Approximate Space-time Symmetry

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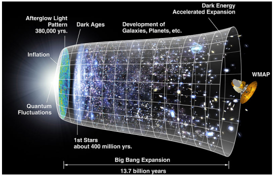
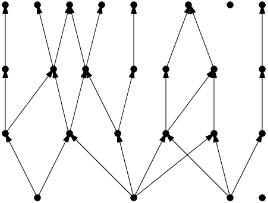
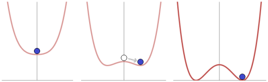
Motivations



Cosmology



Quantum Gravity



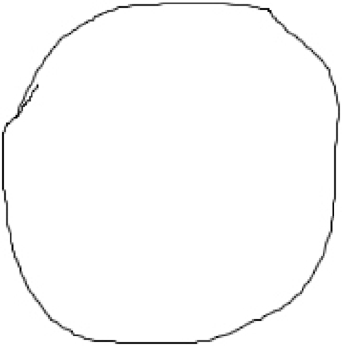
Spontaneous Symmetry Breaking

Outline

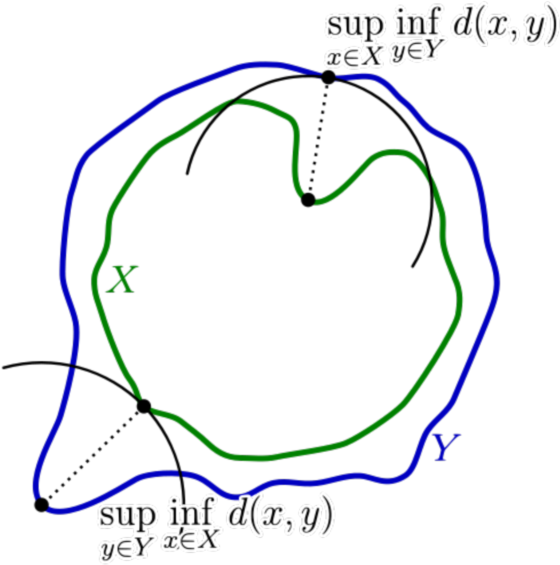
1. Basic Insights and Structures
2. Approximate Symmetry in the Large
3. Approximate Symmetry in the Small
4. Conclusions and Future Work

Outline

Approximately Circular



Hausdorff Distance



Ingredients for Approximate Symmetry

1. Spacetime: (M, g)
2. Putative (approximate) symmetry: subgroup of diffeomorphism group of M
3. Means of comparison: identity on M (or really any diffeomorphism)
4. Standard of comparison: uniform structure = collection of (pseudo)metrics (i.e. distance functions) on the Lorentz metrics on M[1]

[1] Maybe even semi-pseudometrics or less.

Ingredients for Uniform Structure

There are many different choices. Here's one with physical salience:

- Observers: (local) tetrad field $\{\overset{0}{e}{}^a, \overset{1}{e}{}^a, \overset{2}{e}{}^a, \overset{3}{e}{}^a\}$
- (Inverse) Riemannian metric: $h^{ab} = \sum_{i=0}^{3} \overset{i}{e}{}^a \overset{i}{e}{}^b$
- h-fiber norm: $|g|_h = (h^{ab} h^{cd} g_{ac} g_{bd})^{1/2}$
- h-fiber distance: $d_h(g, g') = |g - g'|_h$
- Aggregation to pseudometric: $D_h(g, g') = \sup_{X \subseteq M} d_h(g, g')$

Context-dependent Parts

- Which observers?
- Which fiber norm?
- Which fiber distance?
- How to aggregate?

Or one could take some other approach to quantify or structure how spacetimes are similar to one another.

Examples

Sobolev $W^{0,2}(M)$

- Observers: All observers on M
- Fiber norm: h-fiber norm
- Fiber distance: h-fiber distance
- Aggregation: Sobolev norm $\left(\int_M [d_h(g, g')]^2 \, d\sigma\right)^{1/2}$

Uniform Compact-Open $C^2(\mathcal{C})$

- Observers: All observers on compacta $\mathcal{C}$
- Fiber norm: h-fiber norm
- Fiber distance: max. of h-fiber distances for g, g' and their first and second derivatives
- Aggregation: uniform norm $\sup_C d_h(g, g')$

Outline

Definition: Approximate Symmetry in the Large

- Let (M, g) be a spacetime.
- Let G be a subgroup of the diffeomorphism group of M.
- Let h be an observer on M.
- Let D_h be an h-fiber distance aggregation on M.
- Let ϵ_h be a positive number.

G is a (D_h, ϵ_h)-approximate symmetry in the large when $\sup_{\psi \in G} D_h(g, \psi_*(g)) < \epsilon_h$.

Outline

Motivation

Consider a homogeneous globally hyperbolic space-time on whose foliation into space-like hypersurfaces there is a scalar field, constant on each hypersurface, slowly changing from negative to positive infinity.

Can it be an approximately stationary space-time, that is, have a approximate time-like Killing vector field?

Any observer whose four-velocity is close to being orthogonal to the hypersurfaces will not observe the scalar field change appreciably over small durations.

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There are many different choices. Here's one with physical salience:

- Observers: (local) tetrad field $\{\overset{0}{e}{}^a, \overset{1}{e}{}^a, \overset{2}{e}{}^a, \overset{3}{e}{}^a\}$
- (Inverse) Riemannian metric: $h^{ab} = \sum_{i=0}^{3} \overset{i}{e}{}^a \overset{i}{e}{}^b$
- Riemannian h-distance:
  $\tilde{d}_h(x, x') = \inf_{\gamma:[0,1]\to M}\{||\gamma|| : \gamma(0) = x, \gamma(1) = x'\}$
- h-Distance on diffeomorphisms:[2]
  $\tilde{D}_h(\psi, \psi') = \sup_{x\in M} \tilde{d}_h(\psi(x), \psi'(x))$

Let then $G_\delta = \{\psi \in G : \tilde{D}_h(I, \psi) < \delta\}$

[2] There is some flexibility in this aggregation, as with symmetries in the large.

Definition: Approximate Symmetry in the Small

- Let (M, g) be a spacetime.
- Let G be a subgroup of the diffeomorphism group of M.
- Let h be an observer on M.
- Let D_h be an h-fiber distance aggregation on M.
- Let G_δ be as before for some h-distance on diffeomorphisms and positive number δ.
- Let ϵ_h be a positive numbers.

G is a (D_h, ϵ_h)-approximate G_δ-symmetry in the small when $\sup_{\psi \in G_\delta} D_h(g, \psi_*(g)) < \epsilon_h$.

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Discussion

Approximate symmetry comes in degrees and is relative to observers; it also comes in two types (large and small), depending on whether one considers all or merely small elements of the symmetry group.

Still to do: application to particular cases (e.g., approx. FLRW space-times).

The general approach can probably be extended to other (non-spacetime) symmetries.

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