How different are classical and relativistic spacetimes?

Hajnal Andréka and István Németi
Joint research with Madarász, J. and Székely, G.

Imagine …

Compare classical and relativistic spacetimes with the methods of mathematical logic.

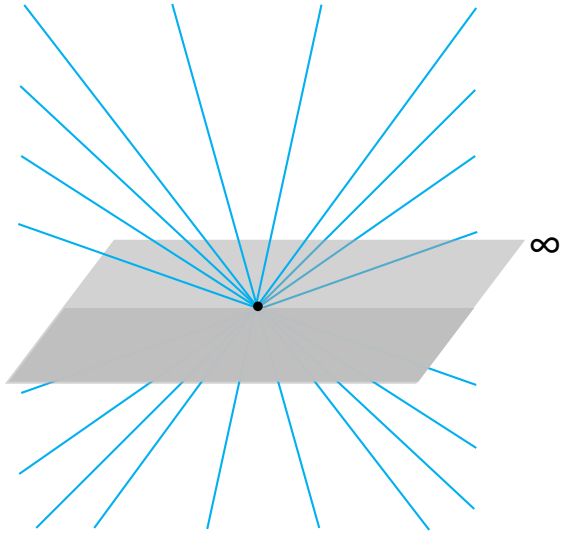
1. Wildly different.

2. Reconstructible within each other.

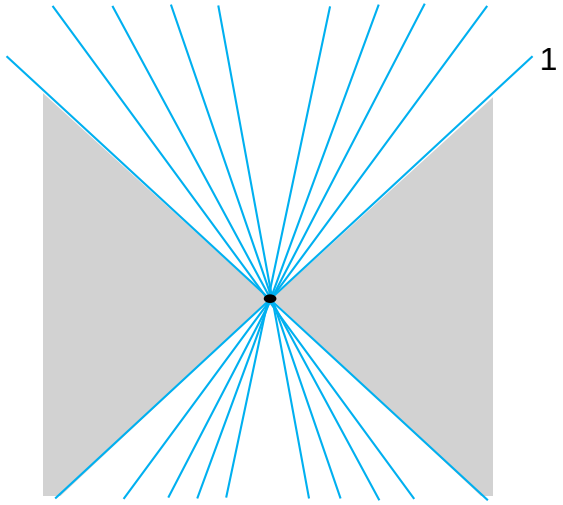
3. Not ms-definitionally equivalent.

1.

Newton spacetime
$\langle\ R^4\ ,\ Col^{\infty}\ \rangle$

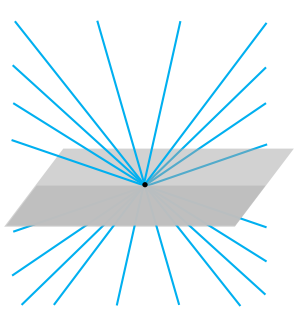


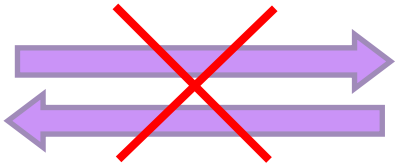
Einstein spacetime
$\langle\ R^4\ ,\ Col^{1}\ \rangle$

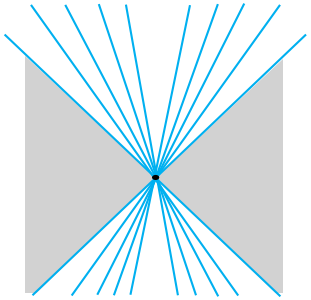


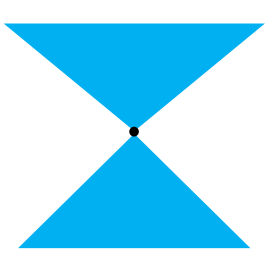
Newton spacetime
$\langle\ R^4\ ,\ Col^0\ \rangle$

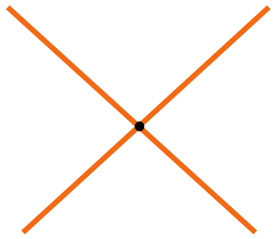
Einstein spacetime
$\langle\ R^4\ ,\ Col^1\ \rangle$







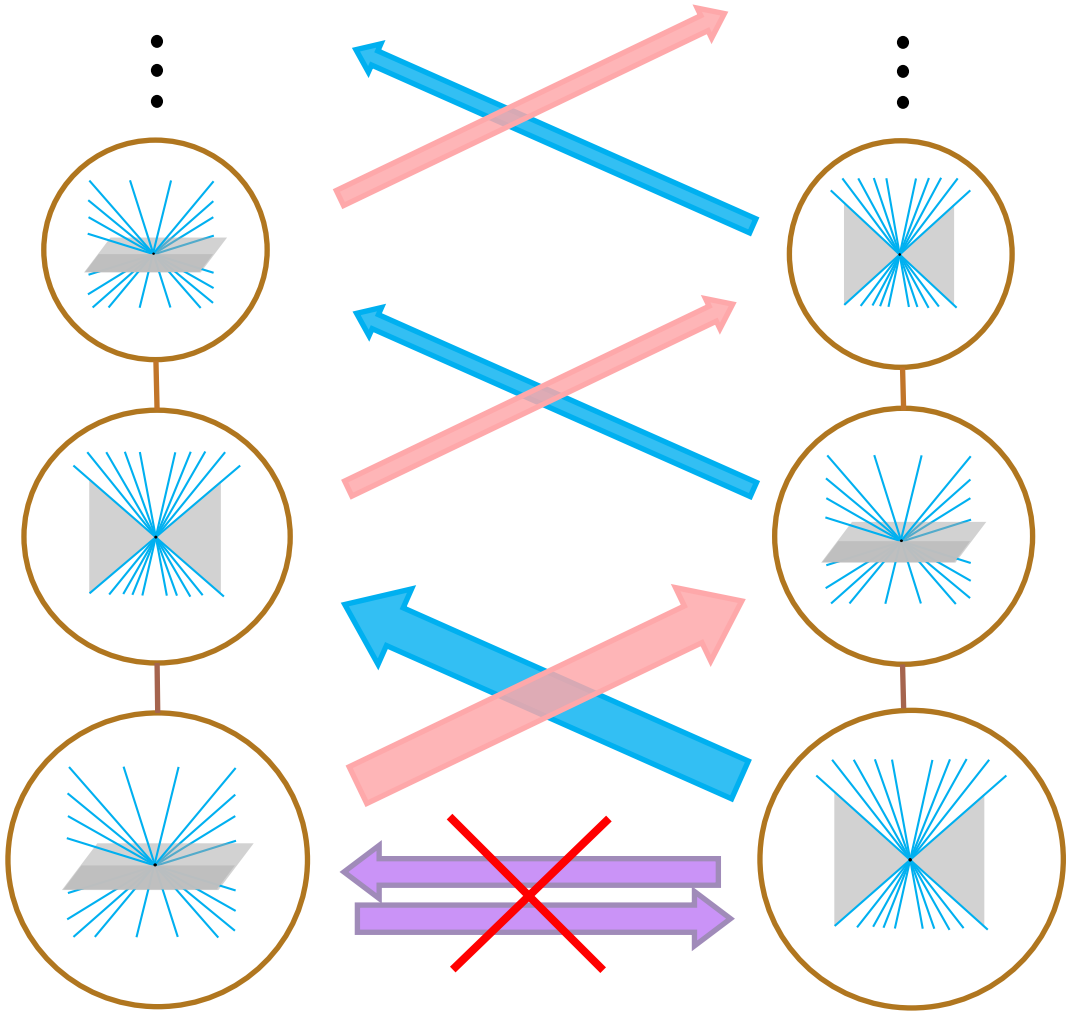




these are all definitionally equivalent

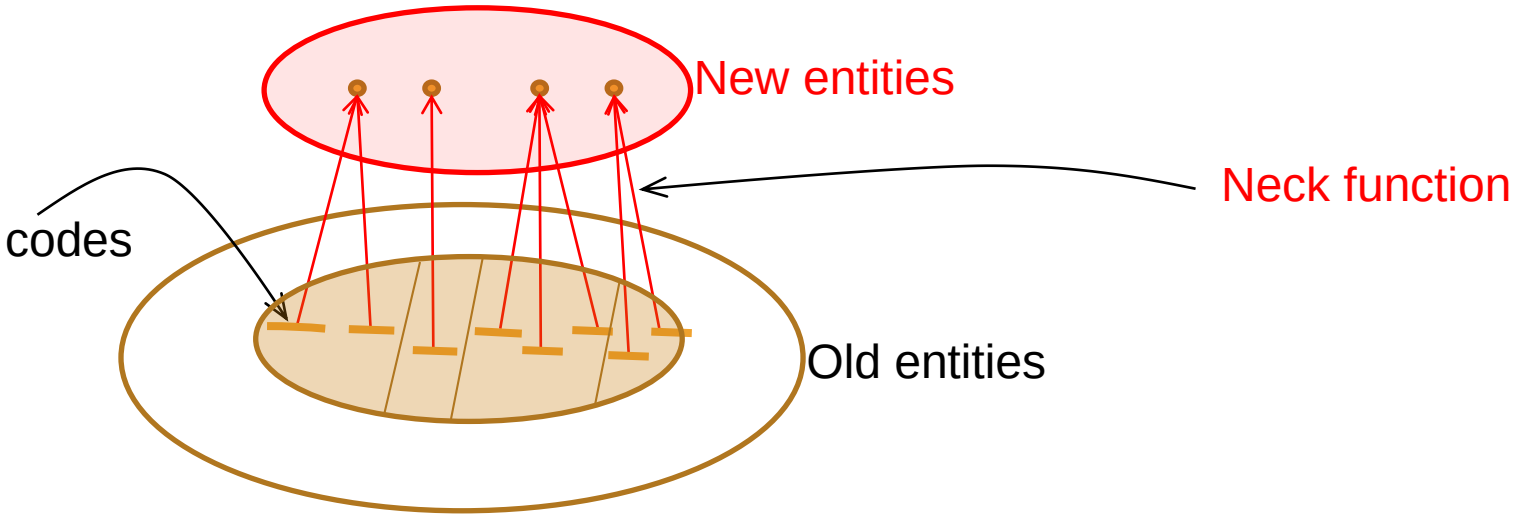
2.

Defining abstract entities



Many-sorted definability theory

New entities are coded by finite sequences of old entities.
A “neck function” tells which new entities are coded by which codes.



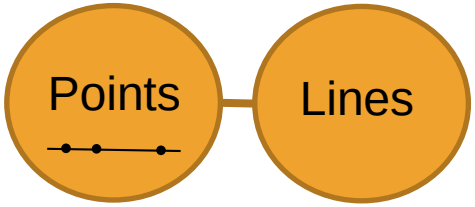
The only requirement is that the kernel of the neck-function has to be nonempty and definable in the old language. Shelah

Many-sorted definability theory

Examples:

1. Lines as new sort in geometry:

   A line is coded by any pair of its points.



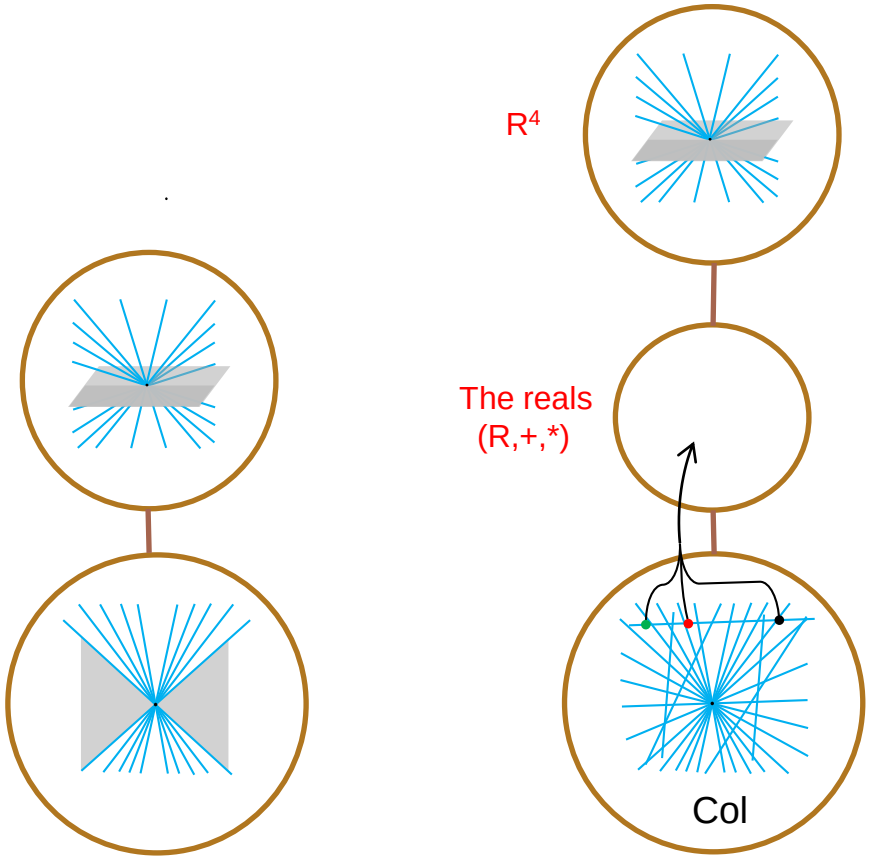
2. Rational numbers from integers:

$$3/4$$

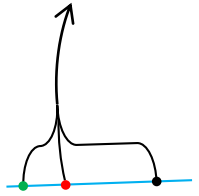
3. General relativistic spacetime manifolds from purely coordinate approaches:

   Events as equivalence classes of coordinate points

How do we define Newton spacetime over Einstein spacetime?

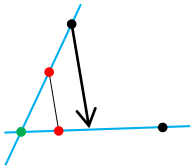


The neck function



Codes are: (o,e,r)

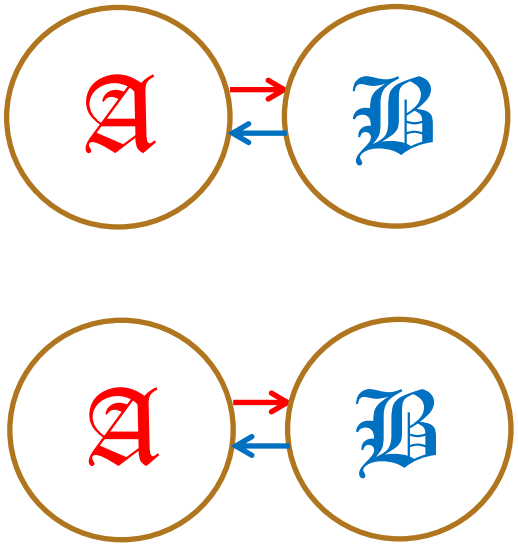
Code the same real:



3.

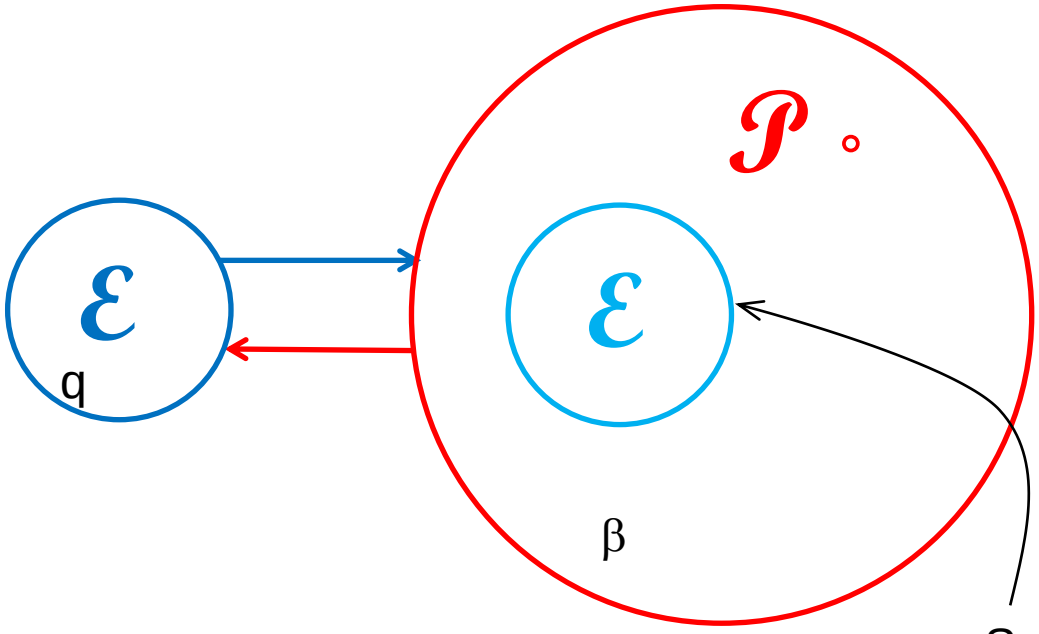
Definitional equivalence

Many-sorted definitional equivalence



Thm1.
Einstein-spacetime is ms-definitionally equivalent to the Poincare-group.

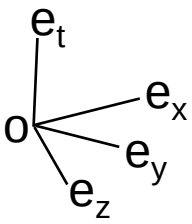
Idea of proof:



Red neck: q is coded by “reflection to q”

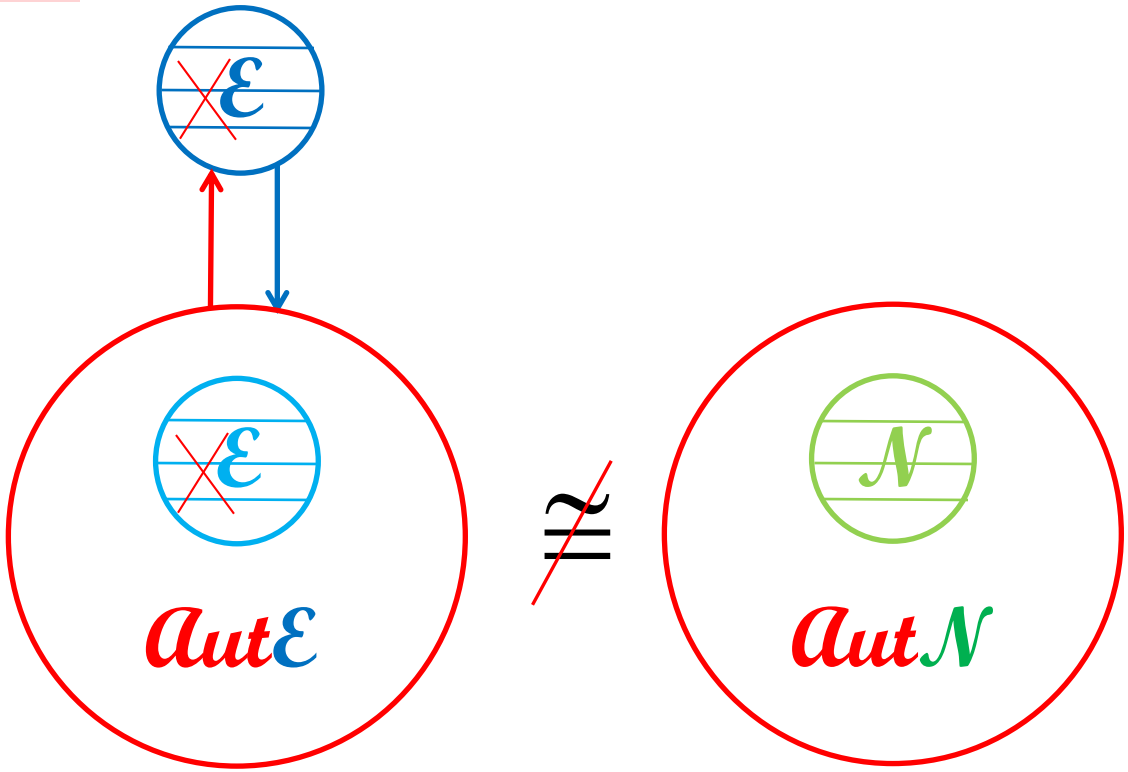
Blue neck: β is coded by “o,e_t,e_x,e_y,e_z, $\beta(o,e_t,e_x,e_y,e_z)$”

orthonormal basis



Thm2.
Einstein-spacetime is not ms-definitionally equivalent to Newton-spacetime.

Idea of proof:



Thank you for your attention!