Continuity of time, movement and locality

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0. Introduction

Some general philosophical nonsense.

Is movement possible?

What is continuity?

How are time and space adequately characterized in a formal-mathematical language?

(adequate to what?)

Do our formal theories capture everything we want to know about time, space and movement?

1. What is movement?

In daily life, movement is well known and dealt with. Yet movement seems to be hard to define.

Dictionary (Webster): movement is the act or process of moving people or things from one place or position to another.

So, in considering movement, we presuppose concepts of space and time. Movement is the covering of space over time, more precisely, the covering of a space interval over a time interval.

Mathematically, space and time intervals are represented by intervals of real numbers. Spatial position as a function of time x is postulated. We speak of movement if that function x is continuous in the sense of high-school mathematics, that is, for any pair of spatial positions $x_1(t_1)$, $x_2(t_2)$ in said space interval,

(C) for each ε, $\varepsilon > 0$, there is some δ, $\delta > 0$, such that if $|t_2 - t_1| < \delta$, then $|x_2(t_2) - x_1(t_1)| < \varepsilon$.

What we have is a series of positional snapshots, which, taken at sufficiently high frequency, reveal position differences as tiny as desired.

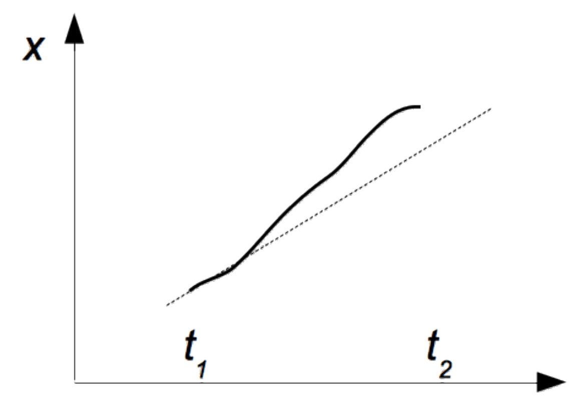
What we have is a description of positions at time points to any desired accuracy.

What we do not have, given the dictionary definition, is a description of movement.

As Zeno's arrow paradox reminds us, a series of spatial positions as a function of time (we add: however dense, even if arithmetically complete, even while satisfying (C)) does not constitute movement.

What about velocity?

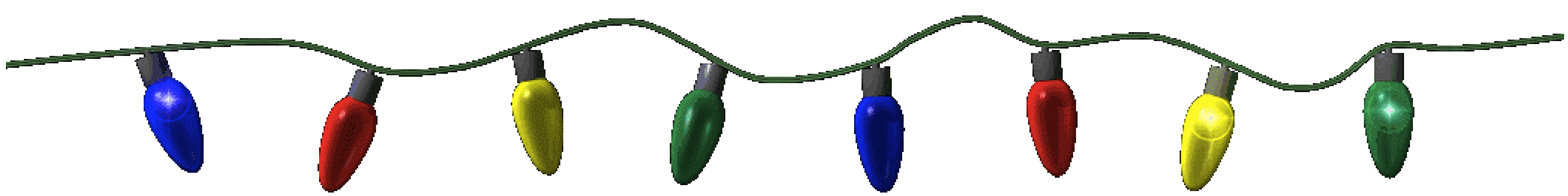
Calculus is supposed to allow us to speak of the velocity at a given time point t_1. It does so by linear approximation of the position function x, yielding an average velocity $(x_2(t_2) - x_1(t_1))/(t_2 - t_1)$ around t_1 for some t_2, which may come as close to t_1 as desired, but has to be distinct from t_1.



So, in calculus, the defined notion of velocity at a time point t_1 does not help to resolve the Zenonian difficulty.

A movie (in old film technology) is a series of still pictures, where neighbors closely resemble each other. The series of still pictures simulates movement, but does not constitute it.

We perceive movement upon rough and incomplete sense impression because we already have a concept of movement.
The phi movement phenomenon (Kolers and von Grünau 1976), the perception of the subsequent flashing of a row of lights as movement, confirms our tendency to perceive movement where there is none.



However, we note that thereby we obtain no evolutionary advantage.

2. Continuity

In mathematics, one speaks of continuity of the set of real numbers. We call it "c-continuity". "C-continuity" involves individual objects and comes about by set-theoretical relations between individual objects.

We introduce a notion of continuity in an intuitive sense, which we call "i-continuity". A line, drawn by a child on a blackboard, is i-continuous. The child ignores the line's physical imperfection and immediately understands i-continuity.

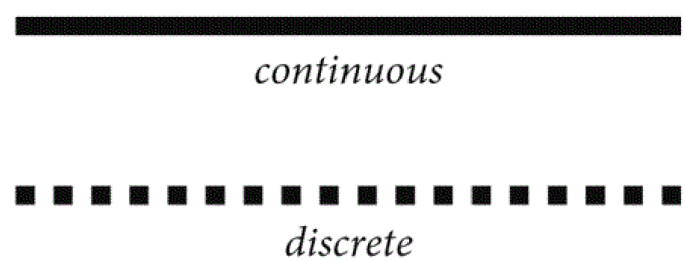
I-continuity is smooth (not in the sense of calculus) transition without leaps or stages. Our natural intuition conceives continuity as i-continuity.

If c-continuity is all we need to understand continuity, then the question “how many points constitute a line?” (Gödel 1948) makes sense. Yet no juxtaposition of points ever generates a line, since

(i) points have extension zero and lines have finite extensions and in no arithmetic, finite or infinite, that has been brought up the neutral element of multiplication is anything but zero;

> But: couldn't a totally ordered set of $\aleph_1$ many points just “fill” a line?
>
> No: for every totally ordered set *a* of $\aleph_\alpha$ many points, there is a totally ordered set *b* of $\aleph_\beta$ many points, with $\alpha < \beta$ and $a \subset b$, where the largest and smallest points of *a* and *b* coincide.

(ii) points are individuals, whereas having lines be sets of individuals would break their i-continuity, destroy their smooth, leapless proceeding.



3. Continuity of time

Time is the parameter of change, in an informal sense.

> Why informal? Because every formal statement talks about individuals and only those. However, we restrict our discourse if we presuppose individuality of change stages, time points etc.

To be able to undergo change, the changing object needs to keep its identity. A series of similar objects is no changing object.

We perceive change similarly to movement. If we perceive a series of sufficiently similar, plausibly arranged objects as a changing object, we apply on that series a concept of change that we already possess.

A property or magnitude E capable of change is usually mathematically described as a function whose domain is a totally ordered set t of time points $t_1, t_2, \ldots$.

To represent change of E adequately, one naturally stipulates that t be

(i) infinite;
(ii) complete under addition to allow for homogeneity of time;
(iii) complete under multiplication to allow for stretching of time;
(iv) totally ordered;
(v) well-ordered to allow change to begin at any point, in particular, provide next neighbors for each of its elements.

However, the conjunction of above requirements cannot be satisfied. Except (iii), above requirements are satisfied by $\mathbb{Z}$, but that implies a fundamental time unit ("Planck time"). With a finite time unit, change would be a series of leaps, counter to our intuition.

So change is formally not characterizable.

Does that matter? It does not matter for (biological) evolutionary success. I can care about my temporal successor just as well as about myself in a future (changed) state.

But we have strong intuitions about change.

We propose as a principle:

(P) Strong intuitions we have which are not conducive to evolutionary success reflect, at least roughly, subjects of discourse of metaphysical relevance.

Taking thus change serious, we are compelled to regard change as i-continuous and time as i-continuity.

4. Is there movement?

Parmenides and Zeno: There is no movement.
Heraclitus: πάντα ῥεῖ, everything flows.

Heraclitus appeals to our intuition of i-continuous change.
I-continuous change is not formalizable because any formal language is about nothing but individuals.
Formalization breaks i-continuity and provides us with snapshots of any desired accuracy.
With formalization, there is no movement.

5. Locality

Locality in physics is understood in various senses.

In an often employed sense, which we call “c-locality”, locality of a physical magnitude E as a function of spacetime is the absence of a direct causal effect from $E(x_1, t_1)$ to $E(x_2, t_2)$, where x_1, t_1 and x_2, t_2 are distinct spacetime points (”events”) which are distant from each other in some formal sense.

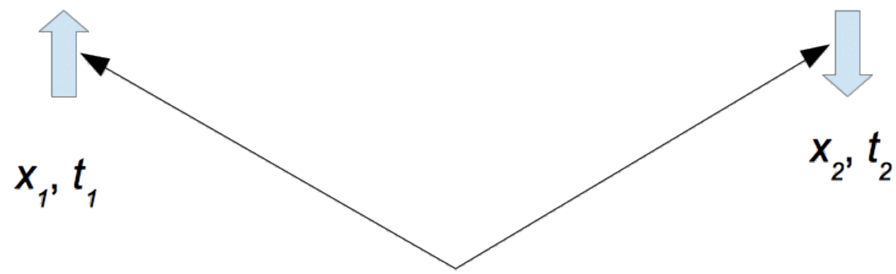
In Relativity Theory, the condition of c-locality is strengthened:

(R1) no direct causal effect between distinct spacetime points; and

(R2) no causal effect (direct or indirect) between spacelike separated spacetime points.

C-locality is well described by formal theories, if direct causal effect is formalizable.

By (R2), Relativity Theory disallows the (apparent) causality in EPR scenarios.



However, already (R1) asks for mediation of cause between any two spacetime points.

The motive behind c-locality is locality in an intuitive sense, which we call “i-locality”, the idea of gradual spatial transition. But, like change over time, i-locality is not formally characterizable. What is more, in any formal setting, every physical magnitude *E* is non-c-local because any two spacetime points are distinct from each other.

We propose as a moral: let us not be concerned about formal theories that are non-c-local.

5. Conclusion

Our intuition about i-continuity cannot be captured by any formal theory, since formal theories are about individuals.
Our intuitions about movement, temporal change and locality are not satisfied by formal theories.

We are left with the choice to discard our intuitions or to accept a basic incompleteness of formal theories.

A proposal at this point:

Keep the intuition about i-continuity;
regard temporal change to be i-continuous;
accordingly, regard time to be i-continuous.

But admit that there is no movement and no i-locality;
regard spacetime to be i-discontinuous.