How generalized Minkowski four-force leads to scalar-tensor gravity

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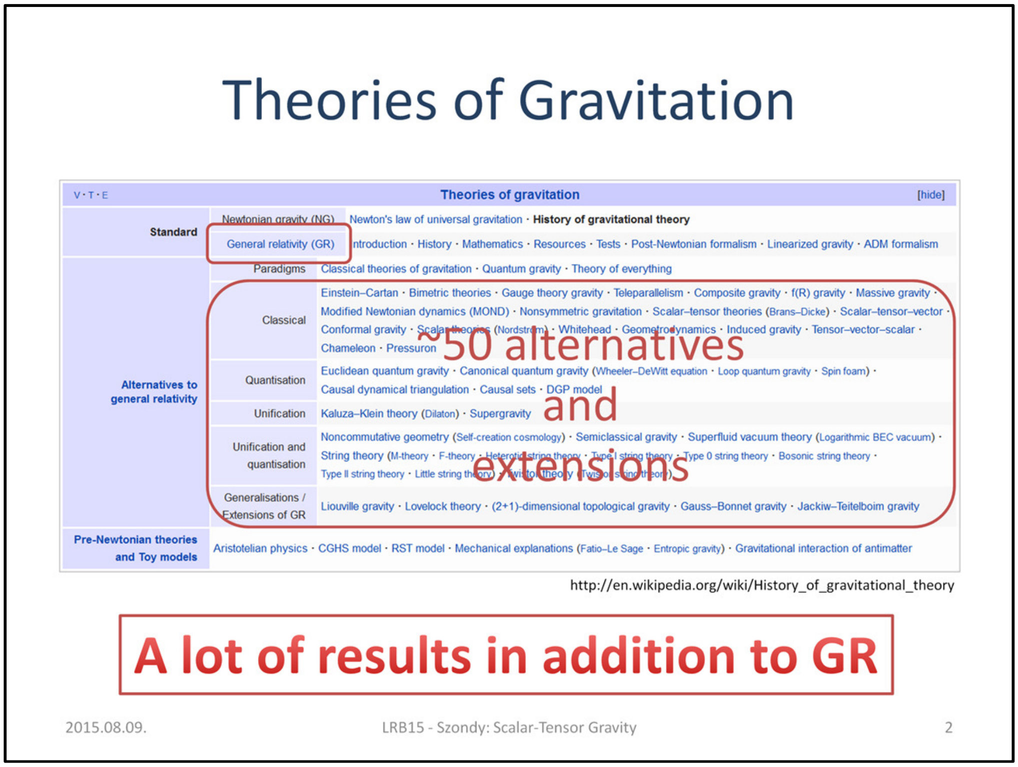
The title of my presentation may suggest that an **alternative scalar-tensor theory of gravitation** is presented here.

My opinion is that we have some freedom is to choose the **set of definitions** that we use, and the result is that we can have a slightly different view, or different interpretation of the same physical reality.

Coexistence of these approaches and comparing the view they present might be useful to understand our world better and overcome problems that we are facing at this field.



General Relativity (GR) is considered so far the best theory of gravitation.
But due to its's limitations, like nonlinearity and lack of QG all agree, that this is not the final theory.

This table from Wikipedia lists alternatives and extensions of GR.

The huge amount of alternative theories indicates another thing as well: there are a lot of useful results outside the scope of GR, that are worth to know.

Topics

- Special Relativity and Minkowski four-force
- Scalar-Tensor Gravity
- Quantum effects (causing this Scalar field)

In my presentation

- I will bulid on the fundamentals and Special Relativity,
- I will mainly deal with a possible Scalar-Tensor theory and
- I will also mention a kind of quantum gravity.

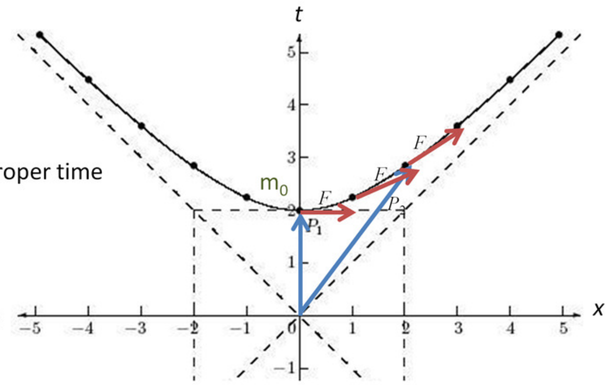
Minkowski (four)-force

$c = 1$

$\boxed{\overline{p} = (E, p) = m_0 \overline{\boldsymbol{u}}}$

$\boxed{m_0 = \sqrt{E^2 - p^2}}$

$\boxed{\overline{\boldsymbol{F}} = m\overline{\boldsymbol{a}} = m\frac{d\overline{\boldsymbol{u}}}{d\tau} = \frac{d\overline{\boldsymbol{p}}}{d\tau}}$ τ →proper time



REST MASS is INVARIANT

$\overline{\boldsymbol{a}} = \frac{d\overline{\boldsymbol{u}}}{d\tau}$ $a_i u^i = 0 \leftrightarrow \overline{\boldsymbol{a}} \perp \overline{\boldsymbol{u}}$

$\overline{\boldsymbol{F}} \perp \overline{\boldsymbol{p}} \leftrightarrow F_i p^i = 0$ orthogonal

$\boxed{\overline{\boldsymbol{F}} \perp \overline{\boldsymbol{p}} \rightarrow \textbf{Special case}}$

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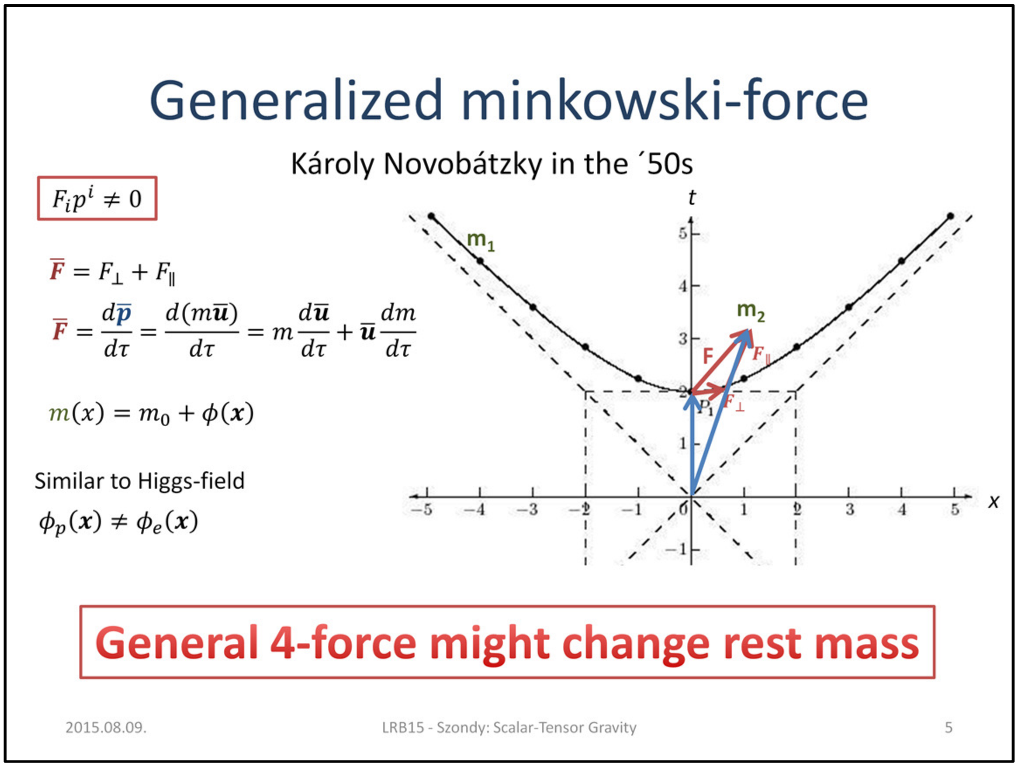
This is a Minkowski diagram.
I will user c=1 measure for simplicity.
The blue vector is the four-momentum of a test particle.
The Minkowski norm of the four-momentum vector (with c=1) is the rest mass.

1. Minkowski 4 force defined similar to the 3D force as the change of the four-momentum vector. (with proper time)
2. In usual case the rest mass (M) is invariant, so F will not change the norm of the four momentum vector → F & p are orthogonal
3. Károly Novobatzky in the 50s said that this is a SPECIAL CASE



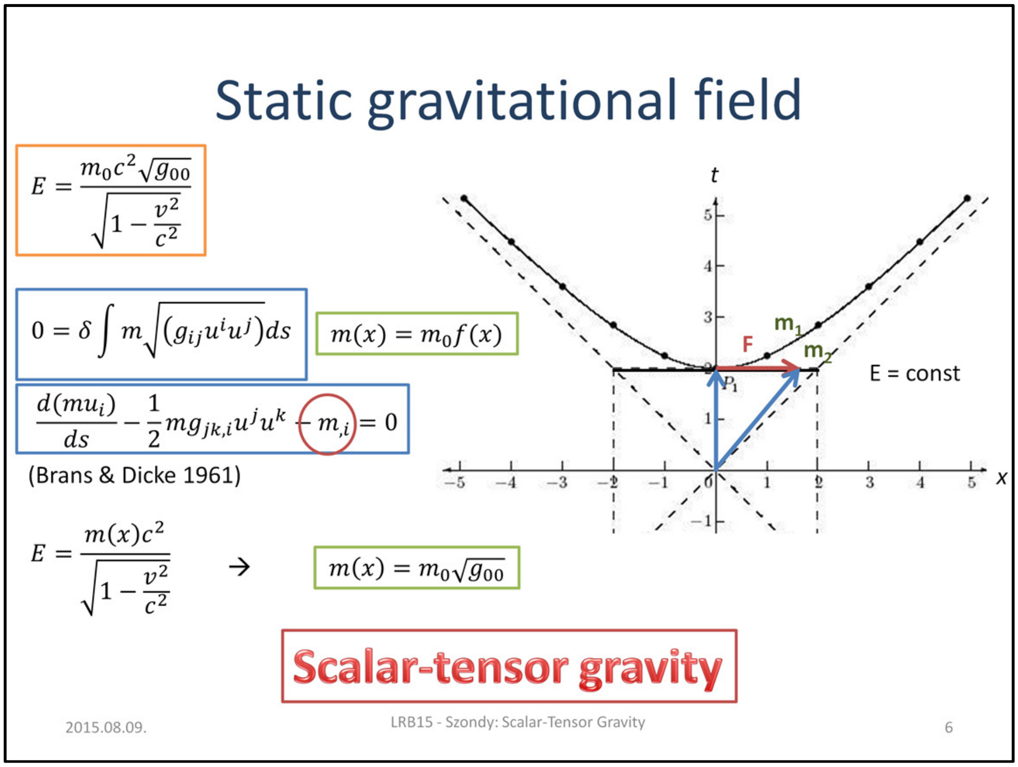
To explain Generalized minkowski-force we will start from the minkowski diagram and the 4momentum vector again.

1. Károly Novobátzky said that: a general Minkowski 4-fource is not necessarily othogonal to the 4-momentum vector

→ a general Minkowski 4-fource might change the norm of the 4-momentum vector,
- so it might change the rest mass

→ In this general case the fource field interacts with the rest mass
- Usual example is carottage effect, where the rest mass of electron (e-) changes
- We have seen such field → Higgs field, where the whole mass of particle is coming from the field
  - Worth to mention that in case of Higgs field this function is different for different elementary particles

→ I will give another, well known example



I will will give another example…

→ the free falling test particle in static gravitational field.

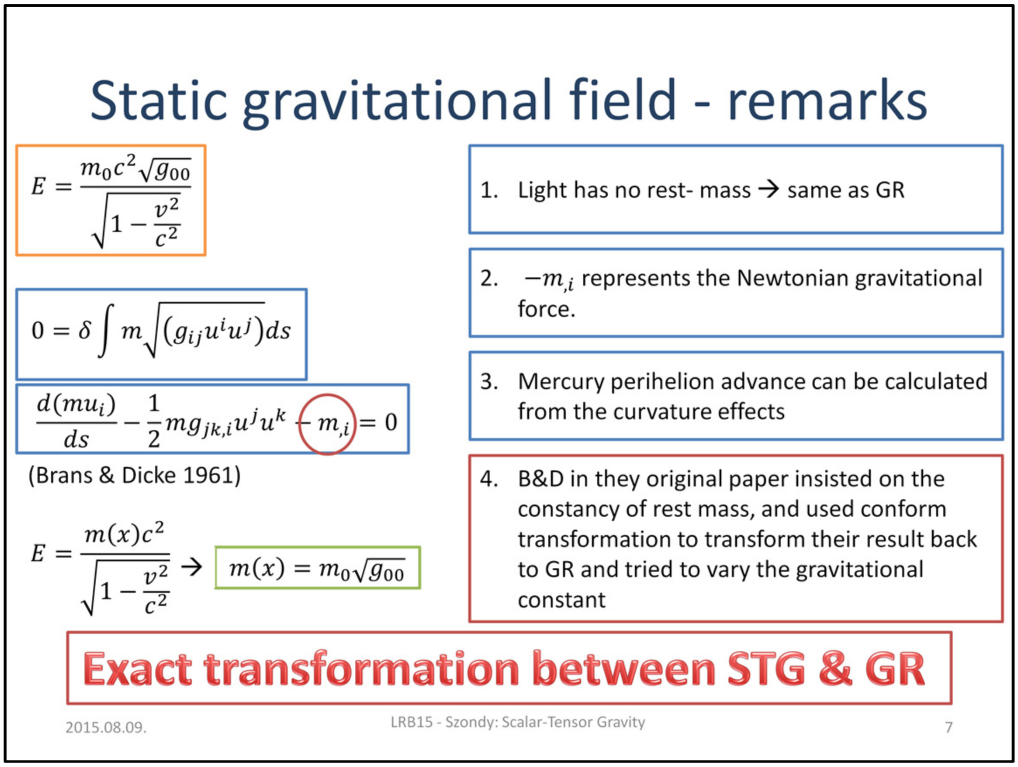
The direction of time axis of Minkowski diagram is bound to the central massive object of the static field.

1. It is well known that the total energy of the free-falling particle is described with the equation at the upper left:
   - here we can see the Lorentz factor
   - as well as the effect gravitational field that is represented by the g_00 component of the metric tensor
2. The total energy of the free falling particle remains constant
3. It means (we can read out from the diagram that) if the 4 momentum changes, the force will also change the rest mass of the particle
   - (end of the 4momentum vector is not on the m=const hyperbolic)
4. We can describe the movement of such particle
   - starting from the Variational principle
   - Considering that the rest mass will change via a scalar function
   - The resulting equation of motion looks like this.
   - This form has been published by Brans & Dicke in the early ´60s
5. The only diffence from the Einsteinian Equation of Motion is an additional term – the change of rest mass

6. In our example we can express this scalar function

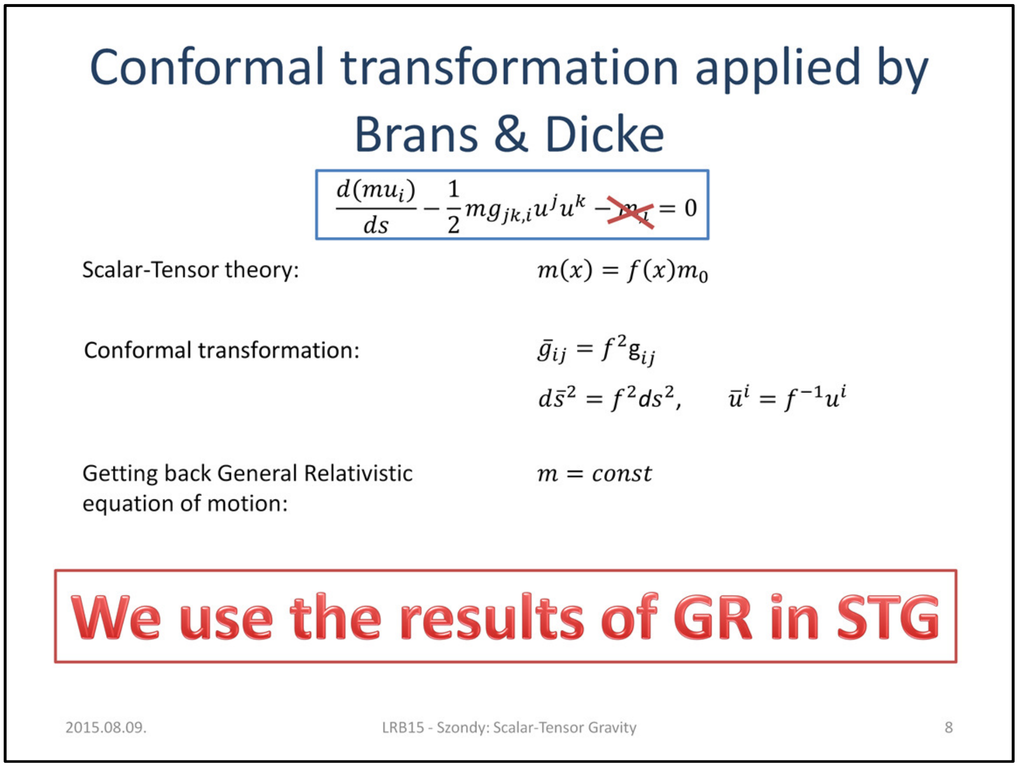
- if get the Lorenz formula
- and we substitute the variable rest mass in it

- g_00 component of metric tensor represents scalar gravitational effect → change of rest mass



Remarks on this Equation of Motion (EoM) and Brans-Dicke (BD) theory

1. Light has no rest mass – so it moves on geodesics – EoM is just like in case of GR
2. $-m_{,i}$ represents the Newtonian gravitaional force. In case of matter there are 2 separated effects:
    - Newtonian gravity
    - Curvature effects
3. perihelion advance of Mercury can be calculated accordingly
    - SR effects + curvature effects (represented by the deflection of light along the orbit.)
4. B&D in they original paper insisted on the constancy of rest mass, and used comform transformation to transform their result back to GR and tried to vary the gravitaional constant.



We have the rest mass varied with this location-dependent scalar function „f”.

1. If we apply a conformal transformation, where we multiply all members of the metric tensor with the factor f-square
2. The measures (including the rest mass) are rescaled with the factor f -1
3. The result is that rest-mass became invariant again and we got back the Einsteinian EoM

This was the original approach of Brans & Dicke.

But we do it the other way around: we transform the results of GR to the Scalar-Tensor Gravity (STG) and observe the results.

How to transform Schwarzscild solution to Scalar-Tensor gravity

$$\bar{g}_{ij} = f^2 g_{ij}$$

Schwarschild solution uses standard coordinates

$$ds^2 = A(r)dt^2 + B(r)dr^2 + \boxed{r^2(d\theta^2 + \sin^2\theta d\varphi^2)}$$

Coordinate transformation $\Downarrow$ $\boxed{r = \rho(1 + \frac{r_s}{4\rho})^2}$

Isotropic coordinates

$$ds^2 = A'(\rho)dt^2 + \boxed{B'(\rho)(d\rho^2 + \rho^2 d\theta^2 + \rho^2 \sin^2\theta d\varphi^2)}$$

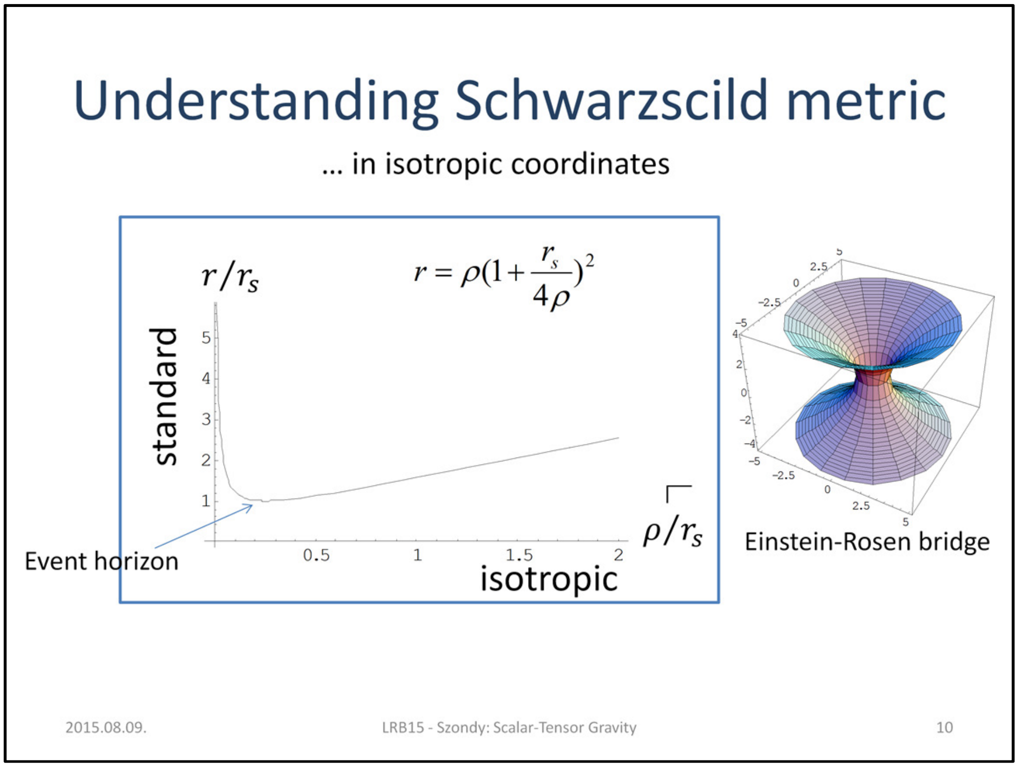
Schwarzschild metric: $ds^2 = \left(\frac{1 - \frac{r_g}{4\rho}}{1 + \frac{r_g}{4\rho}}\right)^2 dt^2 + \left(1 + \frac{r_g}{4\rho}\right)^4 (d\rho^2 + \rho^2 d\theta^2 + \rho^2 \sin^2\theta d\varphi^2)$

We would like to apply conformal transformation on the Schwartschild solution…
The problem is that Schwarschild solution uses standard coordinates… which are not ideal for the conformal transformation.

If we could have isotriopic coordinates, then it would be much better.

Fortunately the transformation is well known from textbooks (e.g. Landau) and the metric tensor of Schwarschild solution in isotriopic coordinates is also available there.

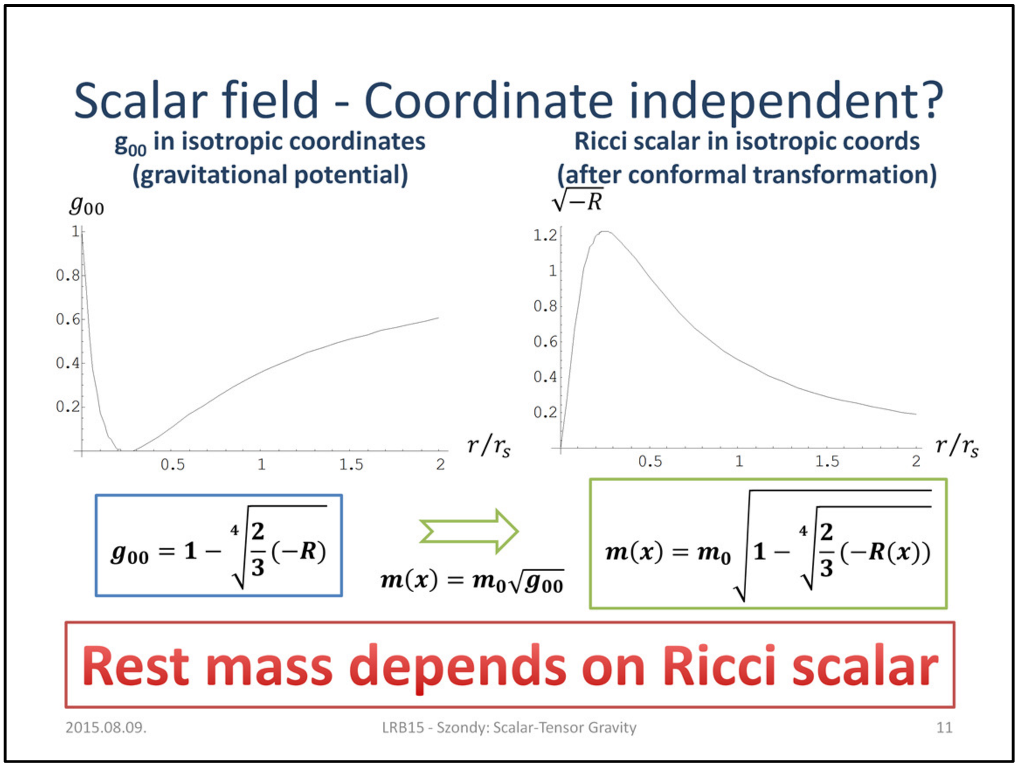
If we have a closer look at the transformation we will find interesting things: →



To have a deeper understandig
I created the diagram of this transformation. (standard→isotropic)

1. On the vertical axis we can see the Standard radial coordinate
2. On the horizontal axis we can see the Isotropic radial coordinate

3. We can see the event horizon in standars coordinates
4. The solution is automatically extended beyond the event horizon
5. The result seems to be an Einstein-Rosen bridge
6. It seems that there is nothing inside the event horizon (in Standard coordinates r<R_s), as it is not part of the domain



We have a scalar field constructed from the metric… but we would like this scalar field being coordinate independent.
Using standard coordinates Brans and Dicke concluded that scalars of the metric tensor are not suitable.

Using isotropic coordinates and this scalar-tensor approach I got a different conclusion:
I created 2 diagrams:
- 1. g_00 – representing the gravitational potential
- 2. square-root of the Ricci scalar (R) after the conformal transformation

It seems that there might be correlation…

… more precisely, we can express g_00 with the Ricci scalar…

Considering, that rest mass changes with g_00, we can express change of rest mass with the scalar curvature…
So it seems, that rest mass is in correspondence with the local background curvature of the spacetime.

This correspondence is a result (I suspect new result) of this Scalar-Tensor approach.

Still a question raises: How can background curvature change rest mass?

In the last 2 slides I will share a model that might explain this.

How can curvature effect Rest-Mass?

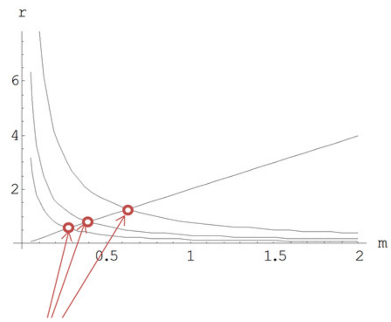
Particle model from caracteristic lengths

Schwarzschild-radius

$$r_s = \frac{2Gm}{c^2}$$

Quantum-radius

$$r_Q = \frac{k}{2\pi}\lambda_{Compton} = \frac{hk}{2\pi mc}$$



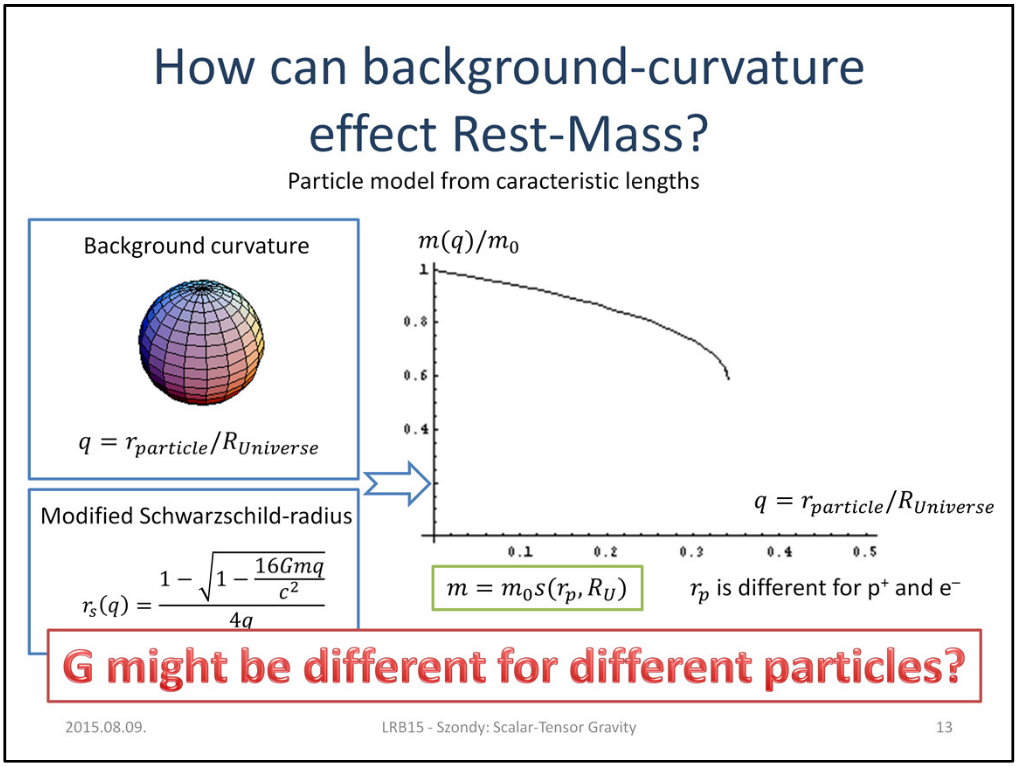
Rest-mass and size depends on ‚k' quantum number

Different possible particles

How can curvature effect rest mass?
For this we created a simple particle model from 2 characteristic lengths.

1. the Schwarzschild radius (proportional to the rest mass)
2. quantum radius (constructed from the Compton wavelength and an arbitrary quantum number k)

With this approach, we got discrete solutions for the rest-mass and size of such theoretical particles depending on the on the quantum number.



How can background-curvature change Rest-Mass?

- In this model we deal with curvature as if the universe would be uniformly curved and finite size.
- We define a relative curvature ‚q' as the rate of the size of the test particle and the size of the Universe.

We found, that **Schwarzschild radius** (more precisely the size of the photon sphere) **depends** on the relative **background curvature** – q

So we have a modified Schwarzschild radius, and we calculate the rest mass of our „quantum particles" using this modified equation.

Solving the equation **analytically** we got what we expected: that the rest mass of our „quantum particle" depends on this relative background curvature.

As „q" depends on the quantum number, this effect will possibly different for different kind of particles (p+ and e-) → violating the Equivalence Principle (EP) and having different gravitational constant (G) for different elementary particles.
→ About Eötvös experiment: usual material consists of the same kind of elementary particles in the same rate → can not measure a difference.

Conclusion

- General force field can change rest mass
- In scalar-tensor gravity Newtonian gravity and curvature-effects are seperated
- Background curvature changes rest mass
- Rest mass change are due to quantum effects
- Gravitational constant is different for different elementary particles

There is a lot to do

References

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Thank you for your attention!

Comments & Questions