Splitting methods in algebraic logic in connection to non-atom–canonicity and non-first order definability

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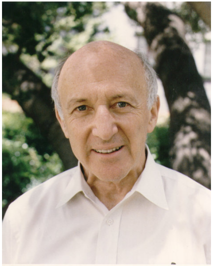
Theorem

Every locally finite CA_ω is representable.

Let $n \geq 3$ be a finite number.



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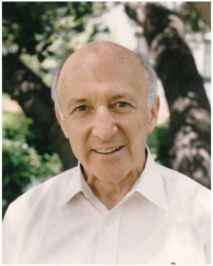
- Finite variable fragment of FOl.



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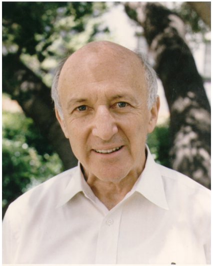

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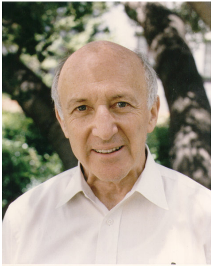

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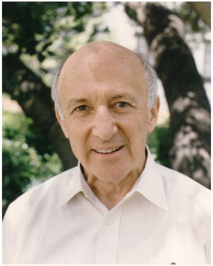

Theorem

Neat Embedding Theorem For any ordinal $\alpha > 0$*,* $\mathbf{S}\mathrm{Nr}_\alpha CA_{\alpha+\omega} = RCA_\alpha$*. In particular,* $\mathbf{S}\mathrm{Nr}_n CA_\omega = RCA_n$*.*

Theorem

$RCA_n \subsetneq \mathbf{S}\mathrm{Nr}_n CA_{n+k}$, for every finite k.
Hence, RCA_n is not finitely axiomatizable.

Theorem (Hirsch, Hodkinson and Maddux)

$$\mathbf{S}\mathrm{Nr}_n CA_{n+k+1} \subsetneq \mathbf{S}\mathrm{Nr}_n CA_{n+k}, \textit{ for all positive } k.$$

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Theorem (Hirsch and Hodkinson)

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Theorem (Hodkinson)

RCA_n is not atom-canonical.

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Theorem

- *Any class K between* $\mathrm{Nr}_n CA_\omega \cap CRCA_n$ *and* $\mathbf{S}_c \mathrm{Nr}_n CA_{n+3}$ *is not elementary.*

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- *Any class K between* $\mathrm{Nr}_n CA_\omega \cap CRCA_n$ *and* $\mathbf{S}_c \mathrm{Nr}_n CA_{n+3}$ *is not elementary.*
- $\mathbf{S}\mathrm{Nr}_n CA_{n+k}$, $k \geq 3$, *is not atom canonical.*