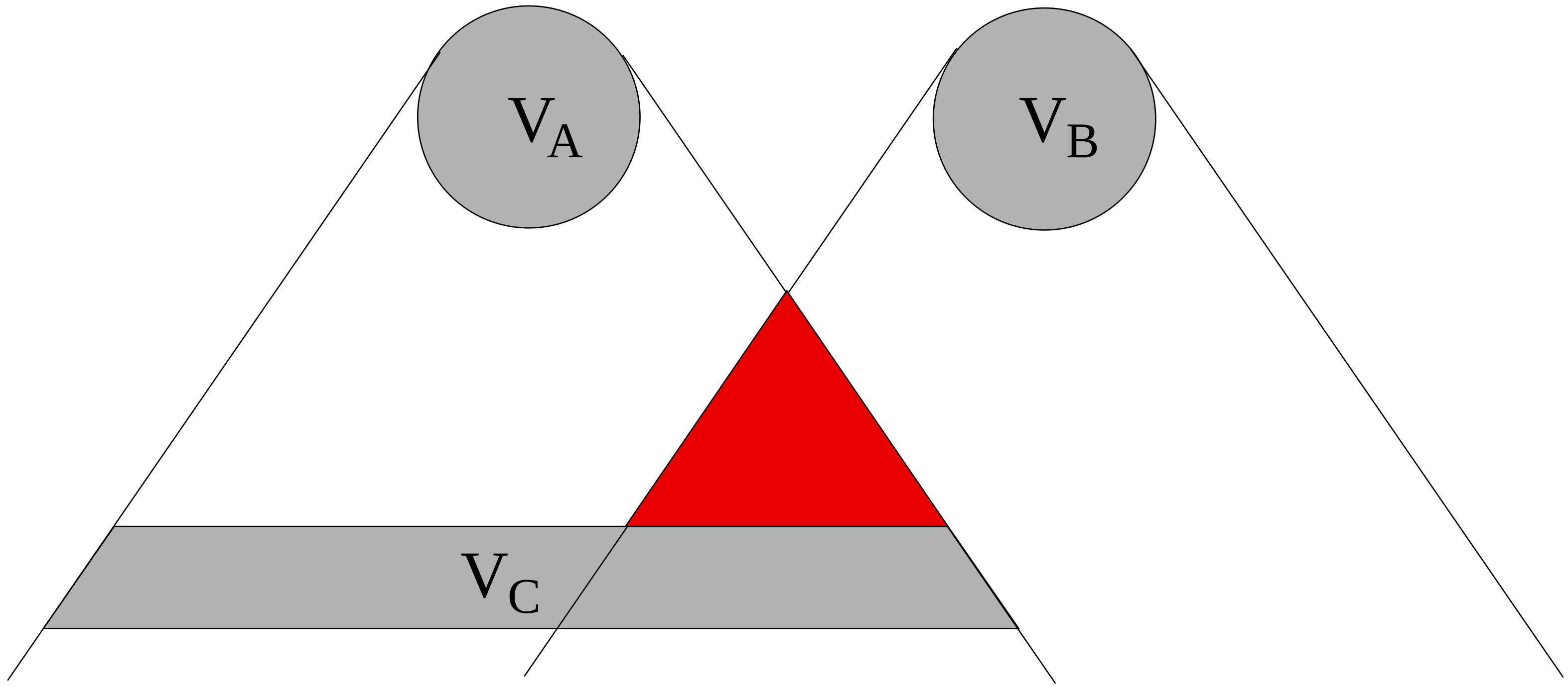
III. Other locality and causality concepts

- **Localization of V_C**



III. Other locality and causality concepts

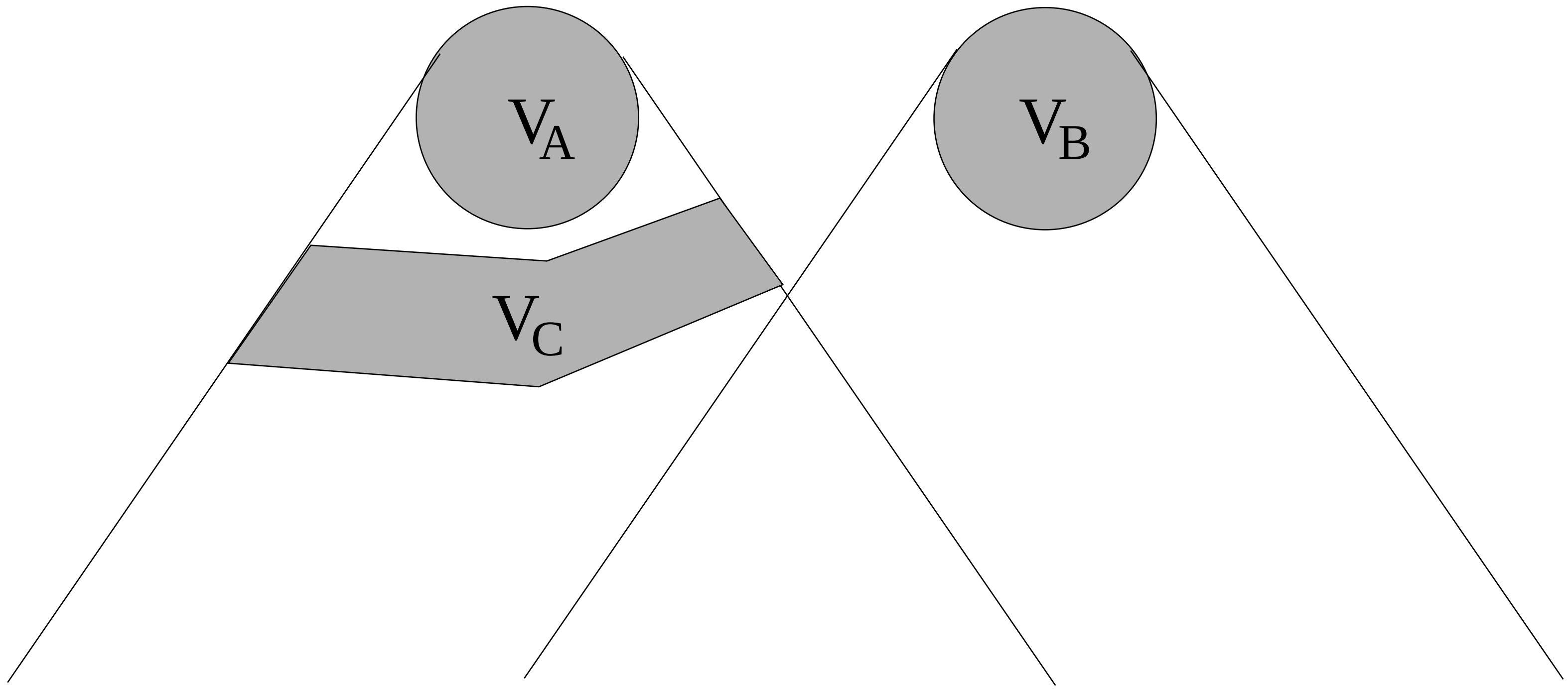
- **Localization of V_C**



III. Other locality and causality concepts

Proposition:

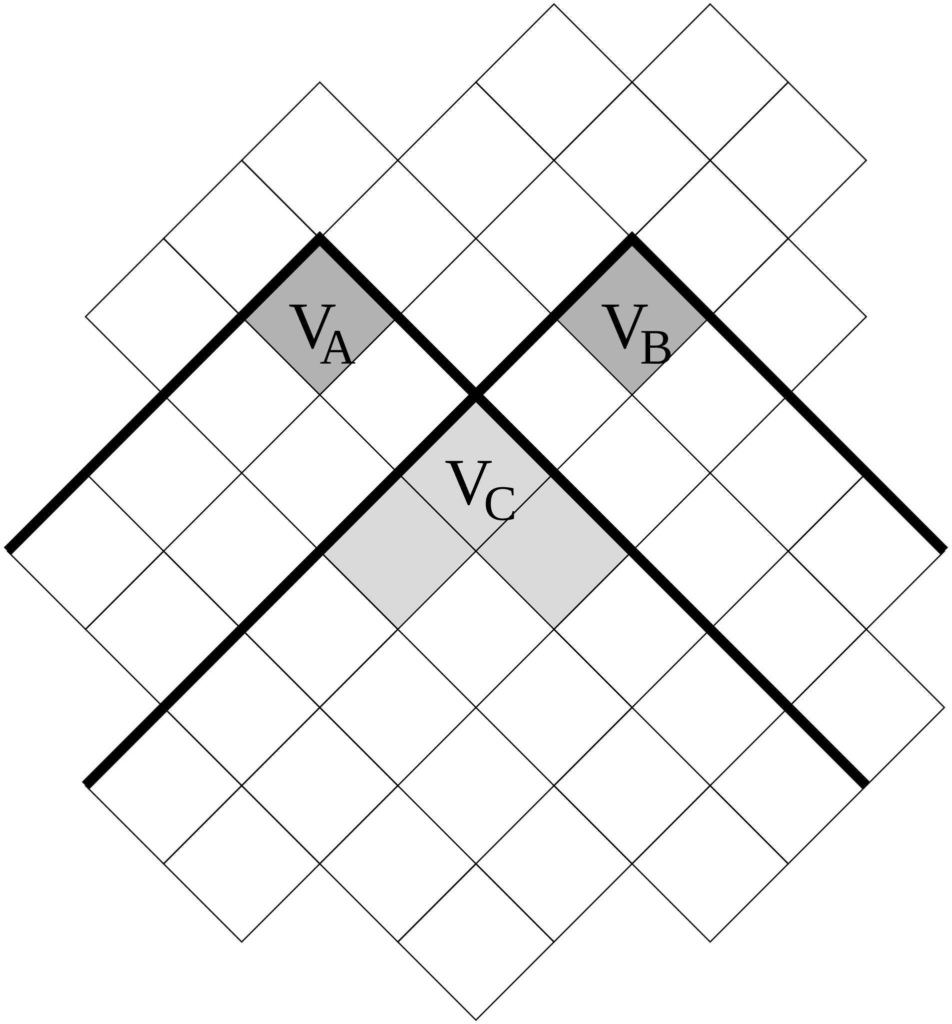
- A covering collection gives rise to a causal set.
  Any shielder-off region is a d-separating set.



IV. Bell inequalities

IV. Bell inequalities

- **Set of correlations** $\longrightarrow$ **Joint common cause**



III. Local causality and the Bell inequalities

- A nice **parallelism**:

$$\begin{array}{rcl} \text{Local causality} & \Longrightarrow & \text{Bell inequalities} \\ \text{Common Cause Principle} & \Longrightarrow & \text{Bell inequalities} \end{array}$$

IV. Bell inequalities

Proposition:

- Joint common cause $\not\Longrightarrow$ Bell inequalities
- Joint common cause + **commutativity** $\Longrightarrow$ Bell inequalities

IV. Bell inequalities

Proposition:

- Local causality $\not\Rightarrow$ Bell inequalities
- Local causality + **commutativity** $\Longrightarrow$ Bell inequalities

Conclusions

- Bell's notion of local causality presupposes a clear-cut framework integrating probabilistic and spatiotemporal entities. This goal can be met by introducing the notion of a LPT.
- In this general framework one can define Bell's notion of local causality and show sufficient conditions on which a LPT will be locally causal.
- There is a nice parallelism between local causality and the CCPs: Bell's inequalities cannot be derived from neither unless the LPT is classical or the common cause is commuting.

References

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II. Bell's local causality in a LPT

- **Complete specification** of beables in a region $V \in \mathcal{K}$:

$$\phi(X) \mapsto \phi_{\mathcal{T}}(X) := \frac{\phi \circ \mathcal{T}}{(\phi \circ \mathcal{T})(\mathbf{1})}$$

by a completely positive map $\mathcal{T}$ on the quasilocal observables obeying the following properties:

P$_1$: the restriction of $\phi_{\mathcal{T}}$ to the local algebra $\mathcal{N}(V)$ is pure,

P$_2$: $B\mathcal{T}(\mathbf{1}) = \mathcal{T}(B) = \mathcal{T}(\mathbf{1})B$ hold for local observables B supported in V'.

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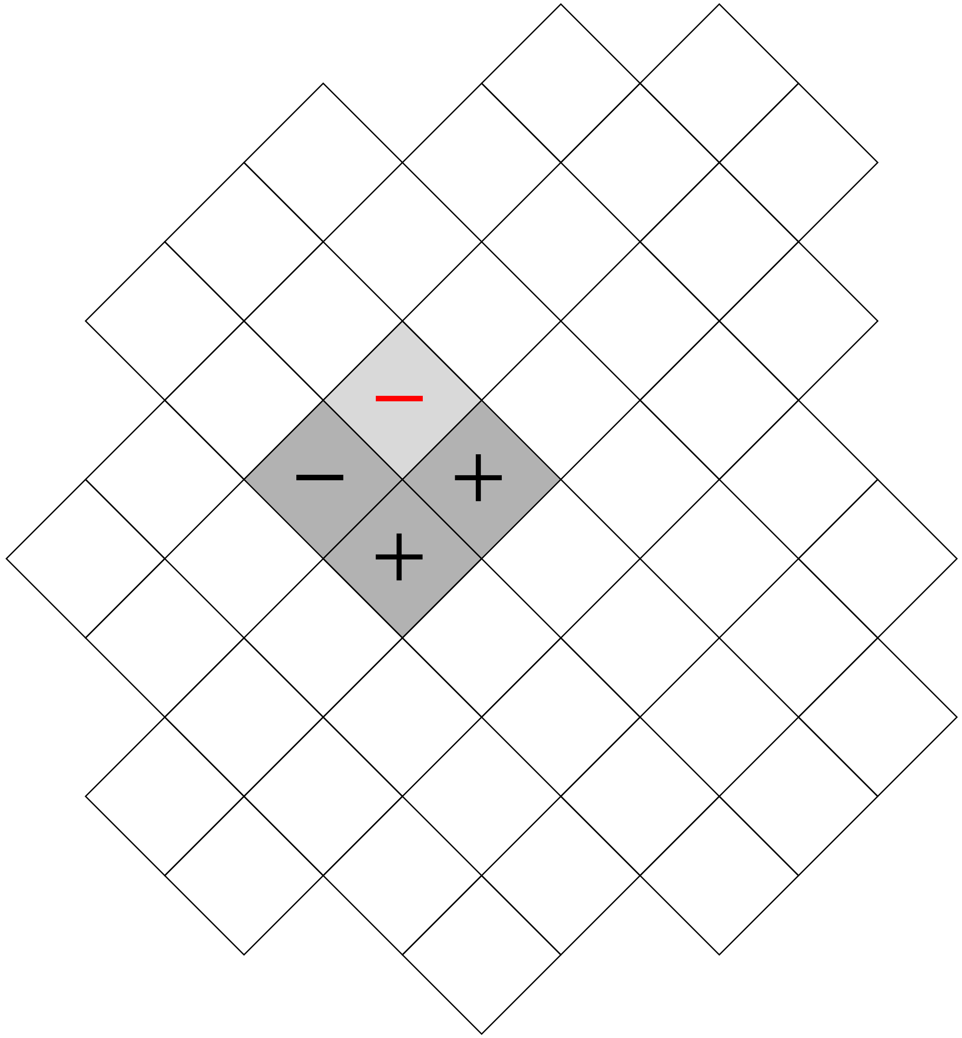
  - for any *spacetime region* V_C satisfying **L**$_1$- **L**$_3$, and
  - for any *operation* $\mathcal{T}$ obeying **P**$_1$ and **P**$_2$

$$\phi_{\mathcal{T}}(AB) = \phi_{\mathcal{T}}(A)\phi_{\mathcal{T}}(B).$$

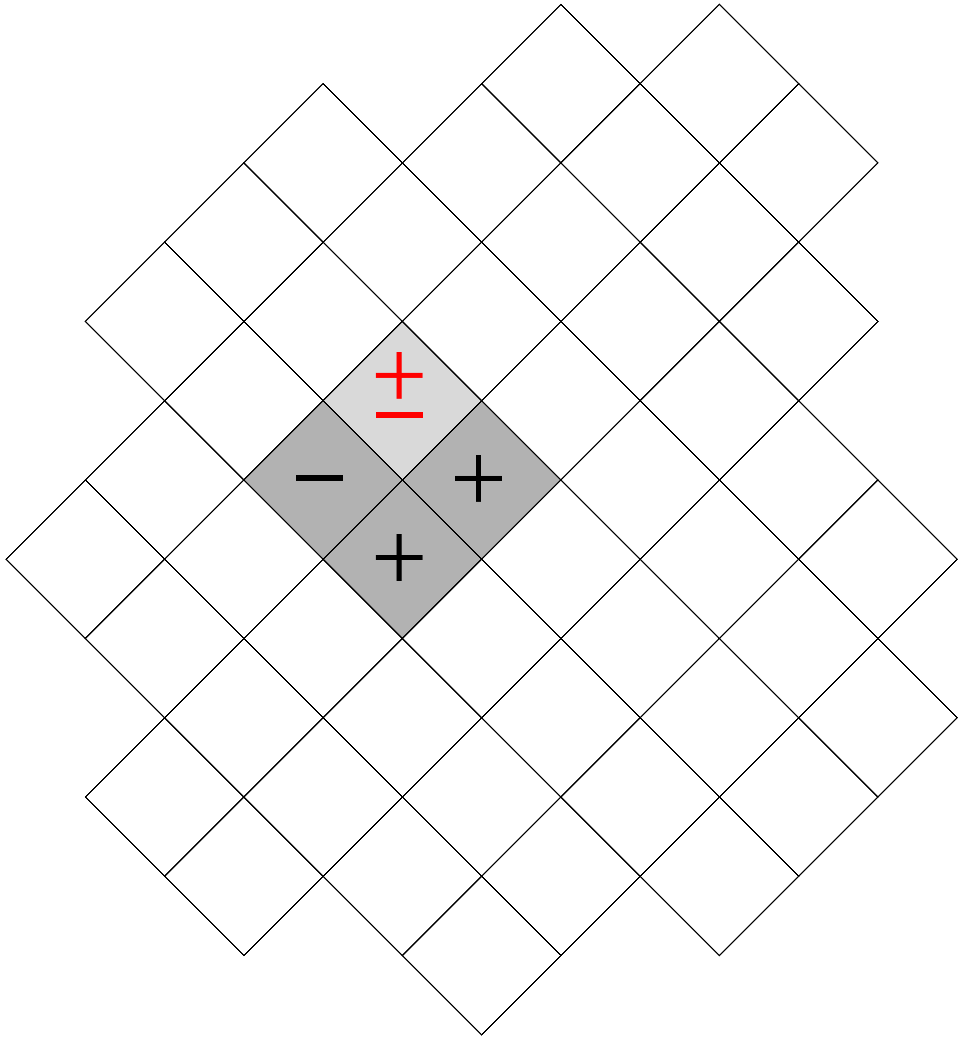
III. Other locality and causality concepts

- **Local primitive causality:** holds in deterministic LCTs



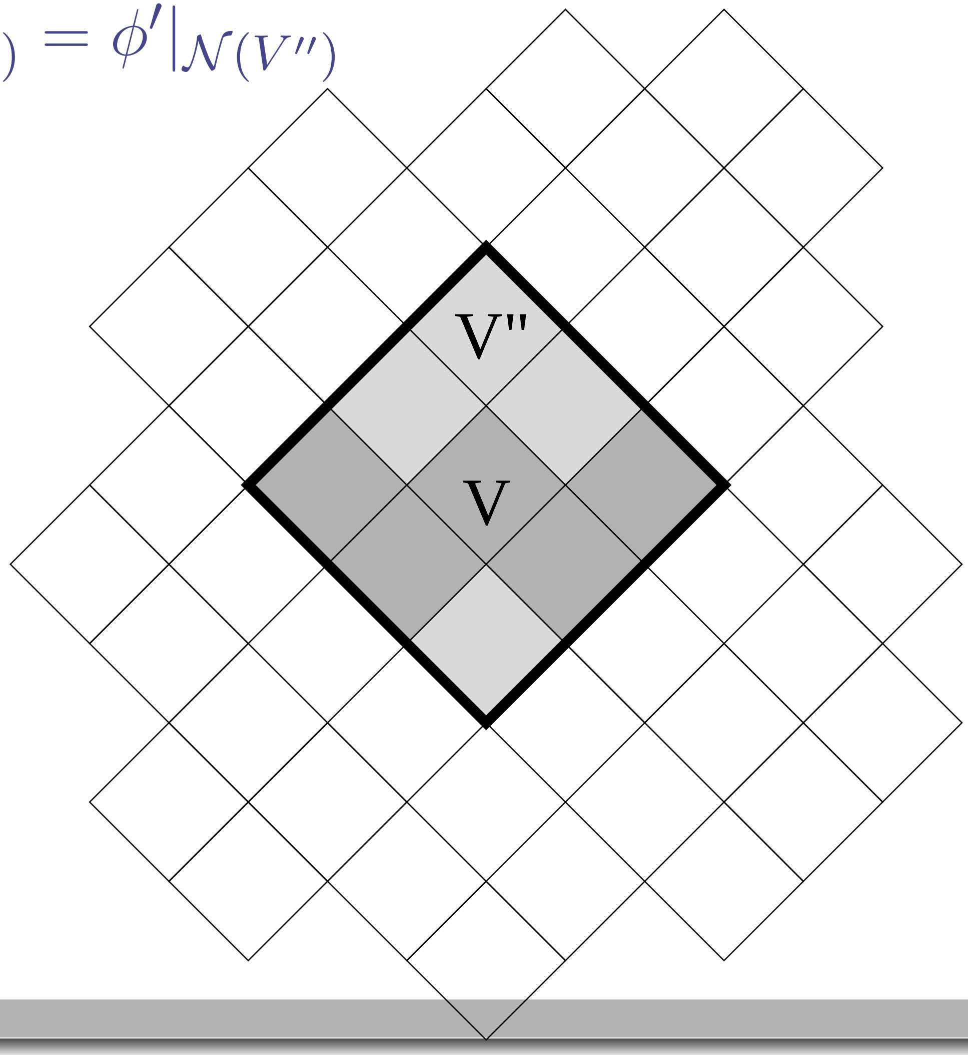
III. Other locality and causality concepts

- **Local primitive causality:** does not hold in stochastic LCTs



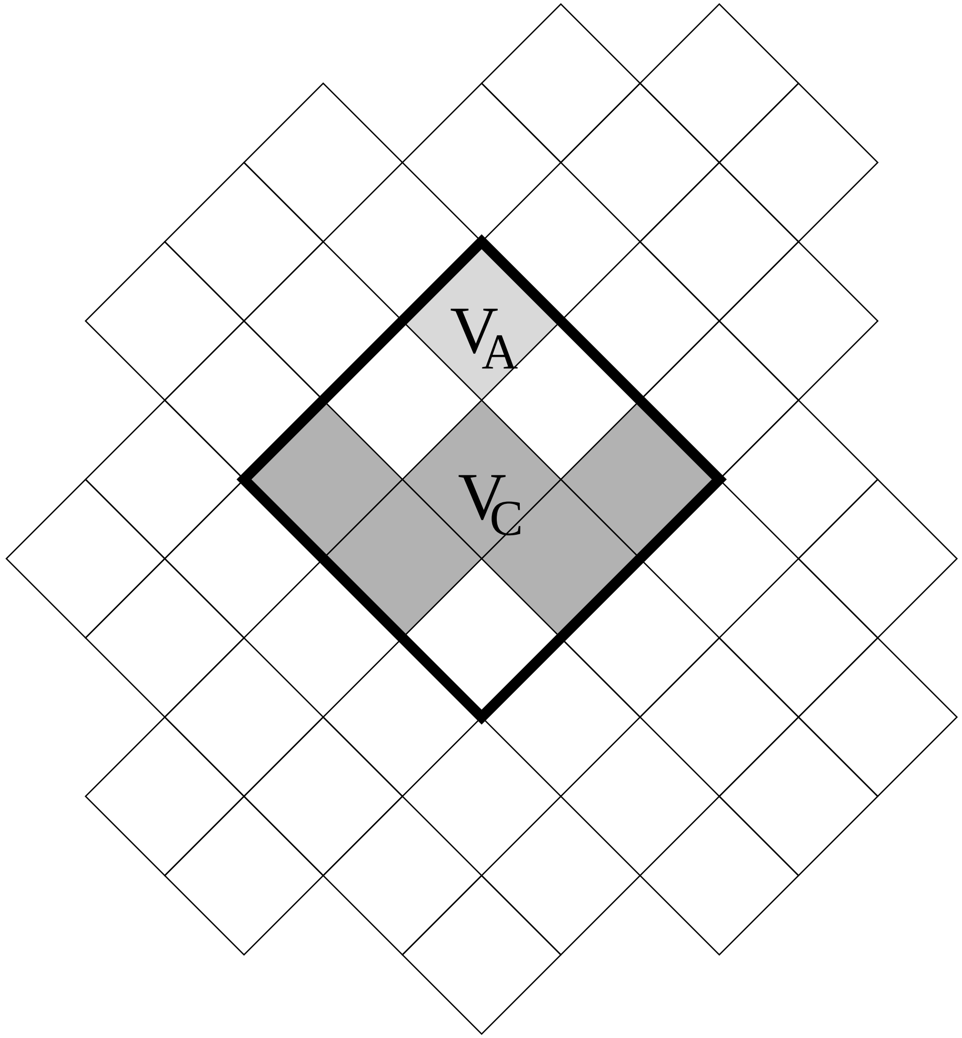
III. Other locality and causality concepts

- **Local primitive causality** implies **local determinism:** for any two states ϕ and ϕ' and $V \in \mathcal{K}$, if $\phi|_{\mathcal{N}(V)} = \phi'|_{\mathcal{N}(V)}$ then $\phi|_{\mathcal{N}(V'')} = \phi'|_{\mathcal{N}(V'')}$



III. Other locality and causality concepts

- . . . which further implies **stochastic Einstein locality**



III. Other locality and causality concepts

Remarks:

- Local primitive causality is a **dependence relation**.
  Local causality is an **independence relation**.
- Local primitive causality **does not rely on the notion of state**, it is exclusively a property of the net.
  Local causality **does depend on the state**.

III. Other locality and causality concepts

Similarities:

1. Both local causality and the CCPs are **properties of a LPT** represented by a net $\{\mathcal{N}(V), V \in \mathcal{K}\}$.
2. The core mathematical requirement of both principles is the **screening-off condition**:

$$\frac{\phi(C_k ABC_k)}{\phi(C_k)} = \frac{\phi(C_k AC_k)}{\phi(C_k)} \frac{\phi(C_k BC_k)}{\phi(C_k)}$$

3. **Bell's inequalities** can be derived from both principles. (But see below.)

III. Other locality and causality concepts

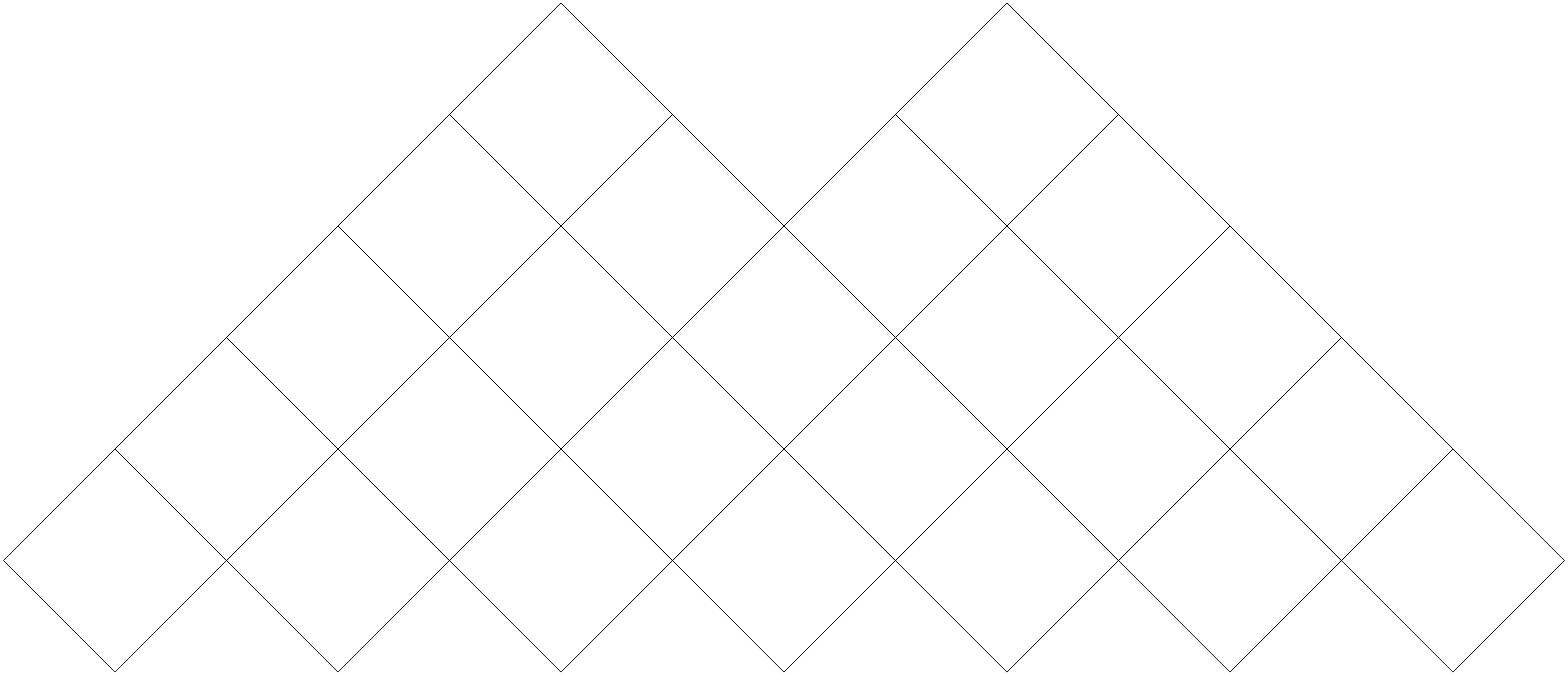
Differences:

1. For local causality the screening-off condition is required for **every** atomic event. For the CCPs it is required only for events of **one** partition.
2. For local causality the screening-off condition is required only for **atomic** events. For the CCPs one is looking for **nontrivial** common causes.
3. For local causality screener-offs are localized '**asymmetrically**' in the past of V_A (or V_B). For the CCP they are localized '**symmetrically**' in the joint / common past of V_A and V_B.

III. Other locality and causality concepts

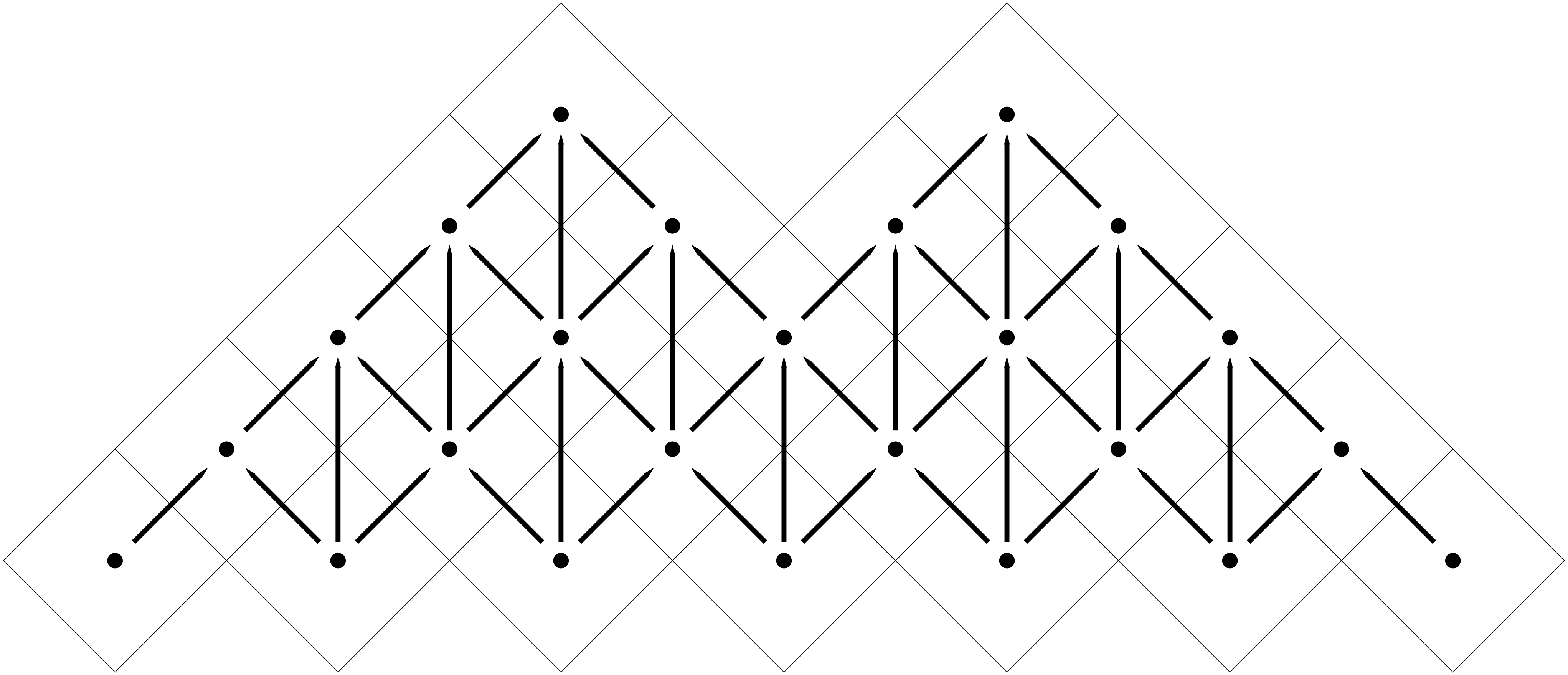
Stochastic LCT	**Theory of causal graphs**
covering collection $\mathcal{K}$	causal graph $\mathcal{G}$
spacetime regions	vertices
...	arrows
...	parents
...	descendants
shielder-off region	d-separating set

- **Covering collection $\mathcal{K}$**



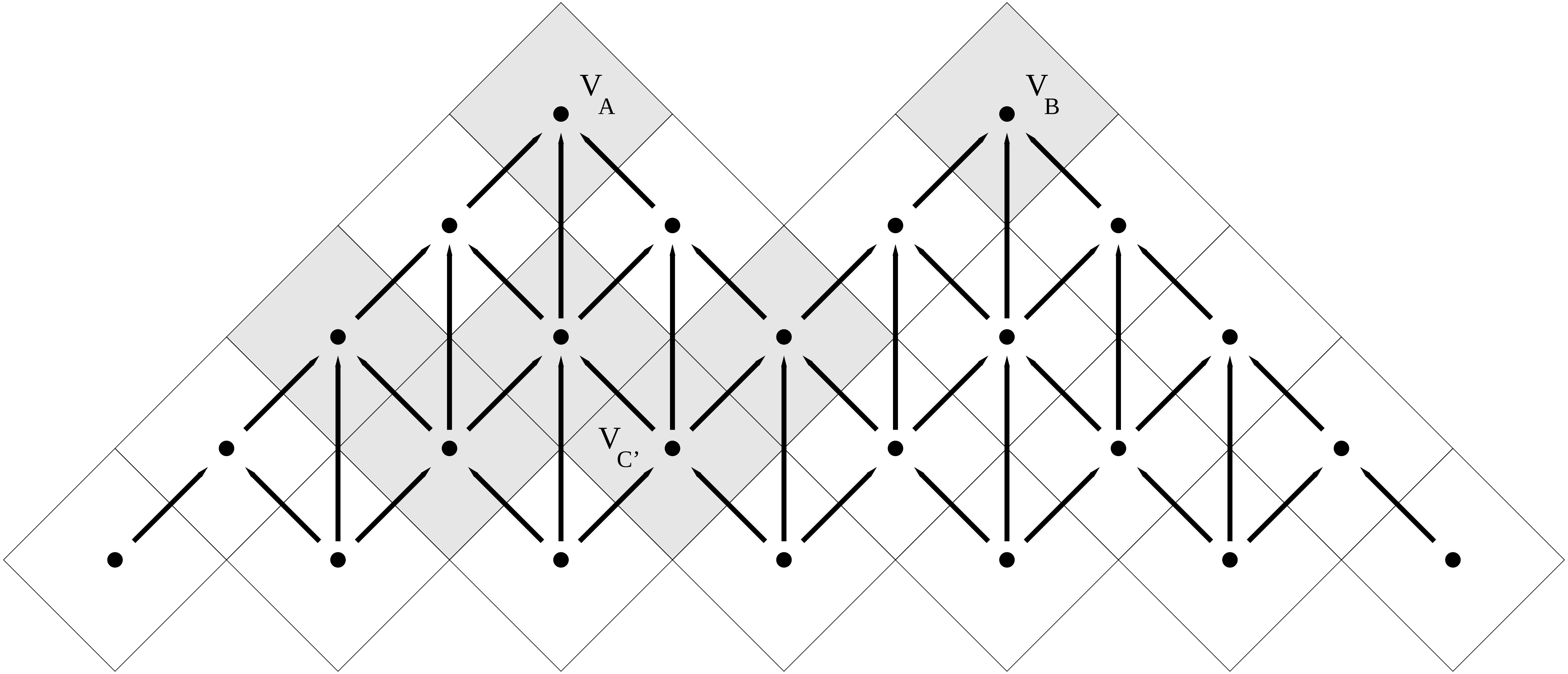
III. Other locality and causality concepts

- **Causal graph** $\mathcal{G}$

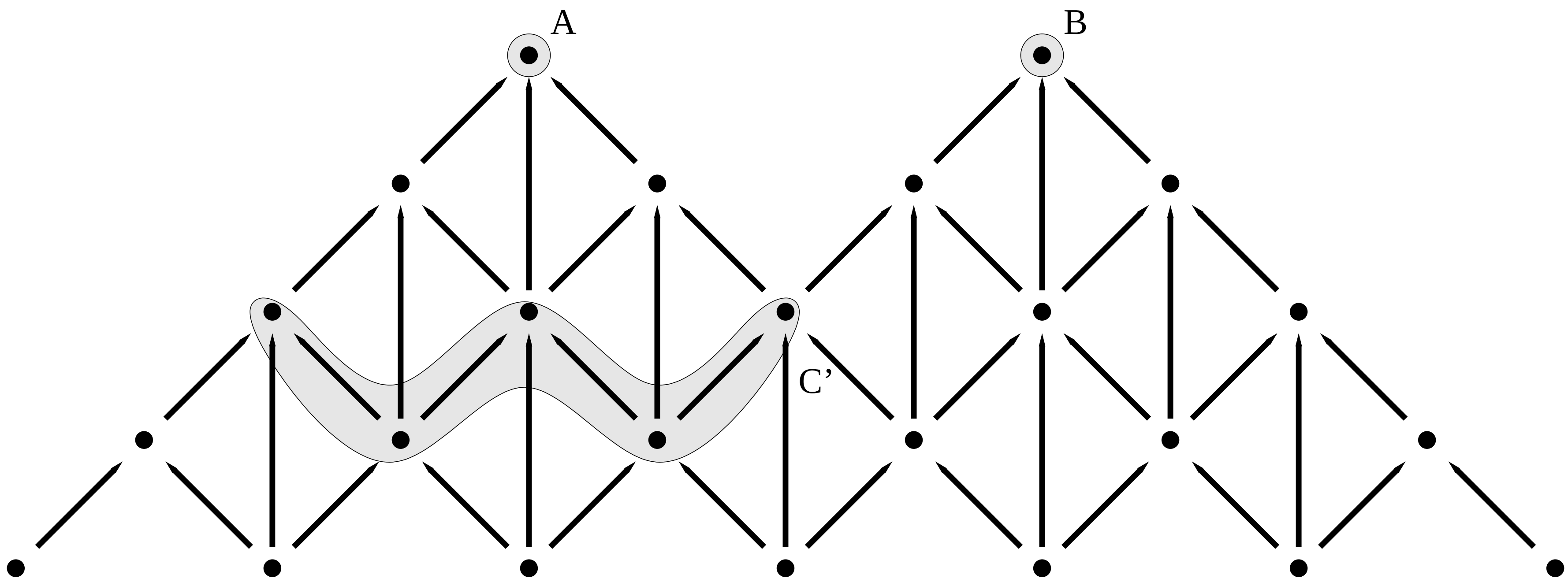


III. Other locality and causality concepts

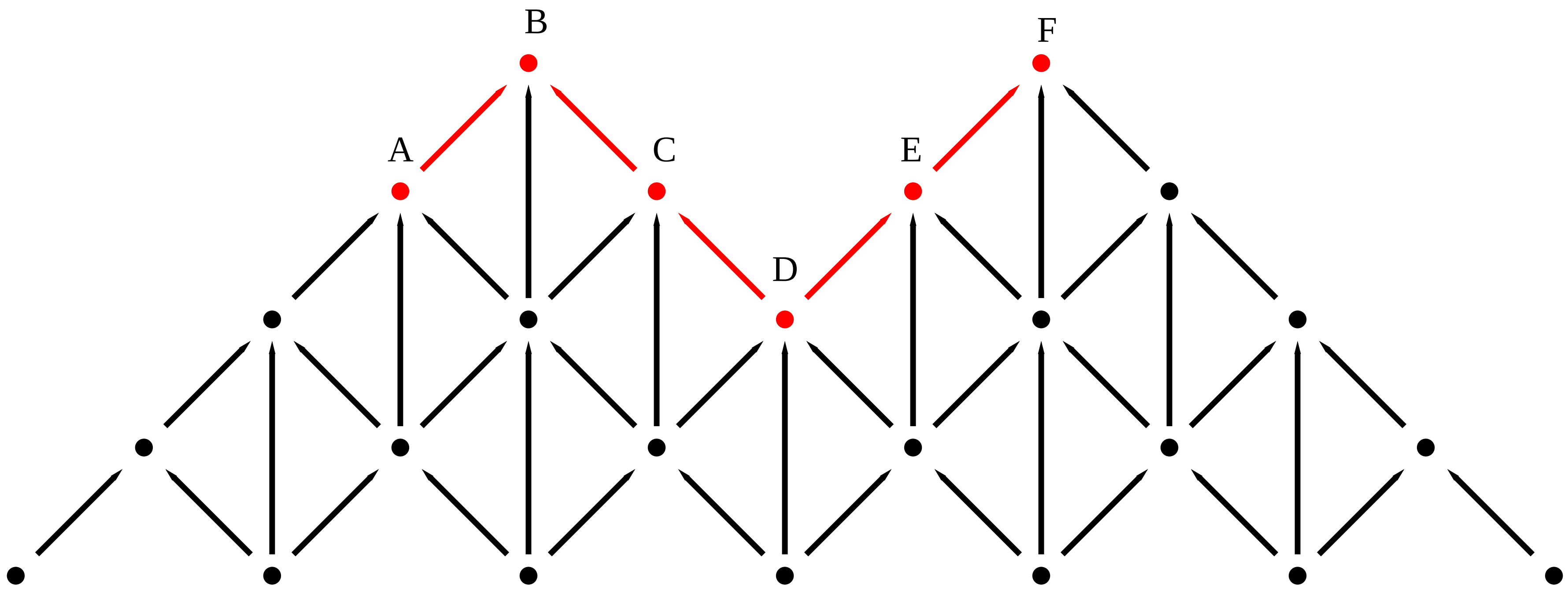
- V_C is **shielding off** V_A from V_B



- C is **d-separating** A from B



A path $\mathcal{P}$: $A \to B \leftarrow C \leftarrow D \to E \to F$



III. Other locality and causality concepts

A path $\mathcal{P}$: $A \to B \leftarrow C \leftarrow D \to E \to F$

Three types of vertices:

- Common effect (collider): $A \to B \leftarrow C$
- Common cause: $C \leftarrow D \to E$
- Intermediary cause: $D \to E \to F$

III. Other locality and causality concepts

A path $\mathcal{P}$: $A \to B \leftarrow C \leftarrow D \to E \to F$

Three types of vertices:

- Common effect (collider): $A \to B \leftarrow C$
- Common cause: $C \leftarrow D \to E$
- Intermediary cause: $D \to E \to F$

Idea:

- Only common causes and intermediary causes **transmit** causal dependence; colliders do not.
- Conditioning on non-colliders **blocks**, conditioning on colliders **introduces** causal dependence.

III. Other locality and causality concepts

A path $\mathcal{P}$: $\quad A \rightarrow B \leftarrow C \leftarrow D \rightarrow E \rightarrow F$

- The causal dependence between A and B is **blocked** by a set S on $\mathcal{P}$ if there is
  - at least one non-collider in S, or
  - at least one collider E such that either E or a descendant of E is *not* in S.
- For example:
  - $S = \{B, C\}$ is blocking, since C is a non-collider in S
  - $S' = \{B\}$ is not blocking, since there is no collider outside S and non-collider inside S'

III. Other locality and causality concepts

- The two vertices are **d-separated** by S if causal dependence is blocked on every path connecting them.
- Denotation: $A \perp\!\!\!\perp_d B \,|\, S$
- Bayesian networks: all probabilistic independencies are **implied** by the Causal Markov Condition via d-separation:

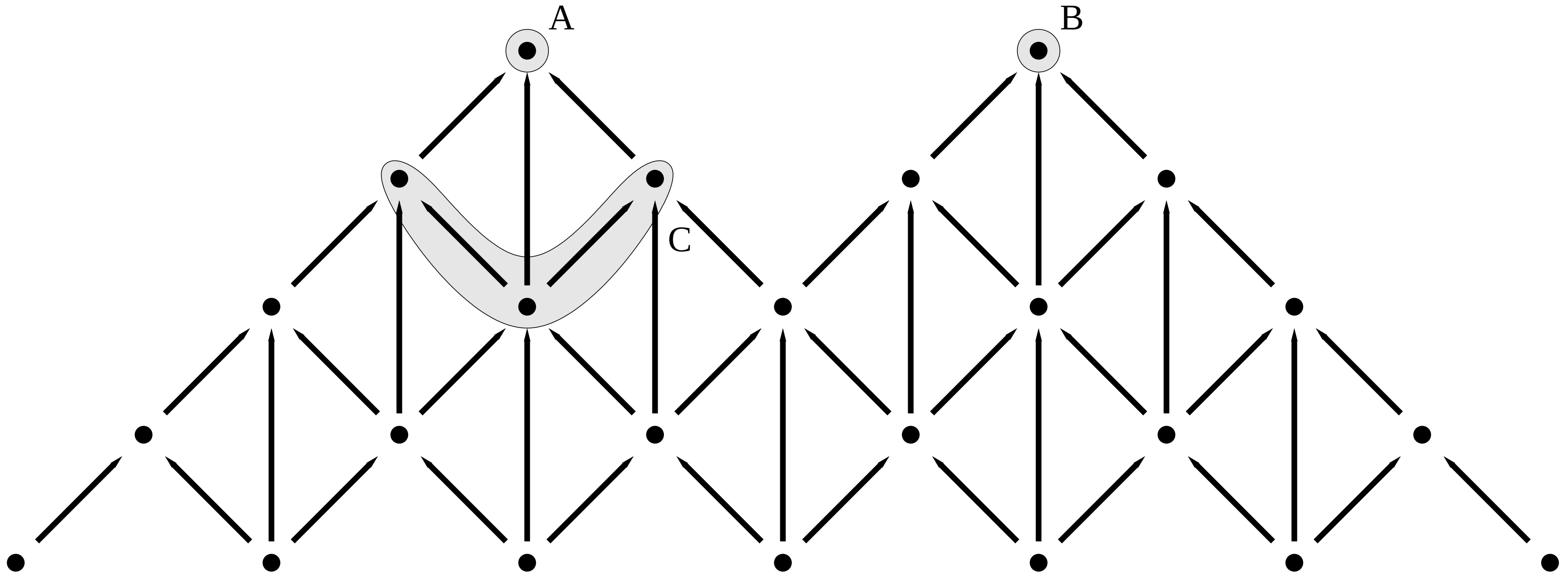
$$A \perp\!\!\!\perp_d B \,|\, S \quad \Longrightarrow \quad A \perp\!\!\!\perp_p B \,|\, S$$

- The causal graph is **faithful** if

$$A \perp\!\!\!\perp_d B \,|\, S \quad \Longleftrightarrow \quad A \perp\!\!\!\perp_p B \,|\, S$$

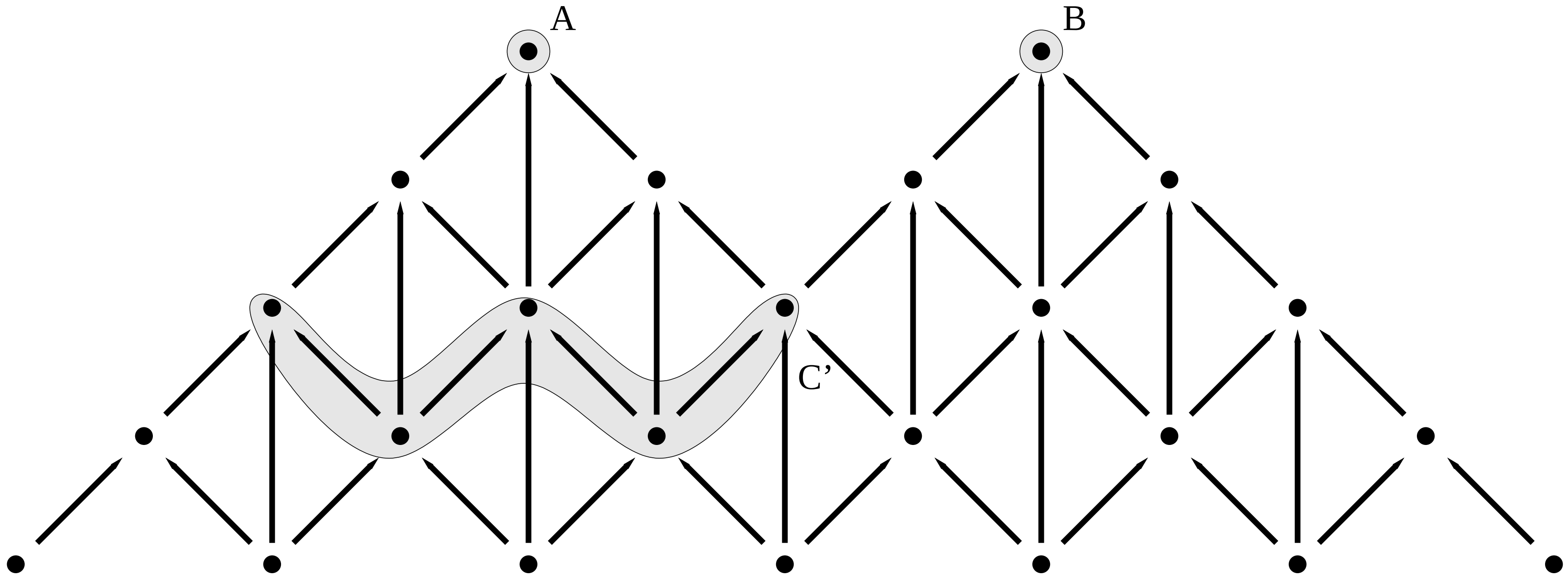
III. Other locality and causality concepts

A and B are d-separated by C:



III. Other locality and causality concepts

A and B are d-separated by $\mathcal{C}'$:



III. Other locality and causality concepts

A and B are *not* d-separated by C'':