Towards a Formal Theory of Digital Physics: Digital Multiverses

Logic, Relativity, and Beyond 2017 – Renyi Institute

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Talking Points

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1. Digital Physics
2. Multiverse Hierarchy
3. Higher-Order Cellular Automata (HOCA)
4. Toy Model of Physics on CA
5. Multiverses as HOCA
6. Philosophical Implications of HOCA

Foundations of Digital Physics

Definition: Discrete Structure

- $(Disc(D) \iff (Obj(D) \subset Obj(Set) \wedge \forall x(x \in Obj(D) \implies \forall y(y \in Obj(D) \implies \exists N((x \in N) \wedge \neg(y \in N)))))))$

Definition: Digital Structure

- $(Dig(D) \iff (Disc(D) \wedge \exists B(B \subseteq D_A \wedge \forall d_i(d_i \in D_A \implies \exists z(z = (b_1, \cdots, b_k) \in B^k \wedge d_i = conc(z))))))).$

- **Digitalism** – Physical Reality $\iff$ Digital Structure
- **Pancomputationalism** – All physical processes computable
- **Zuse Thesis** – Physical reality is ontologically a digital computer

Digitalism Implies Pancomputationalism

Theorem

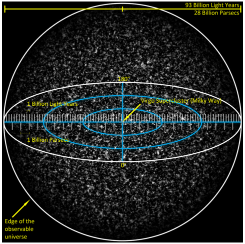
$Digitalism \implies Pancomputationalism$

Heuristic Proof.

(Digitalism $\wedge$ Kreisel Thesis $\wedge$ Church Thesis $\wedge$ Church-Turing Thesis) $\implies$ Pancomputationalism ■

Level 0 Multiverse

- Our Hubble Volume (Observable Universe)



Level 1 Multiverse

- Induced by cosmic inflation
- Infinite space of Hubble Volumes realizing all initial conditions
- Each universe has the same physical laws / constants

Level 2 Multiverse

- Infinite space of "finite" Level 1 Multiverses
- Induced by spontaneous symmetry-breaking predicted by chaotic inflation
- Each universe may have different physical laws / constants

Level 3 Multiverse

- Similar to Modal Realism
- Everettian Many-Worlds Interpretation of Quantum Mechanics
- Branching histories; all possible worlds consistent with the wavefunction
- Every world shares the same physical laws, is in a different dimension of Hilbert Space (worlds are orthogonal)

Level 4 Multiverse

- Platonism
- Physical Existence $\iff$ Mathematical Existence (self-consistent)

Usual Definition of Classical CA

- John Milnor:

$$CA = (\mathcal{K}, \mathscr{L}^n, f)$$

- $\mathcal{K}$ is a finite set of "alphabet symbols", i.e. the atomic constituents of the automaton; this includes at least two symbols, namely the empty symbol e and at least one other arbitrary symbol
- $\mathscr{L}^n \subseteq \mathbb{R}^n = \sum_{k=1}^{n} (a_k v_k) | a_k \in (A \subseteq \mathbb{Z})$
- $f : \tau \to \tau$ is a *cellular automaton map*, which maps configurations onto configurations
- $\tau : \mathscr{L}^n \to \mathcal{K}$ is a *configuration* that maps alphabet symbols onto lattice points

Statically-Typed HOCA

Suppose we allowed our configurations instead to look like this: $\tau : \mathscr{L}^n \to CA$. Then we would have another definition:

$$CA^r = (\mathcal{K}^\lambda, \mathscr{L}^n, q_0^r, Q, f^r, r)$$

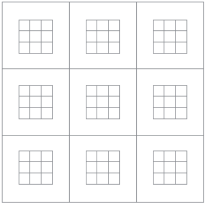
in which:

- $\lambda = r - 1$
- $r \in \mathbb{N}$ is a *type*,
- $\mathcal{K}^\lambda$ is an alphabet, the set generated by all CA^λ. (i.e. type-λ cellular automata).
- ECA^0 is the canonical empty symbol.
- ECA^r is the empty type-r automaton whose elements are ECA^λ, where $\lambda = r - 1$.

Statically-Typed HOCA (cont.)

- $\mathscr{L}^n$ is the same as it is in classical CA models.
- $f^r : \tau^\lambda \to \tau^\lambda$ is a type-r CA map, or a CA^r map
- $\tau^\lambda : \mathscr{L}^n \to CA^\lambda$ is a type-λ configuration
- q_0^r is a particular (initial) type-r configuration

Example – Fractal Tic-Tac-Toe



Freely-Typed HOCA

Intuitively, Type-n automata whose configurations not restricted by type (except that the types of its elements strictly less than n).

$$CA_x^r = (\mathcal{K}^\lambda, \mathscr{L}^n, q_x^r, Q, f_x^r, x, r)$$

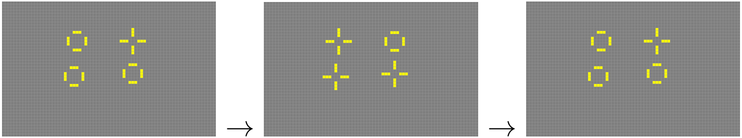
- Most is the same; distinction lies in the configurations:
- $\tau^\lambda : \mathcal{K}_x^\lambda \to \mathscr{L}^n$ maps objects of $(\lambda = r - 1)$-type objects or objects of lower type (down to x) onto lattice points. In general, $\mathcal{K}_x^y$ is the set $\bigcup_{k=x}^{y} CA^k$.
- Such automata are called CA_x^r.

Emergent HOCA

- Maximally type-n automata with differently-ruled maximally type-n sub-automata
- $\exists(\mathscr{L}^m \in CA')[(\mathscr{L}^m \subseteq \mathscr{L}^n) \wedge (m \leq n) \wedge (\exists(x', ((\mathcal{K}^{\lambda}_{x'})' \subseteq \mathcal{K}^{\lambda}_{x}), (q^r_{x'})', Q', (f^r_{x'}))' [((K^{\lambda}_{x'})', \mathscr{L}^m, (q^r_{x'})', Q', f^r_{x'}, x', r) \in CA^r_{x'}])].$
- Analogous to Object-Oriented Programming

Example of Emergent HOCA – Conway's Game of Life

1. (U): If a cell is alive and has less than 2 live neighbors, it dies
2. (L): If a cell is alive and has 2 or 3 live neighbors, it lives on
3. (O): If a cell is alive and has more than 3 live neighbors, it dies
4. (R): If a cell is dead and has exactly 3 live neighbors, it becomes alive



What the Modern Physics Might Look Like on a CA

$$\left\{ \begin{array}{ccc} \textbf{Modern} & \Longleftrightarrow & \textbf{Digital} \\ Spacetime & \Longleftrightarrow & \mathscr{L}^n \\ ElementaryParticles & \Longleftrightarrow & \mathcal{K} \\ InitialConditions & \Longleftrightarrow & q_0 \\ NaturalLaws & \Longleftrightarrow & f \\ SpatialHistory & \Longleftrightarrow & Q \end{array} \right\}$$

Level 1 Multiverse as CA

- Type-1 Emergent HOCA
- Each (disjoint) neighborhood represents a Hubble Volume and has different initial conditions

Level 2 Multiverse as CA

- Type-1 Emergent, whose sub-automata are Level-1 CA Multiverses with different laws
- Each Type-1 element has a different evolution function

Level 3 Multiverse as CA

- Similar to Modal Realism
- Contains all possible worlds consistent with some stochastic evolution function
- Type-t Statically-Typed for time t (branching spacetime)

Level 4 Multiverse as CA

- Neo-Platonism
- Classical $\bigcup$ ST $\bigcup$ FT $\bigcup$ Emergent
- "All is computation"

Philosophical Implications

- Ontological formalization of the simulation hypothesis (emergent HOCA)
- Epistemic representation of Everettian wavefunction realism (static type-2 automaton)
- New way to think of modal realism

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