incorporating relativity in categorical models of abstract physical theories

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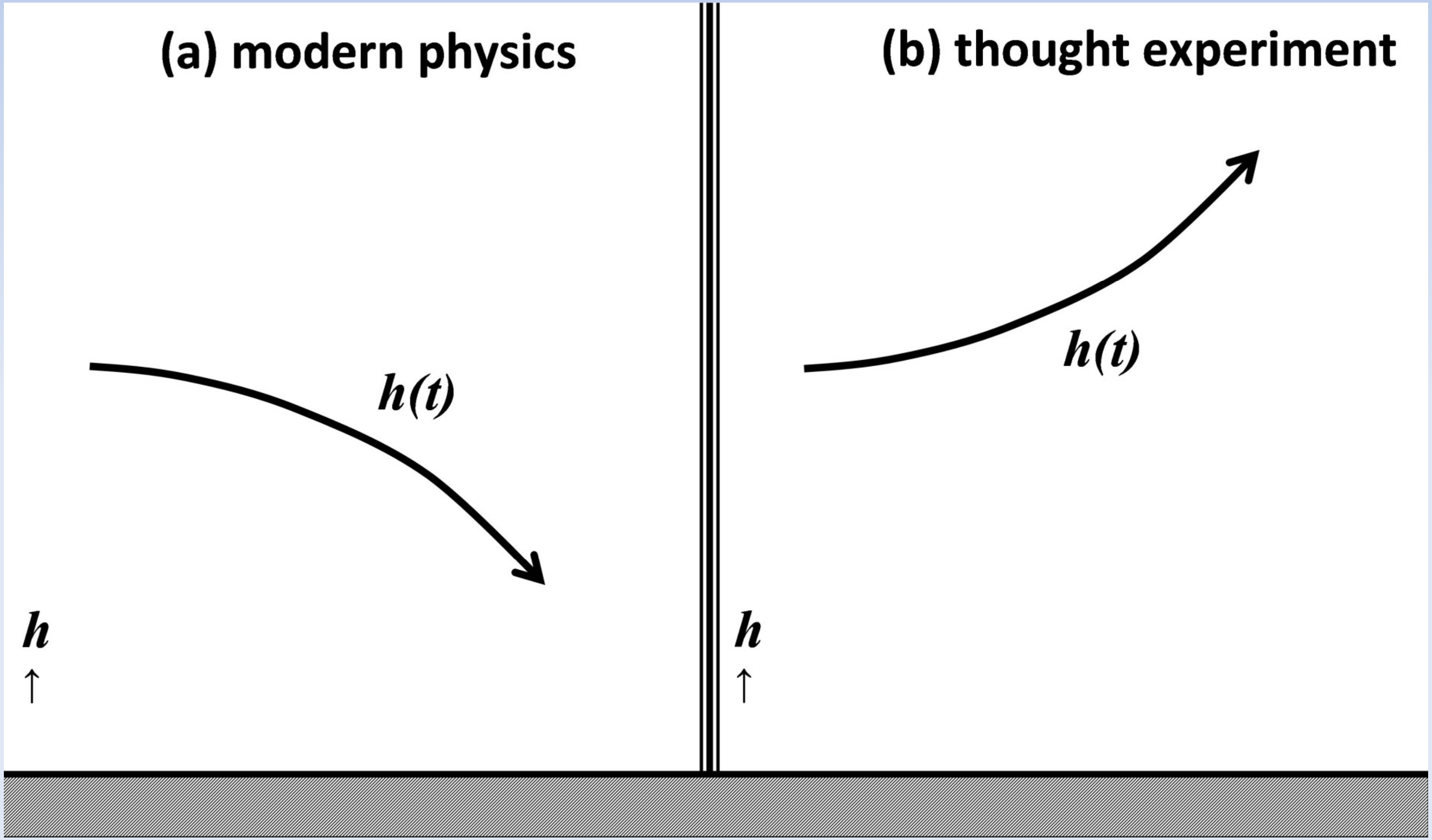
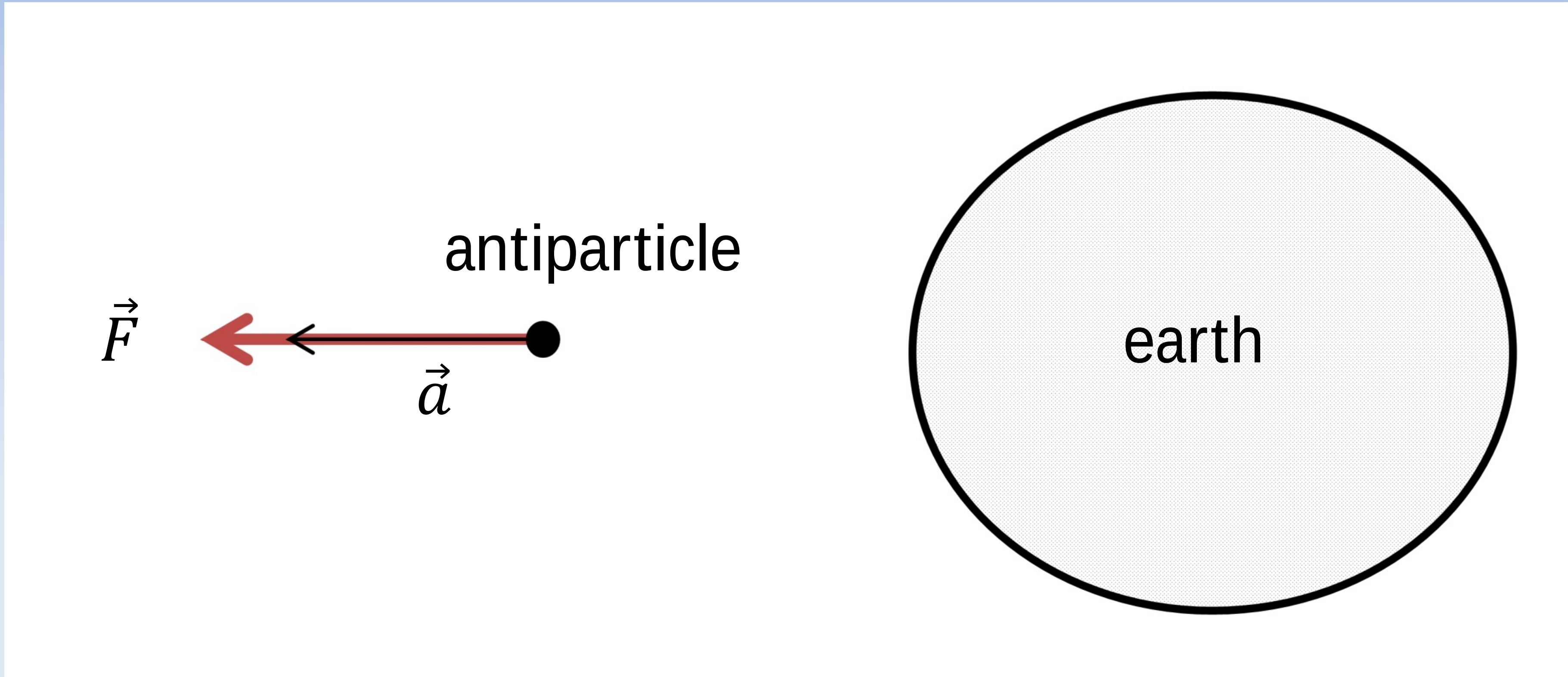
overview

- **background**
  some words about my research
- **abstract physical theories**
  what are they and why are they interesting?
- **problem**
  agreement with SR: standard model theory inapplicable
- **solution**
  apply categorical model theory!

background (1/3)



background (2/3)



$$\bar{m}_g = -\bar{m}_i$$

(Morrison & Gold, 1957)

background (3/3)

- **GR, QED, QCD are incompatible with repulsive gravity:**
  $\bar{m}_g = -\bar{m}_i$ is <u>impossible</u>
- $\therefore$ **repulsive gravity $\rightarrow$ GR/QED/QCD <u>emergent</u>**
  these theories are then <u>not fundamental</u>
- **what lies underneath?**
  which physical principles underlie repulsive gravity?
- **I've developed a theory**
  Ann. Phys. **522**:699 (2010); **523**:990 (2011); **528**:626 (2016)
- **unfortunately, no low hanging fruit**

abstract physical theories (1/5)

an **abstract physical theory T formalized in ZF** consists of:

1. the language L(T), a sublanguage of L(ZF) given by:
   i. the **individual constants** of T
   ii. the **relations** of T
2. the **formal axioms** of T:
   i. for every individual constant φ: $\exists x(x = \varphi)$
   ii. for every relation R: $\exists v(v = R)$
3. the **physical axioms** of T:
   wffs in L(T)
4. the **interpretation rules** of T
   add physical meaning to constants and relations of T

abstract physical theories (2/5)

essential feature of an abstract physical theory T:

constants interpeted as real-world things are abstract sets

proper designator/definite description

- designates a thing by an interpretation rule
- but does not represent its physical state

an abstract physical theory T is to be true
regardless of the properties of the things designated

abstract physical theories (3/5)

non-examples of abstract physical theories:

- special relativity

  event: concrete element of $\mathbb{R}^4$
  world line: concrete function on $\mathbb{R}^4$

- quantum mechanics

  wave function: concrete element of $\mathcal{H}$
  spectrum of observable: concrete set of values

an abstract physical theory is to express the most general principles, even more general than SR and QM

abstract physical theories (4/5)

toy example of abstract physical theory:

- <u>language</u>

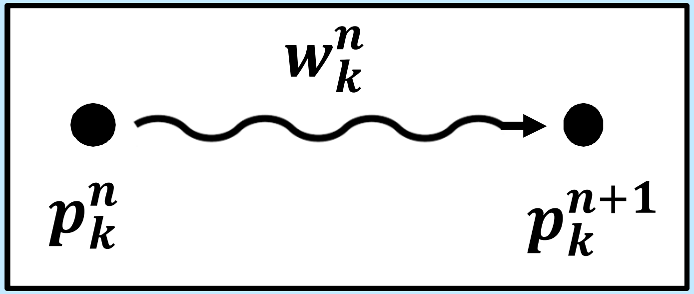
  for $n, k \in \mathbb{Z}$, constants p_k^n, w_k^n
  binary relation (.) → (.)

- <u>interpretation rules</u>

  p_k^n: particle state #*n* in process #*k*
  w_k^n: wave state #*n* in process #*k*
  $\alpha \rightarrow \beta$: α turns into β by a discrete transition

- <u>physical axioms</u>

  $\forall n, k \in \mathbb{Z}: p_k^n \rightarrow w_k^n$
  $\forall n, k \in \mathbb{Z}: w_k^n \rightarrow p_k^{n+1}$



abstract physical theories (5/5)

why are abstract physical theories **interesting**?

- empirical reduction (Rosaler 2015)

  a theory T **reduces empirically** to a theory T' *iff*
  T reproduces the empirically successful predictions of T'

- Unifying Scheme

  an abstract physical theory T is a **Unifying Scheme** *iff* T has a model M that reduces empirically to GR and QED

- Grand Unifying Scheme (GUS)

  an abstract physical theory T is a **GUS** *iff* T has a model M that is empirically adequate

problem (1/3)

- agreement with SR
  an abstract physical theory T **agrees** with SR *iff* it has a model M that reduces empirically to SR

standard tool: specify a **concrete set-theoretic model of T**

- an interpretation of the constants and relations of T in a concrete set such that for every physical axiom A of T

$$M \vDash I(A)$$

- if φ designates a thing, then I(φ) represents the physical state of that thing in the coordinate system of an observer

problem (2/3)

suppose you have specified a set-theoretic model M of T

- M predicts the motion of object φ for one observer

HOWEVER

- M does not predict the motion of that object φ in the coordinate system of another observer
- so: M does not reduce empirically to SR

a single set-theoretic model M of T does not predict relativity of spatiotemporal characteristics of motion

problem (3/3)

SET-THEORETIC MODEL M OF THE TOY THEORY

- p_1^1, p_1^2: point particles at (t_1, X_1), (t_2, X_2) in the IRF of $\mathcal{O}$
- in the IRF of $\mathcal{O}$', p_1^1 and p_1^2 will be at (t_1', X_1'), (t_2', X_2')
- M has no info on coordinates of p_1^1, p_1^2 in the IRF of $\mathcal{O}$'

specifying a single set-theoretic model is insuffient for proving that the physical axioms of T agree with SR

solution (1/4)

category $\mathcal{C}$

- ‘objects’ of $\mathcal{C}$
- ‘arrows’ of $\mathcal{C}$
- an arrow *f* connects an object x to an object y

$$f\colon x \to y$$
$$x = \text{dom } f$$
$$y = \text{cod } f$$

- if $f\colon x \to y$, $g\colon y \to z$ then there is an arrow h

$$h = g \circ f \;\wedge\; h\colon x \to z$$

- for every object x there is an identity arrow 1_x

$$1_x\colon x \to x$$

solution (2/4)

Example 1

- ‘objects’ of $\mathcal{C}$ are all groups
- ‘arrows’ of $\mathcal{C}$ are group isomorphisms

- collection of ‘objects’ not necessarily a set
- if so: **small category**

Example 2

- ‘objects’ of $\mathcal{C}$ are all models of a first-order theory T
- ‘arrows’ of $\mathcal{C}$ are model isomorphisms

solution (3/4)

Categorical model $\mathcal{C}$ of an abstract physical theory T

- collection of objects: $\{M_j\}_{j\in F}$ (small category)
- M_j is concrete set-theoretic model of T
  - o $M_j \leftrightarrow (X, \varphi_j)$
  - o M_j's all formulated in the same language L($\mathcal{C}$)
- 'arrows' f of $\mathcal{C}$ are model isomorphisms
  - o $f: M_i \rightarrow M_j \leftrightarrow$ coordinate transformation

solution (4/4)

- $\mathcal{C}$ **reproduces** SR if SR can be **incorporated** in $\mathcal{C}$

$\{M_j\}_{j \in F}$ relativistic theory from semantic point of view

T theory from the syntactic point of view

- **the tool to apply for proving that T agrees with SR:**

specify a categorical model $\mathcal{C}_0$ of T incorporating SR

- **'speculative' research program:**

hard core: T

empirical & theoretical progression: successors $\mathcal{C}_1$, $\mathcal{C}_2$, ...

aim: prove that T is a GUS