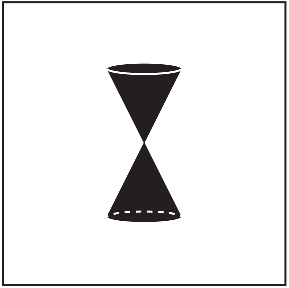
four criteria for theoretical equivalence

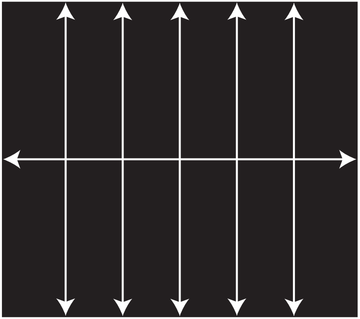
thomas barrett

princeton university

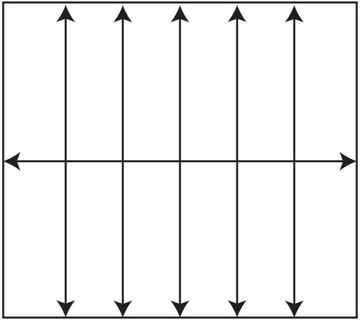
(1-3) General relativity



(3-1) General relativity

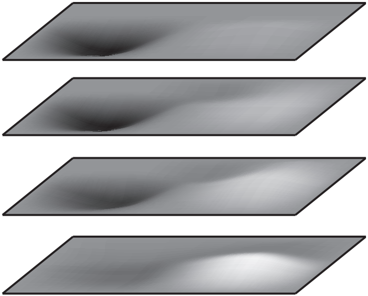


Hamiltonian mechanics

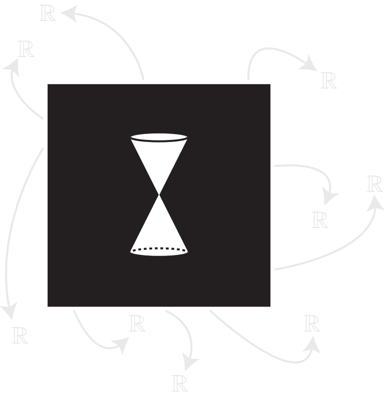


Lagrangian mechanics

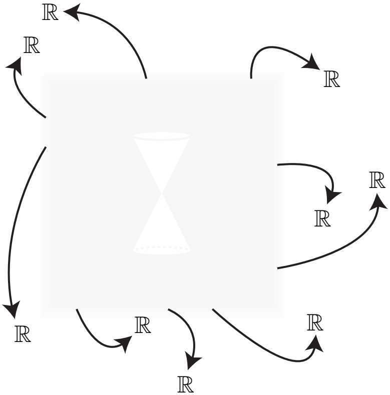
Newtonian Gravitation



Geometrized
Newtonian Gravitation



General relativity



The theory of
Einstein algebras

Philosophers of science have proposed a number of formal criteria for theoretical equivalence.

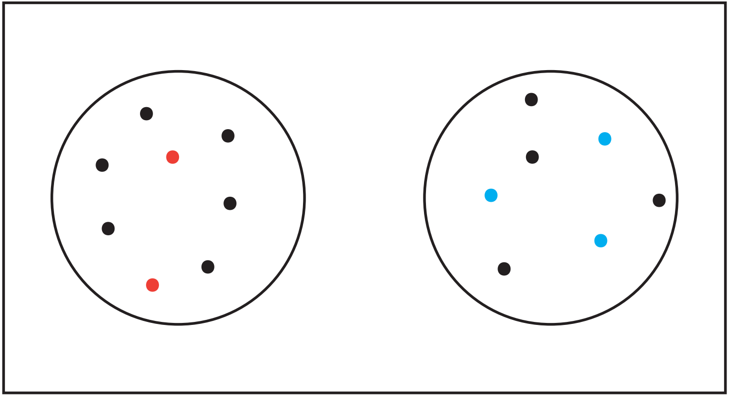
logical equivalence	?	definitional equivalence	?	Morita equivalence	?	categorical equivalence

logical		definitional		Morita		categorical
equivalence	?	equivalence	?	equivalence	?	equivalence

A **signature** Σ is a set of sort symbols, predicate symbols, function symbols, and constant symbols.

$$\Sigma = \{s_1, s_2, p, q\}$$

A Σ-**structure** is an interpretation of these symbols.



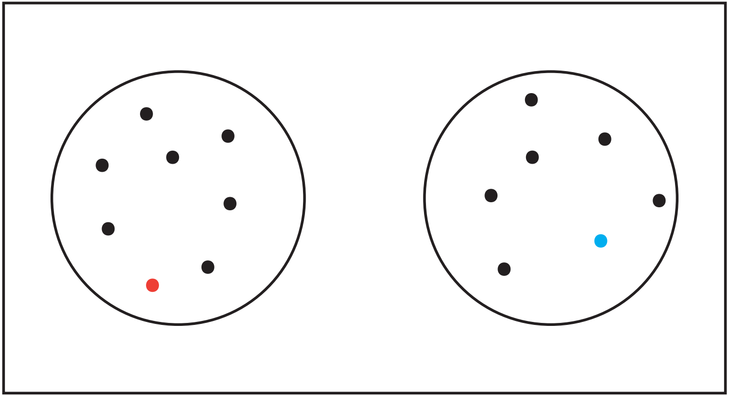
A Σ-structure.

A Σ-**theory** T is a set of sentences in the signature Σ.

- there is a unique x of sort s_1 that is p.
- there is a unique y of sort s_2 that is q.

A Σ-theory T.

A **model** of a Σ-theory T is a Σ-structure in which all of the sentences in T are true.



A model of the Σ-theory T.

Two theories are **logically equivalent** if they have the same class of models.

- there is a unique x of sort s_1 that is p and there is a unique y of sort s_2 that is q.

A Σ-theory T' that is logically equivalent to T.

logical equivalence	?	**definitional equivalence**	?	Morita equivalence	?	categorical equivalence

There are many pairs of theories that are not logically equivalent, but are nonetheless intuitively equivalent.

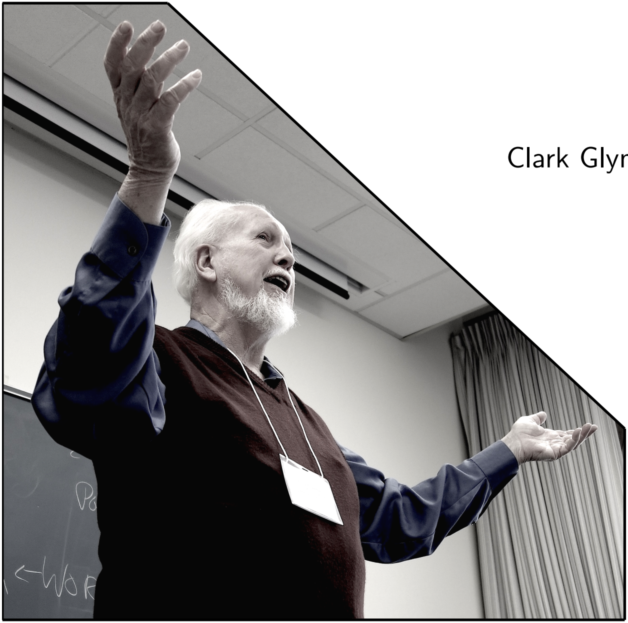
theory of groups_1

theory of groups_2

A theory with signature $\{\mathsf{s}, \cdot, ^{-1}\}$.

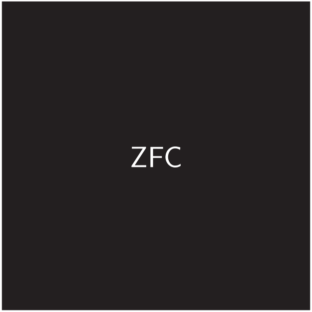
A theory with signature $\{\mathsf{s}, \cdot, \mathsf{e}\}$.

We need a more general criterion for theoretical equivalence than logical equivalence.



Clark Glymour

A **definitional extension** of a theory T is a theory T^+ obtained by adding to T definitions of new predicate symbols, function symbols, and constant symbols.



A theory with signature $\{s, \in\}$.

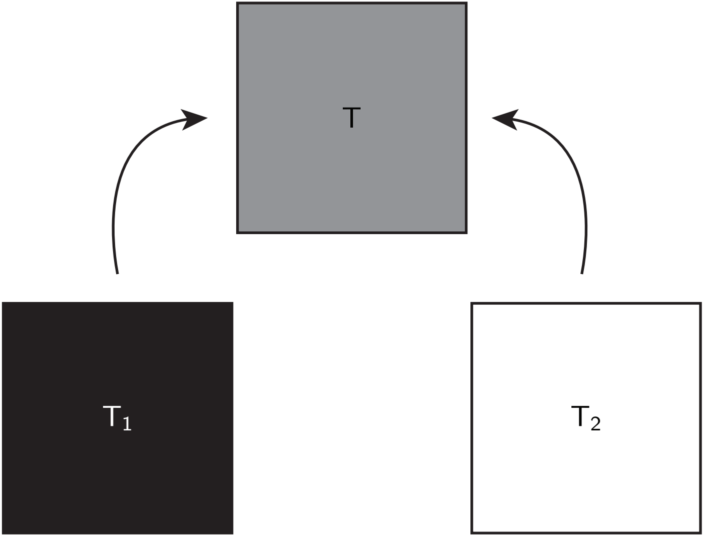
$$\mathrm{ZFC}$$

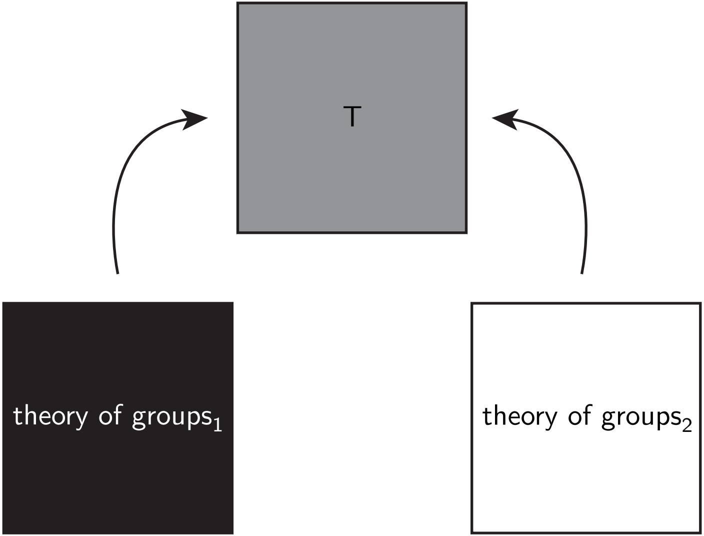
$$\cup$$

$$\{\forall x \forall y (x \subseteq y \leftrightarrow \forall z (z \in x \rightarrow z \in y))\}$$

A definitional extension of ZFC to $\{s, \in, \subseteq\}$.

Two theories are **definitionally equivalent** if they have a common definitional extension.


T
T_1
T_2

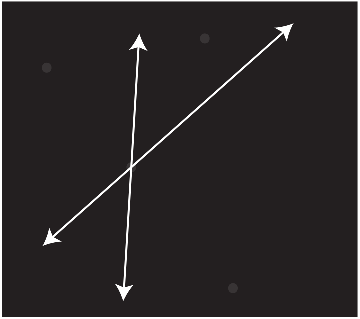

T
theory of groups1
theory of groups2

logical equivalence $\longrightarrow$ definitional equivalence ? Morita equivalence ? categorical equivalence

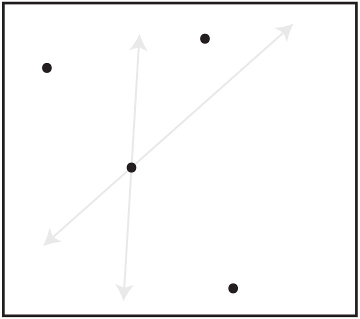
logical equivalence $\not\longleftarrow$ definitional equivalence

logical equivalence	$\longrightarrow$ $\not\longleftarrow$	definitional equivalence	?	**Morita equivalence**	?	categorical equivalence

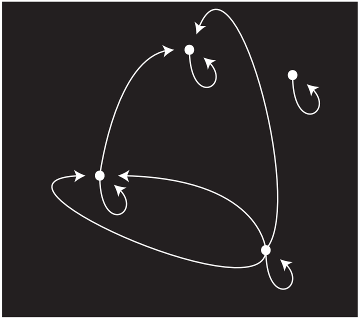
There are some pairs of theories that are not definitionally equivalent, but are nonetheless intuitively equivalent.



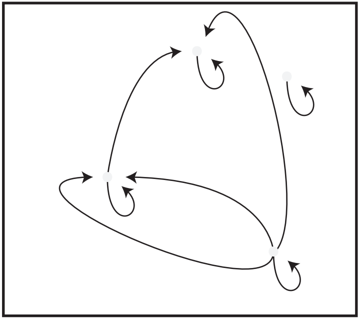
Euclidean geometry
with lines



Euclidean geometry
with points



Category theory
with objects and arrows



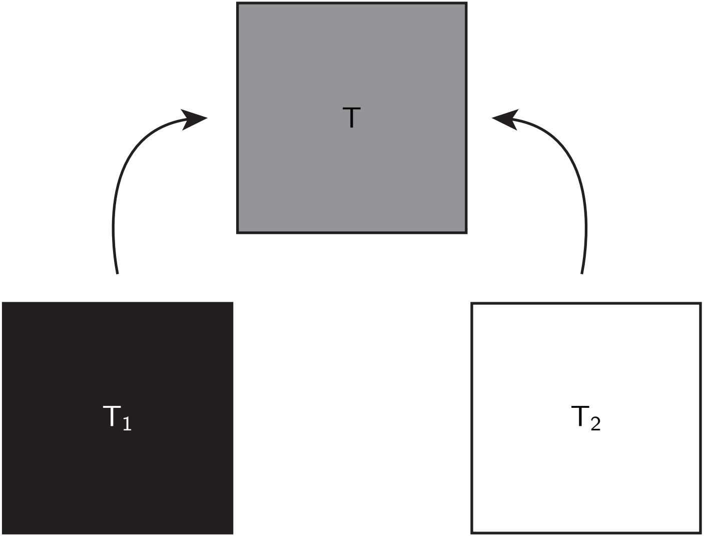
Category theory
with arrows

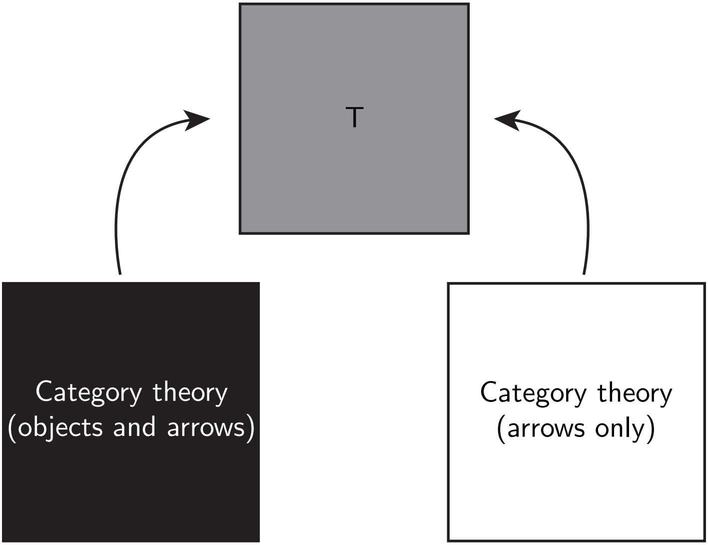
We need a more general criterion for theoretical equivalence than definitional equivalence.

Kiiti Morita

A **Morita extension** of a theory T is a theory T^+ obtained by adding to T definitions of new *sort* symbols, predicate symbols, function symbols, and constant symbols.

Two theories are **Morita equivalent** if they have a common Morita extension.


T
T_1
T_2

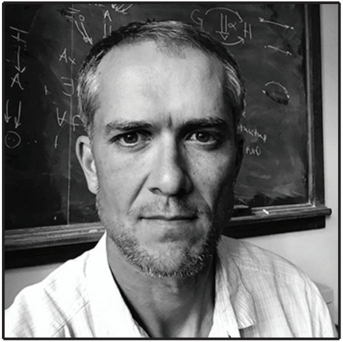

T
Category theory (objects and arrows)
Category theory (arrows only)

logical equivalence	$\longrightarrow$ $\not\longleftarrow$	definitional equivalence	$\longrightarrow$ $\not\longleftarrow$	Morita equivalence	?	categorical equivalence

logical equivalence	$\longrightarrow$ $\not\longleftarrow$	definitional equivalence	$\longrightarrow$ $\not\longleftarrow$	Morita equivalence	?	**categorical equivalence**

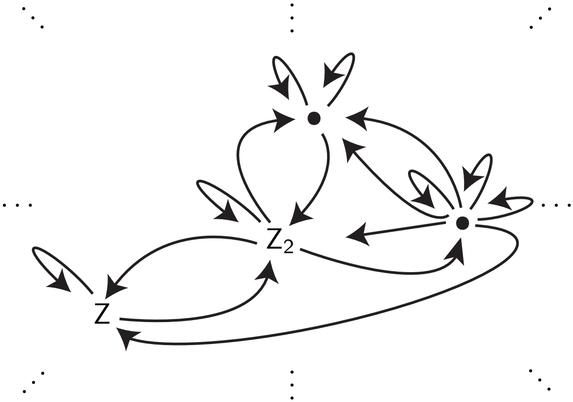
Morita equivalence is a difficult concept to apply to physical theories.

Jim Weatherall



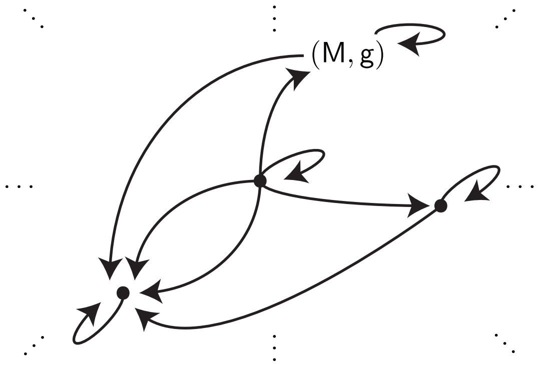
Hans Halvorson

First-order theories have categories of models.



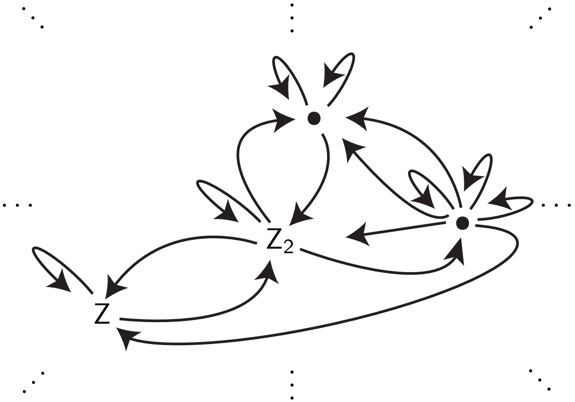
The category of models for the theory of groups$_1$.

Physical theories have categories of models too.

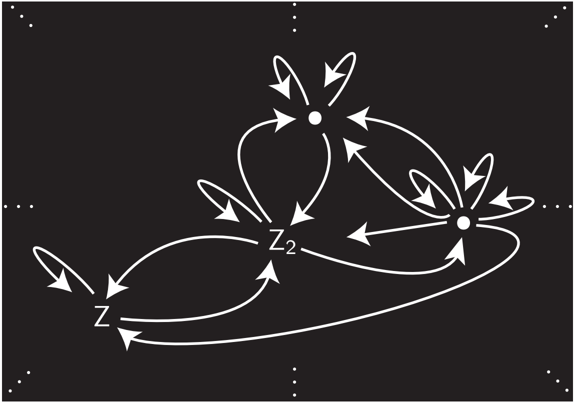


The category of models for general relativity.

Two theories are **categorically equivalent** if they have equivalent (structurally identical) categories of models.



The category of models for the theory of groups$_1$.



The category of models for the theory of groups$_2$.

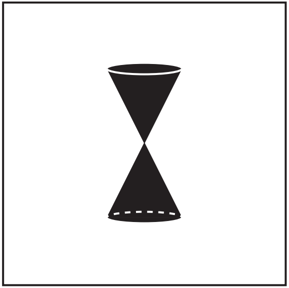
Categorical equivalence captures a sense in which pairs of theories are equivalent.

Theorem 1. If two theories are Morita equivalent, then they are categorically equivalent.

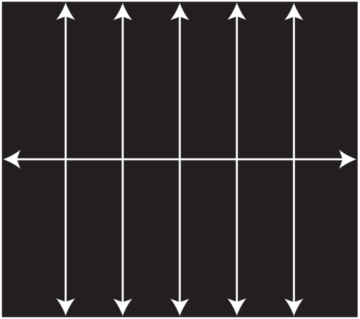
Theorem 2. There are categorically equivalent theories that are not Morita equivalent.

logical equivalence $\underset{\not\longleftarrow}{\longrightarrow}$ definitional equivalence $\underset{\not\longleftarrow}{\longrightarrow}$ Morita equivalence $\underset{\not\longleftarrow}{\longrightarrow}$ categorical equivalence

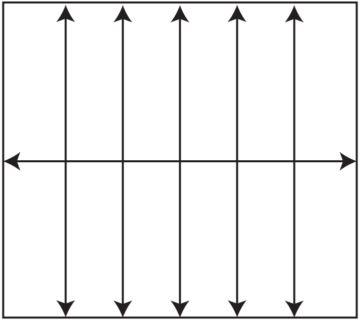
(1-3) General relativity



(3-1) General relativity

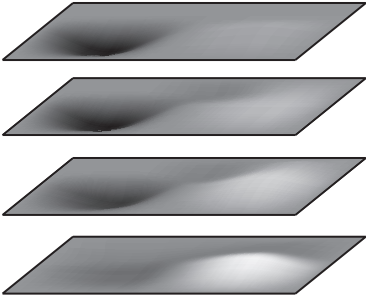


Hamiltonian mechanics

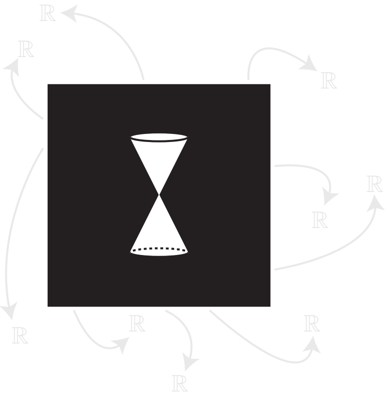


Lagrangian mechanics

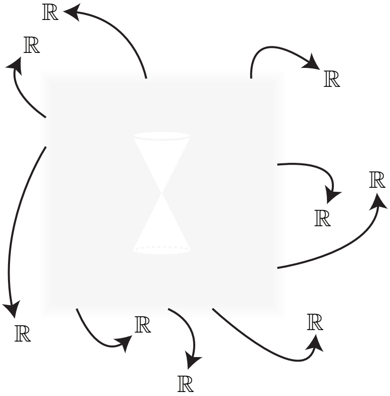
Newtonian Gravitation



Geometrized
Newtonian Gravitation



General relativity



The theory of
Einstein algebras

thank you.