How generalized Minkowski four-force leads to scalar-tensor gravity

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Theories of Gravitation

V · T · E		Theories of gravitation [hide]
Standard	Newtonian gravity (NG)	Newton's law of universal gravitation · **History of gravitational theory**
	General relativity (GR)	Introduction · History · Mathematics · Resources · Tests · Post-Newtonian formalism · Linearized gravity · ADM formalism
Alternatives to general relativity	Paradigms	Classical theories of gravitation · Quantum gravity · Theory of everything
	Classical	Einstein–Cartan · Bimetric theories · Gauge theory gravity · Teleparallelism · Composite gravity · f(R) gravity · Massive gravity · Modified Newtonian dynamics (MOND) · Nonsymmetric gravitation · Scalar–tensor theories (Brans–Dicke) · Scalar–tensor–vector · Conformal gravity · Scalar theories (Nordström) · Whitehead · Geometrodynamics · Induced gravity · Tensor–vector–scalar · Chameleon · Pressuron
	Quantisation	Euclidean quantum gravity · Canonical quantum gravity (Wheeler–DeWitt equation · Loop quantum gravity · Spin foam) · Causal dynamical triangulation · Causal sets · DGP model
	Unification	Kaluza–Klein theory (Dilaton) · Supergravity
	Unification and quantisation	Noncommutative geometry (Self-creation cosmology) · Semiclassical gravity · Superfluid vacuum theory (Logarithmic BEC vacuum) · String theory (M-theory · F-theory · Heterotic string theory · Type I string theory · Type 0 string theory · Bosonic string theory · Type II string theory · Little string theory) · Twistor theory (Twistor string theory)
	Generalisations / Extensions of GR	Liouville gravity · Lovelock theory · (2+1)-dimensional topological gravity · Gauss–Bonnet gravity · Jackiw–Teitelboim gravity
Pre-Newtonian theories and Toy models		Aristotelian physics · CGHS model · RST model · Mechanical explanations (Fatio–Le Sage · Entropic gravity) · Gravitational interaction of antimatter

~50 alternatives and extensions

http://en.wikipedia.org/wiki/History_of_gravitational_theory

A lot of results in addition to GR

Topics

- Special Relativity and Minkowski four-force
- Scalar-Tensor Gravity
- Quantum effects (causing this Scalar field)

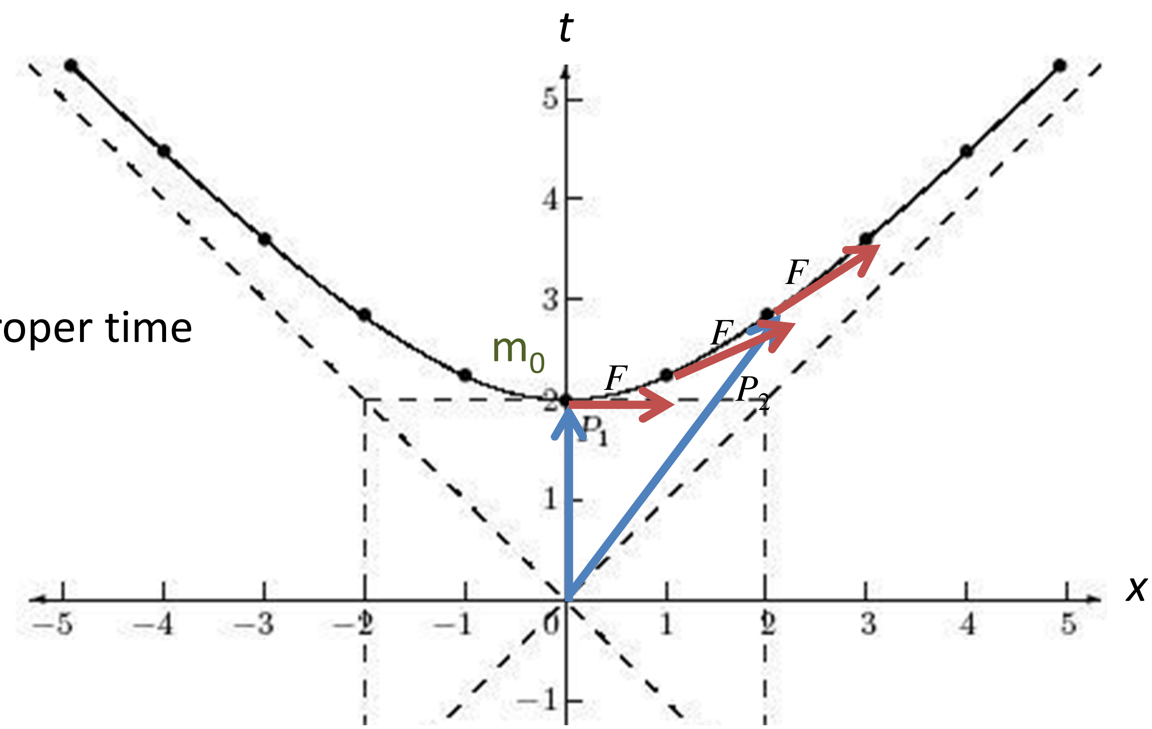
Minkowski (four)-force

$c = 1$

$\overline{\boldsymbol{p}} = (E, p) = m_0 \overline{\boldsymbol{u}}$

$m_0 = \sqrt{E^2 - p^2}$

$\overline{\boldsymbol{F}} = m\overline{\boldsymbol{a}} = m\dfrac{d\overline{\boldsymbol{u}}}{d\tau} = \dfrac{d\overline{\boldsymbol{p}}}{d\tau}$ τ →proper time



REST MASS is INVARIANT

$\overline{\boldsymbol{a}} = \dfrac{d\overline{\boldsymbol{u}}}{d\tau}$ $a_i u^i = 0 \leftrightarrow \overline{\boldsymbol{a}} \perp \overline{\boldsymbol{u}}$

$\overline{\boldsymbol{F}} \perp \overline{\boldsymbol{p}} \leftrightarrow F_i p^i = 0$ orthogonal

$\overline{\boldsymbol{F}} \perp \overline{\boldsymbol{p}}$ → **Special case**

Generalized minkowski-force

Károly Novobátzky in the ´50s

$F_i p^i \neq 0$

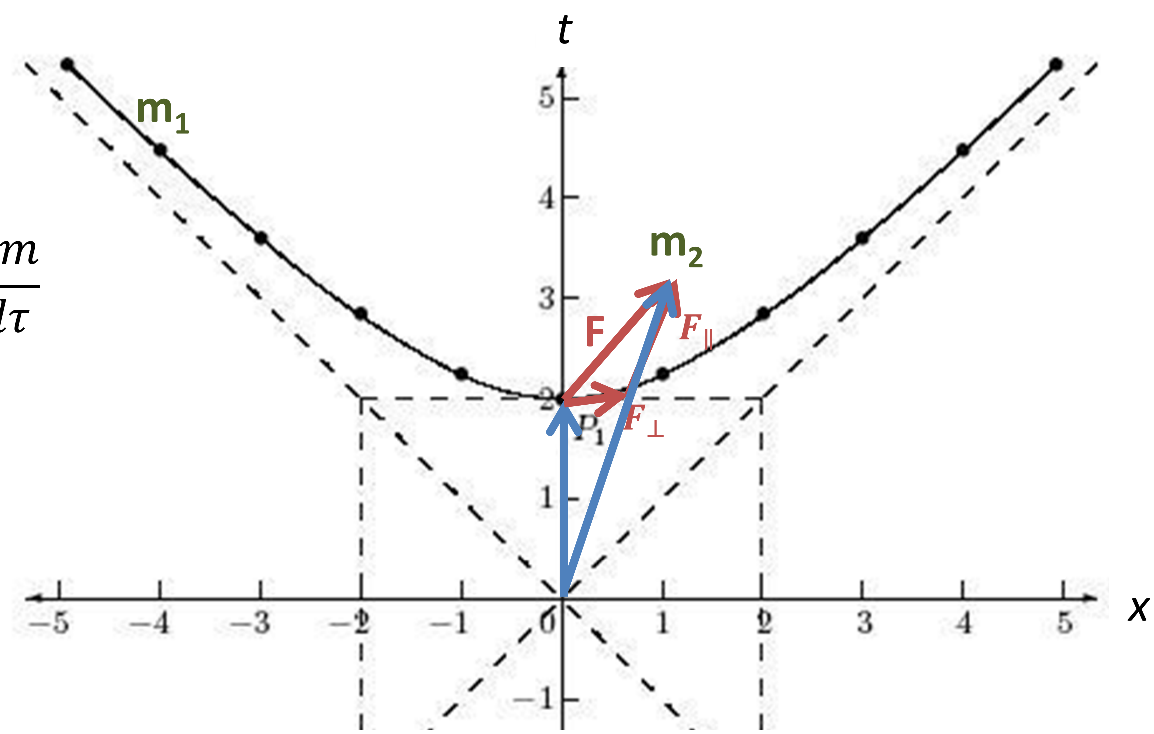
$\overline{F} = F_\perp + F_\parallel$

$\overline{F} = \frac{d\overline{p}}{d\tau} = \frac{d(m\overline{u})}{d\tau} = m\frac{d\overline{u}}{d\tau} + \overline{u}\frac{dm}{d\tau}$

$m(x) = m_0 + \phi(\boldsymbol{x})$

Similar to Higgs-field

$\phi_p(\boldsymbol{x}) \neq \phi_e(\boldsymbol{x})$



General 4-force might change rest mass

Static gravitational field

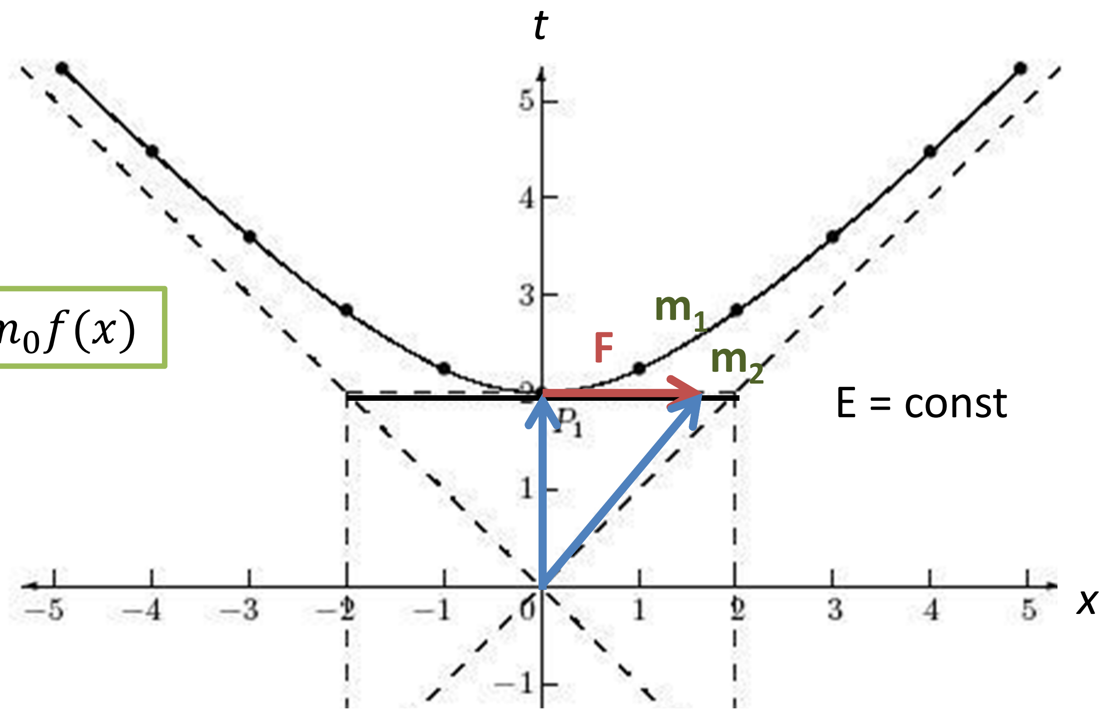
$$E = \frac{m_0 c^2 \sqrt{g_{00}}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$0 = \delta \int m \sqrt{\left(g_{ij} u^i u^j\right)} ds$$

$$m(x) = m_0 f(x)$$

$$\frac{d(mu_i)}{ds} - \frac{1}{2} m g_{jk,i} u^j u^k - m_{,i} = 0$$

(Brans & Dicke 1961)



$$E = \frac{m(x) c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$ → $$m(x) = m_0 \sqrt{g_{00}}$$

Scalar-tensor gravity

Static gravitational field - remarks

$$E = \frac{m_0 c^2 \sqrt{g_{00}}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$0 = \delta \int m \sqrt{(g_{ij} u^i u^j)} ds$$

$$\frac{d(mu_i)}{ds} - \frac{1}{2} m g_{jk,i} u^j u^k - m_{,i} = 0$$

(Brans & Dicke 1961)

$$E = \frac{m(x) c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \rightarrow m(x) = m_0 \sqrt{g_{00}}$$

1. Light has no rest- mass → same as GR
2. $-m_{,i}$ represents the Newtonian gravitational force.
3. Mercury perihelion advance can be calculated from the curvature effects
4. B&D in they original paper insisted on the constancy of rest mass, and used conform transformation to transform their result back to GR and tried to vary the gravitational constant

Exact transformation between STG & GR

Conformal transformation applied by Brans & Dicke

$$\frac{d(mu_i)}{ds} - \frac{1}{2} m g_{jk,i} u^j u^k - \cancel{m_{,i}} = 0$$

Scalar-Tensor theory: $m(x) = f(x) m_0$

Conformal transformation: $\bar{g}_{ij} = f^2 g_{ij}$

$$d\bar{s}^2 = f^2 ds^2, \qquad \bar{u}^i = f^{-1} u^i$$

Getting back General Relativistic equation of motion: $m = const$

We use the results of GR in STG

How to transform Schwarzscild solution to Scalar-Tensor gravity

$$\bar{g}_{ij} = f^2 g_{ij}$$

Schwarschild solution uses standard coordinates

$$ds^2 = A(r)dt^2 + B(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

Coordinate transformation

$$r = \rho(1 + \frac{r_s}{4\rho})^2$$

Isotropic coordinates

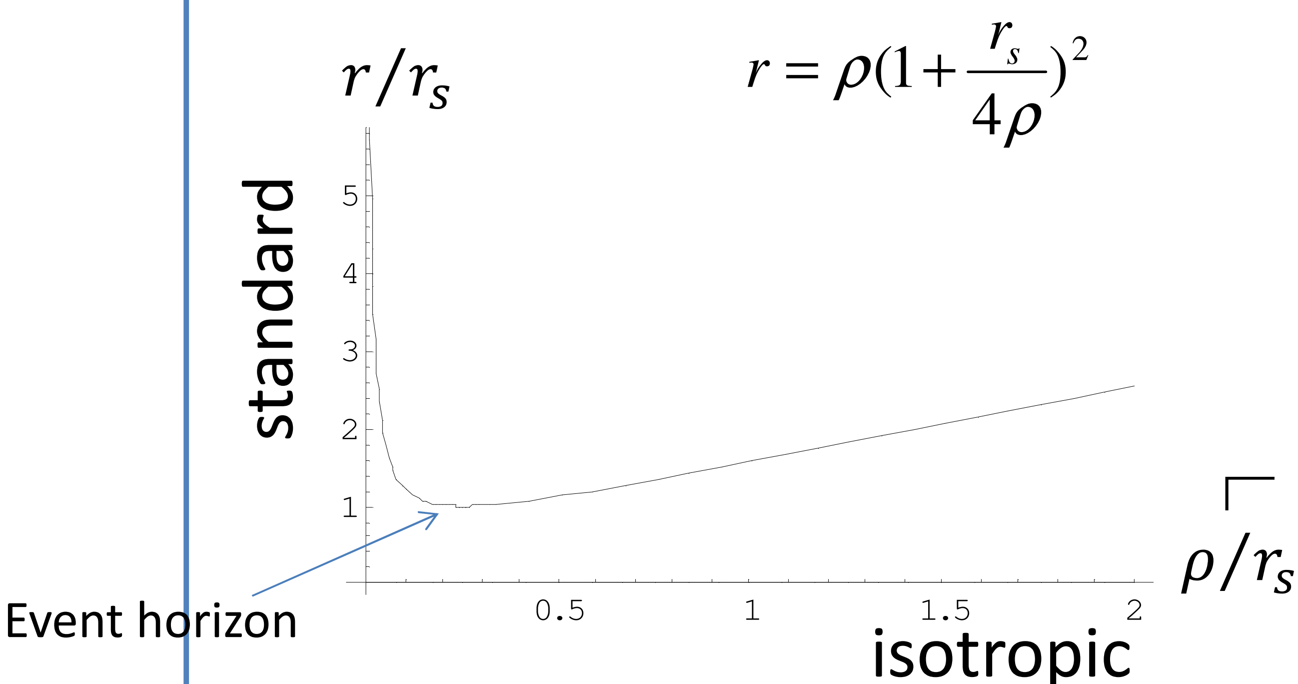
$$ds^2 = A'(\rho)dt^2 + B'(\rho)(d\rho^2 + \rho^2 d\theta^2 + \rho^2 \sin^2\theta d\varphi^2)$$

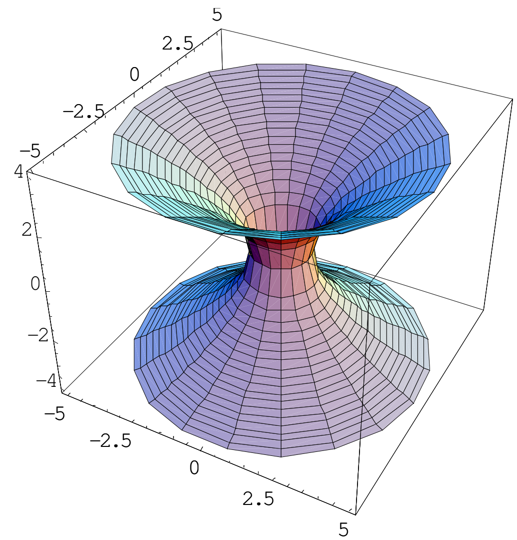
Schwarzschild metric:

$$ds^2 = \left(\frac{1 - \frac{r_g}{4\rho}}{1 + \frac{r_g}{4\rho}}\right)^2 dt^2 + \left(1 + \frac{r_g}{4\rho}\right)^4 (d\rho^2 + \rho^2 d\theta^2 + \rho^2 \sin^2\theta d\varphi^2)$$

Understanding Schwarzscild metric

… in isotropic coordinates

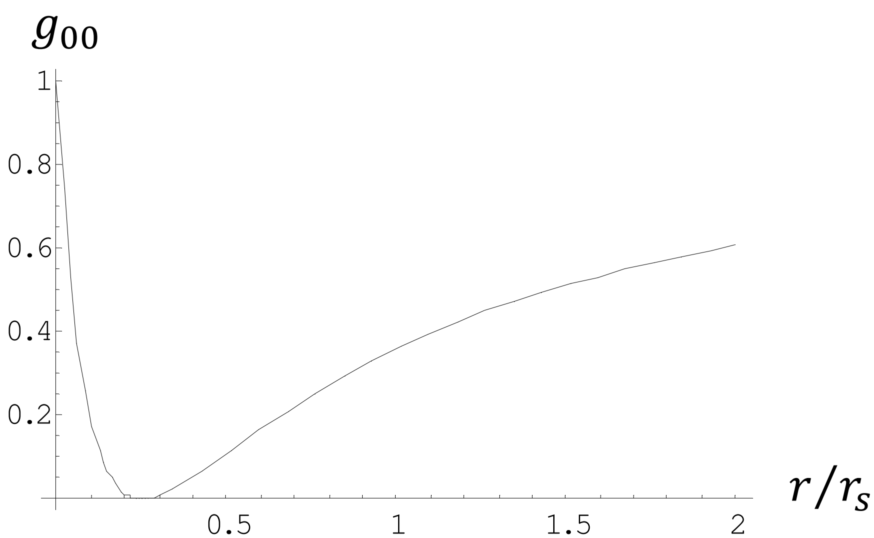




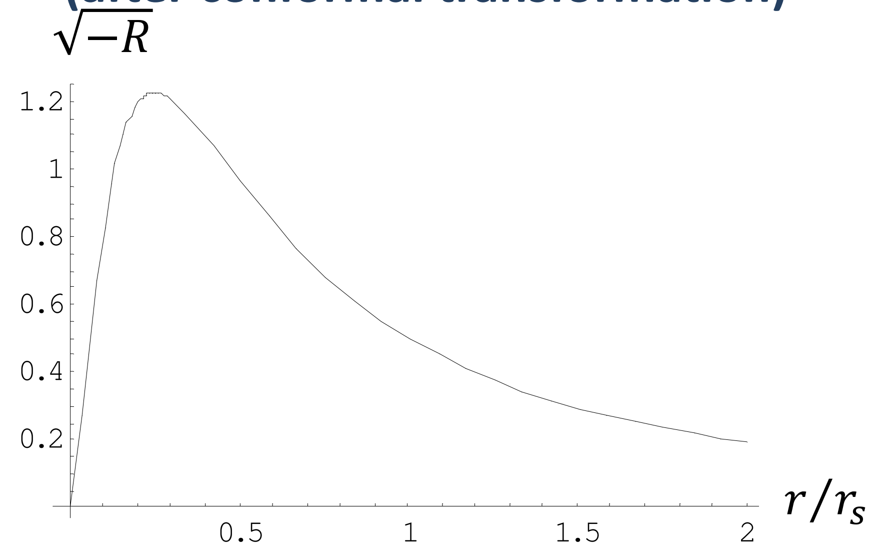
Einstein-Rosen bridge

Scalar field - Coordinate independent?

g_{00} in isotropic coordinates (gravitational potential)



Ricci scalar in isotropic coords (after conformal transformation)



$$g_{00} = 1 - \sqrt[4]{\frac{2}{3}(-R)}$$

$$m(x) = m_0\sqrt{g_{00}}$$

$$m(x) = m_0\sqrt{1 - \sqrt[4]{\frac{2}{3}(-R(x))}}$$

Rest mass depends on Ricci scalar

How can curvature effect Rest-Mass?

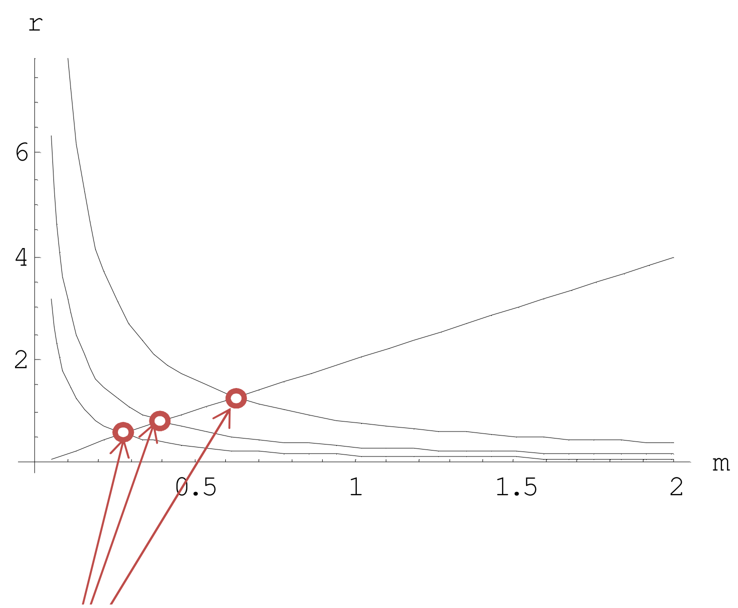
Particle model from caracteristic lengths

Schwarzschild-radius

$$r_s = \frac{2Gm}{c^2}$$

Quantum-radius

$$r_Q = \frac{k}{2\pi}\lambda_{Compton} = \frac{hk}{2\pi mc}$$



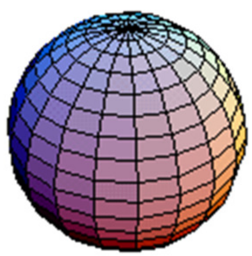
Rest-mass and size depends on ‚k' quantum number

Different possible particles

How can background-curvature effect Rest-Mass?

Particle model from caracteristic lengths

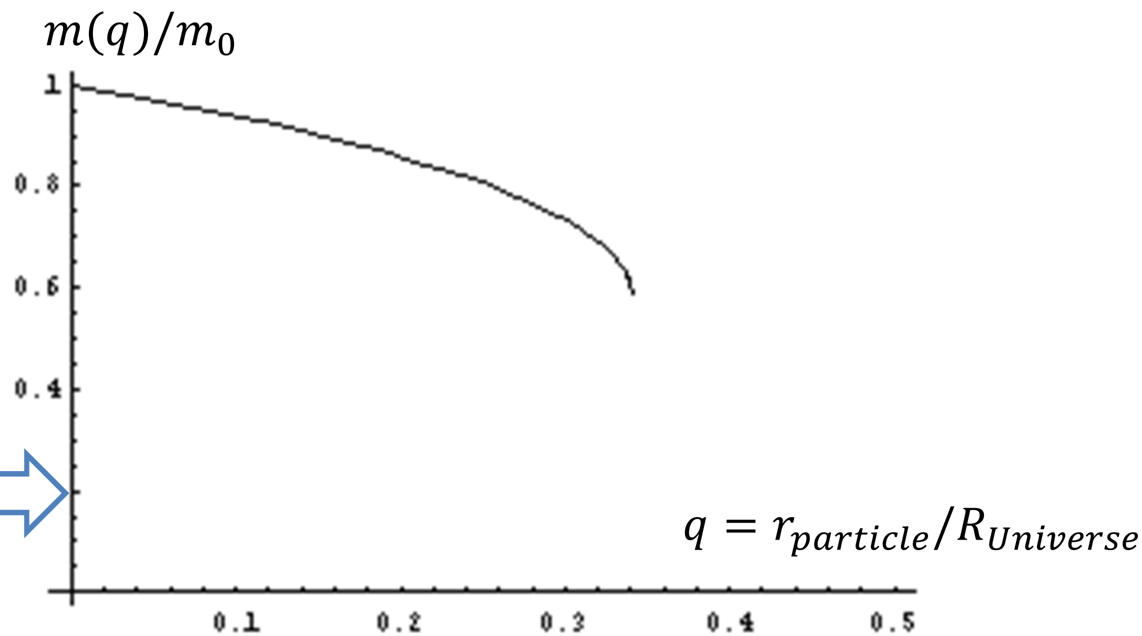
Background curvature



$q = r_{particle}/R_{Universe}$

Modified Schwarzschild-radius

$$r_s(q) = \frac{1 - \sqrt{1 - \frac{16Gmq}{c^2}}}{4q}$$



$m = m_0 s(r_p, R_U)$

r_p is different for p^+ and e^-

G might be different for different particles?

Conclusion

- General force field can change rest mass
- In scalar-tensor gravity Newtonian gravity and curvature-effects are seperated
- Background curvature changes rest mass
- Rest mass change are due to quantum effects
- Gravitational constant is different for different elementary particles

There is a lot to do

References

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Thank you for your attention!

Comments & Questions