The many faces of the constraints in general relativity

István Rácz

Wigner RCP
Budapest

racz.istvan@wigner.mta.hu

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Plan

1. GR has a predictive power
2. Foliations and splittings puts the basic variables in new dress
3. Constraints form evolutionary systems
4. Summary and final remarks

Janus-faced GR:

The arena and the phenomena :

All the pre-GR physical theories provide a distinction between the **arena** in which physical phenomena take place and the **phenomena** themselves.

	arena:	phenomena:
classical mechanics	phase space: δ_{ab}	dynamical trajectories
electrodynamics	Minkowski spacetime: η_{ab}	evolution of F_{ab}
general relativity	curved spacetime: g_{ab}	evolution of g_{ab}

Such a clear distinction between the arena and the phenomenon is simply not available in general relativity

the metric plays both roles.

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- GR is more than merely a field theoretic description of gravity.
  It is a certain body of universal rules:
  - modeling the space of events by a four-dimensional differentiable manifold
  - the use of tensor fields and tensor equations to describe physical phenomena
  - use of the (otherwise dynamical) metric in measuring of distances, areas, volumes, angles ...

The predictive power of GR:

The Cauchy problem in GR (in full generality only ∼six decades ago):

- Choquet-Bruhat Y & Geroch R (1969): there always exists a **maximal Cauchy development** that is unique up to spacetime diffeomorphisms.
- there exists a **continuous "one-to-one"** mapping
- this mapping is also **causal**

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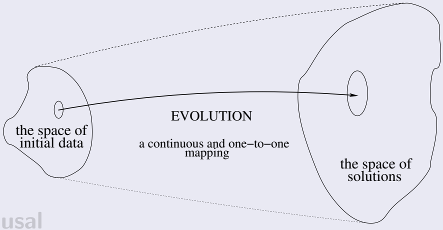

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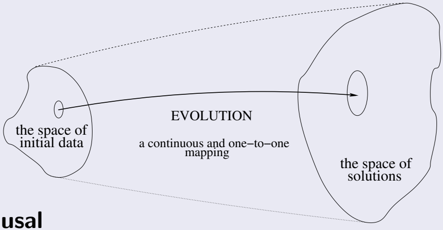

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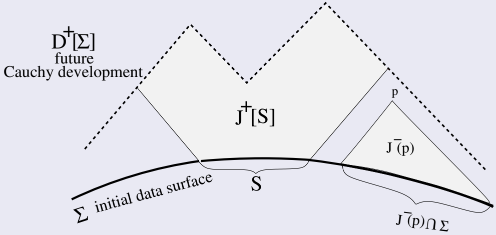

The main conceptual issue:

Assume that suitable initial data is given on some initial data surface Σ:

As a fixed background/arena does not exist in GR neither the base manifold M (where the solution manifest itself) nor the metric g_{ab} (satisfying the Einstein equations) is know in advance to solving the pertinent Cauchy problem

Initial data surface: (Σ, h_{ij}, K_{ij}) (satisfying the constraints)

Spacetime: (M, g_{ab}) (satisfying the Einstein equations)

Σ $\qquad \varphi[\Sigma]$

φ

$(h_{ij}, K_{ij}) \longrightarrow \varphi_* \longrightarrow (\varphi_* h_{ij}, \varphi_* K_{ij})$

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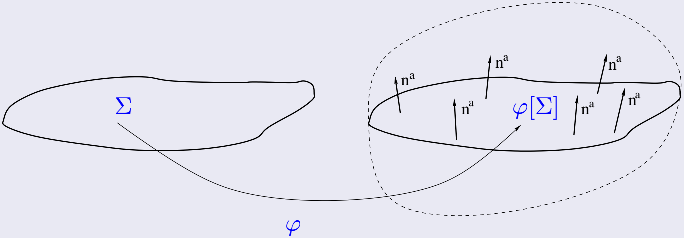

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The initial value problem starts by solving the constraints:

Constraints in the 4-dimensional Lorentzian vacuum case:

- initial data (h_{ij}, K_{ij}) metric and symmetric tensor on Σ_0

$$ {}^{(3)}R + (K^j{}_j)^2 - K_{ij}K^{ij} = 0 \quad \& \quad D_j K^j{}_i - D_i K^j{}_j = 0 $$

D_i denotes the covariant derivative operator associated with h_{ij}.

The conformal (elliptic) method

- the **constraints are solved** by transforming them into a **semilinear elliptic system** replace the fields h_{ij} and $K_{ij} - \frac{1}{3}\,h_{ij}\,\tau$ (where $\tau = K^l{}_l = h^{kl}K_{kl}$) by $\tilde{h}_{ij}$ and $\tilde{K}_{ij}$ as

$$ h_{ij} = \phi^4\,\tilde{h}_{ij} \quad \text{and} \quad K_{ij} - \tfrac{1}{3}\,h_{ij}\,\tau = \phi^{-2}\,\tilde{K}_{ij} $$

- Lichnerowicz equation: $$ \tilde{D}^l\tilde{D}_l\phi - \tfrac{1}{8}\,\tilde{R}\,\phi + \tfrac{1}{8}\,\tilde{K}_{ij}\tilde{K}^{ij}\,\phi^{-7} - \tfrac{1}{12}\,\tau^2\,\phi^5 = 0 $$

- York equation: $$ \tilde{D}^l\tilde{D}_l X_i + \tilde{D}^l\,U_{li} - \tfrac{2}{3}\,\phi^6(\tilde{D}_i\tau) = 0 $$

  where U_{ij} is an arbitrary traceless tensor, and $\tilde{K}_{ij}$ reads as

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Some aspects of the conformal method:

The strong points:

- the conformal method developed by Lichnerowicz and York could, in principle, determine all the possible initial data configurations in general relativity
- there has been derived a great number of existence, non-existence, or uniqueness theorems for the pertinent semilinear elliptic system

Some of the weak points:

- almost all of these theorems require the constancy of $\tau = K^l{}_l$
- the method is highly implicit due to the elliptic character of the basic equations and the replacements $h_{ij} = \phi^4\,\widetilde{h}_{ij}$ and $K_{ij} - \frac{1}{3}\,h_{ij}\,\tau = \phi^{-2}\,\widetilde{K}_{ij}$ $\Longrightarrow$
  - no direct control of the physical parameters of the initial data specifications
  - non-negligible spurious gravitational wave content of the spacetimes evolved from Bowen-York type initial data specifications ($\widetilde{h}_{ij}$ is flat, τ is constant)
- "... **no way singles out** precisely which functions (i.e., which of the 12 metric or extrinsic curvature components or functions of them) can be freely specified, which functions are determined by the constraints, and which functions correspond to gauge transformations. Indeed, **one of the major obstacles to developing a quantum theory of gravity** is the inability to single out the physical degrees of freedom of the theory. " R.M. Wald: General Relativity, Univ. Chicago Press, (1984)

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The main messages:

1. $\mathbf{n+1}$-dimensional $(\mathbf{n \geq 3})$ Riemannian and Lorentzian spaces satisfying the Einstein equations, and some mild topological assumptions, will be considered
2. many of the arguments and techniques developed originally and applied so far exclusively only in the Lorentzian case do also apply to Riemannian spaces
   - **the constraints propagate**: they hold everywhere if ...
   - **contrary to the folklore**: a new evolutionary approach is introduced—as an alternative of the elliptic conformal method—to solve the constraints
   - the coupled set of constraints can be put into the form of **evolutionary systems** to which (local) **existence and uniqueness** of solutions is guaranteed.
3. **!!! regardless** whether the primary space is Riemannian or Lorentzian

Based on some recent papers

- I. Rácz: *Is the Bianchi identity always hyperbolic?*, Class. Quantum Grav. 31 (2014) 155004
- I. Rácz: *Cauchy problem as a two-surface based 'geometrodynamics'*, Class. Quantum Grav. 32 (2015) 015006
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   - **the constraints propagate**: they hold everywhere if ...
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$$\boxed{G_{ab} - \mathscr{G}_{ab} = 0}$$

with source term $\mathscr{G}_{ab}$ having a vanishing divergence $\nabla^a \mathscr{G}_{ab} = 0$

  - or, in a more conventionally looking setup

$$\boxed{[R_{ab} - \tfrac{1}{2}\, g_{ab}\, R] + \Lambda\, g_{ab} = 8\pi\, T_{ab}}$$

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The primary foliation:

No restriction on the topology by Einstein's equations! (local PDEs)

- **Assume** that (apart from centers) M can be foliated by a one-parameter family of homologous codimension-one surfaces. More precisely, we shall assume the existence of a smooth function $\sigma : M \rightarrow \mathbb{R}$ such that its gradient $\nabla_a \sigma$ does not vanish except at centers which are isolated non-degenerate critical points of σ with zero Morse index, i.e. where σ has its local extremum.

- Apart from these centers the $\sigma = const$ level surfaces—they will also be denoted by Σ_σ—are supposed to be orientable either compact and without boundary in M or non-compact and infinite.

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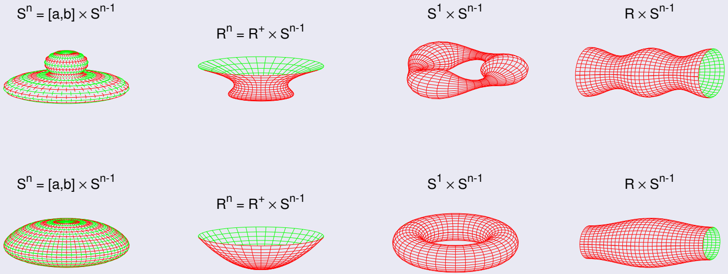

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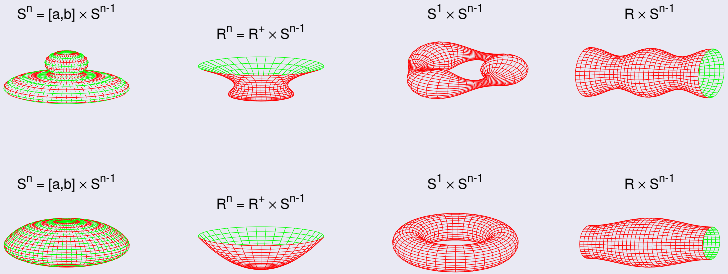

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The primary splitting:

Rephrasing:

- ... (apart from centers) M is foliated by a one-parameter family of homologous hypersurfaces, i.e. $M \simeq \mathbb{R} \times \Sigma$, for some codimension one manifold Σ
  - known to **hold for globally hyperbolic spacetimes** (Lorentzian case)
  - in either case: it is only a mild restriction on the topology of M
    - ... there exists a smooth function $\sigma : M \to \mathbb{R}$ **with non-vanishing gradient** $\nabla_a \sigma$ such that (apart from centers) the $\sigma = const$ level surfaces $\Sigma_\sigma = \{\sigma\} \times \Sigma$ comprise the one-parameter foliation of $M \implies$ $\boxed{n_a \sim \nabla_a \sigma}$

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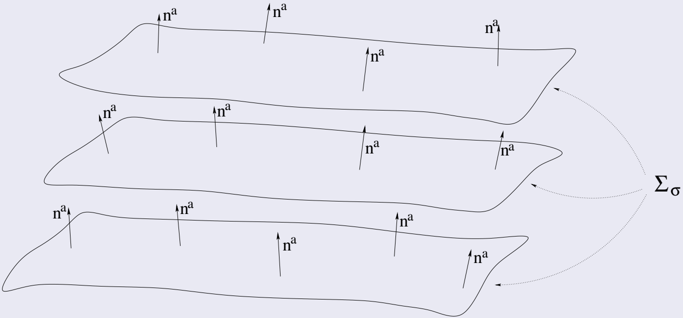

The primary projection operator:

- n^a the 'unit norm' vector field that is normal to the Σ_σ level surfaces

$$n^a n_a = \epsilon$$

  - the sign ϵ of the norm of n^a is not fixed
    takes the value -1 or $+1$ for Lorentzian or Riemannian metric g_{ab}, resp.

- **the projection operator**

$$h^a{}_b = \delta^a{}_b - \epsilon\, n^a n_b$$

  to the level surfaces of $\sigma : M \to \mathbb{R}$

- **the induced metric** on the $\sigma = const$ level surfaces

$$h_{ab} = h^e{}_a h^f{}_b\, g_{ef}$$

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The decomposition of various fields:

Examples:

- a form field: $$L_a = \delta^e{}_a\, L_e = (h^e{}_a + \epsilon\, n^e n_a)\, L_e = \boldsymbol{\lambda}\, n_a + \mathbf{L}_a$$
  - where $\boldsymbol{\lambda} = \epsilon\, n^e\, L_e \quad \text{and} \quad \mathbf{L}_a = h^e{}_a\, L_e$
- “time evolution vector field”

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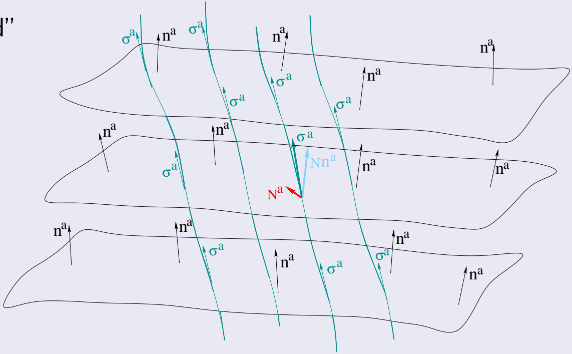

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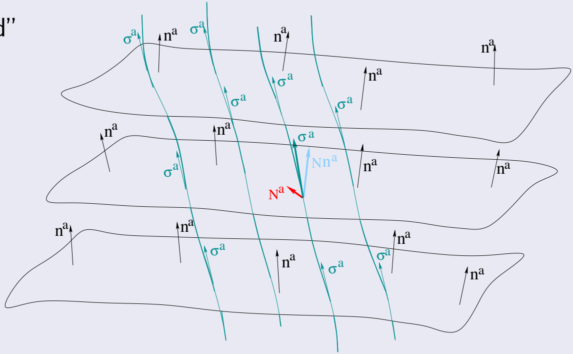

- where N and N^a denotes the 'laps' and 'shift' of $\sigma^a = (\partial_\sigma)^a$:

$$N = \epsilon \, (\sigma^e n_e) \quad \text{and} \quad N^a = h^a{}_e \, \sigma^e$$

Decompositions of various fields:

Any symmetric tensor field P_{ab} can be decomposed

in terms of n^a and fields living on the $\sigma = const$ level surfaces as

$$P_{ab} = \boldsymbol{\pi}\, n_a n_b + [n_a\, \mathbf{p}_b + n_b\, \mathbf{p}_a] + \mathbf{P}_{ab}$$

where $\boldsymbol{\pi} = n^e n^f\, P_{ef}\,,\quad \mathbf{p}_a = \epsilon\, h^e{}_a n^f\, P_{ef}\,,\quad \mathbf{P}_{ab} = h^e{}_a h^f{}_b\, P_{ef}$

It is also rewarding to inspect the decomposition of the contraction $\nabla^a P_{ab}$:

$$\epsilon\,(\nabla^a P_{ae})\, n^e = \mathscr{L}_n \boldsymbol{\pi} + D^e \mathbf{p}_e + [\boldsymbol{\pi}\,(K^e{}_e) - \epsilon\, \mathbf{P}_{ef} K^{ef} - 2\,\epsilon\, \dot{n}^e \mathbf{p}_e]$$

$$(\nabla^a P_{ae})\, h^e{}_b = \mathscr{L}_n \mathbf{p}_b + D^e \mathbf{P}_{eb} + [(K^e{}_e)\, \mathbf{p}_b + \dot{n}_b\, \boldsymbol{\pi} - \epsilon\, \dot{n}^e \mathbf{P}_{eb}]$$

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Decompositions of various fields:

Examples:

- the metric

$$g_{ab} = \epsilon\, n_a n_b + h_{ab}$$

- the "source term"

$$\mathscr{G}_{ab} = n_a n_b\, \mathfrak{e} + [n_a\, \mathfrak{p}_b + n_b\, \mathfrak{p}_a] + \mathfrak{S}_{ab}$$

where $\mathfrak{e} = n^e n^f \mathscr{G}_{ef}, \quad \mathfrak{p}_a = \epsilon\, h^e{}_a n^f \mathscr{G}_{ef}, \quad \mathfrak{S}_{ab} = h^e{}_a h^f{}_b \mathscr{G}_{ef}$

- the l.h.s. of our basic field equation $E_{ab} = G_{ab} - \mathscr{G}_{ab}$

$$E_{ab} = n_a n_b\, E^{(\mathcal{H})} + [n_a\, E^{(\mathcal{M})}_b + n_b\, E^{(\mathcal{M})}_a] + (E^{(\mathcal{EVOL})}_{ab} + h_{ab}\, E^{(\mathcal{H})})$$

$$E^{(\mathcal{H})} = n^e n^f E_{ef}, \quad E^{(\mathcal{M})}_a = \epsilon\, h^e{}_a n^f E_{ef}, \quad E^{(\mathcal{EVOL})}_{ab} = h^e{}_a h^f{}_b E_{ef} - h_{ab}\, E^{(\mathcal{H})}$$

Decompositions of various fields:

Examples:

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The explicit forms:

The various projections of $E_{ab} = G_{ab} - \mathscr{G}_{ab}$:

$$E^{(\mathcal{H})} = n^e n^f E_{ef} = \tfrac{1}{2}\{-\epsilon\,{}^{(n)}\!R + (K^e{}_e)^2 - K_{ef}K^{ef} - 2\,\mathfrak{e}\}$$

$$E^{(\mathcal{M})}_a = h^e{}_a n^f E_{ef} = D_e K^e{}_a - D_a K^e{}_e - \epsilon\,\mathfrak{p}_a$$

$$E^{(\mathcal{EVOL})}_{ab} = {}^{(n)}\!R_{ab} + \epsilon\,\{-\mathscr{L}_n K_{ab} - (K^e{}_e)K_{ab} + 2\,K_{ae}K^e{}_b - \epsilon\,N^{-1}D_a D_b N\} + \tfrac{1+\epsilon}{(n-1)}\,h_{ab}\,E^{(\mathcal{H})} - \left(\mathfrak{S}_{ab} - \tfrac{1}{n-1}\,h_{ab}\,[\mathfrak{S}_{ef}\,h^{ef} + \epsilon\,\mathfrak{e}]\right)$$

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$$\mathfrak{e} = n^e n^f\,\mathscr{G}_{ef}, \quad \mathfrak{p}_a = \epsilon\,h^e{}_a n^f\,\mathscr{G}_{ef}, \quad \mathfrak{S}_{ab} = h^e{}_a h^f{}_b\,\mathscr{G}_{ef}$$

and the **extrinsic curvature** K_{ab} is defined as

$$K_{ab} = h^e{}_a \nabla_e n_b = \tfrac{1}{2}\,\mathscr{L}_n h_{ab}$$

here $\mathscr{L}_n$ stands for the Lie derivative with respect to n^a

The decomposition of $\nabla^a E_{ab} = 0$ where $E_{ab} = G_{ab} - \mathscr{G}_{ab}$:

Relations between various parts of the Einstein equations:

$$\mathscr{L}_n E^{(\mathcal{H})} + D^e E^{(\mathcal{M})}_e + [\, E^{(\mathcal{H})} (K^e{}_e) - 2\,\epsilon\,(\dot n^e\, E^{(\mathcal{M})}_e) - \epsilon\, K^{ae}\,(E^{(\mathcal{EVOL})}_{ae} + h_{ae}\, E^{(\mathcal{H})})\,] = 0$$

back: $\nabla^a P_{ab}$

$$\mathscr{L}_n E^{(\mathcal{M})}_b + D^a (E^{(\mathcal{EVOL})}_{ab} + h_{ab}\, E^{(\mathcal{H})}) + [\,(K^e{}_e)\, E^{(\mathcal{M})}_b + E^{(\mathcal{H})}\, \dot n_b - \epsilon\,(E^{(\mathcal{EVOL})}_{ab} + h_{ab}\, E^{(\mathcal{H})})\,\dot n^a\,] = 0$$

when writing them out explicitly in some local coordinates $(\sigma, x^1, \dots, x^n)$ adopted to the vector field $\sigma^a = N\, n^a + N^a$: $\sigma^e \nabla_e \sigma = 1$ and the foliation $\{\Sigma_\sigma\}$, read as

$$\left\{ \begin{pmatrix} \frac{1}{N} & 0 \\ 0 & \frac{1}{N}\, h^{ij} \end{pmatrix} \partial_\sigma + \begin{pmatrix} -\frac{1}{N}\, N^k & h^{ik} \\ h^{jk} & -\frac{1}{N}\, N^k\, h^{ij} \end{pmatrix} \partial_k \right\} \begin{pmatrix} E^{(\mathcal{H})} \\ E^{(\mathcal{M})}_i \end{pmatrix} = \begin{pmatrix} \mathscr{E} \\ \mathscr{E}^j \end{pmatrix}$$

- the source terms $\mathscr{E}$ and $\mathscr{E}^j$ are linear and homogeneous in $E^{(\mathcal{H})}$ and $E^{(\mathcal{M})}_i$ $\Longrightarrow$ **if** the metric h_{ab} is Riemannian it is a **first order symmetric hyperbolic** system for $(E^{(\mathcal{H})}, E^{(\mathcal{M})}_i)^T$, and it is **linear** and **homogeneous** in these variables
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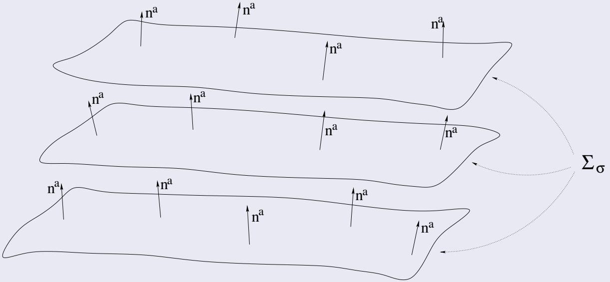
The propagation of the constraints:

Theorem

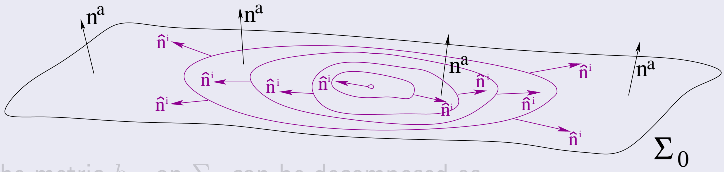
Let (M, g_{ab}) be as specified above and assume that the metric h_{ab} induced on the $\sigma = const$ level surfaces is Riemannian. Then, **regardless whether g_{ab} is of Lorentzian or Euclidean signature**, *any solution to the reduced equations $E^{(\mathcal{EVOL})}_{ab} = 0$ is also a solution to the full set of field equations $G_{ab} - \mathscr{G}_{ab} = 0$ provided that the constraint expressions $E^{(\mathcal{H})}$ and $E^{(\mathcal{M})}_{a}$ vanish on one of the $\sigma = const$ level surfaces.*



The secondary foliation and splittings:

Assume that on one of the $\sigma = const$ level surfaces (say on Σ_0) there exists a smooth function $\rho : \Sigma_0 \to \mathbb{R}$ gradient of which does not vanish (except at centers)

the $\rho = const$ level surfaces $\mathscr{S}_\rho$ are suppose to be homologous to each other and assume (for simplicity) that they are orientable compact without boundary in Σ_0



- The metric h_{ij} on Σ_0 can be decomposed as

$$h_{ij} = \hat{\gamma}_{ij} + \hat{n}_i \hat{n}_j$$

in terms of the positive definite metric $\hat{\gamma}_{ij}$, induced on the $\mathscr{S}_\rho$ level surfaces

- the unit norm field, normal to the $\mathscr{S}_\rho$ level surfaces, can be decomposed as

$$\hat{n}^i = \hat{N}^{-1} \left[(\partial_\rho)^i - \hat{N}^i \right]$$

where $\hat{N}$ and $\hat{N}^i$ denotes the 'laps' and 'shift' of an 'evolution' vector field $\rho^i = (\partial_\rho)^i$ on Σ_0

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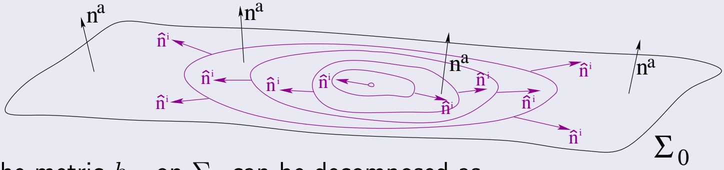

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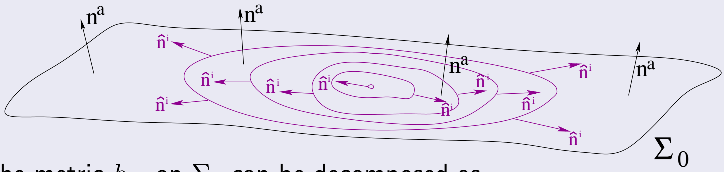

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One needs various secondary splittings:

The momentum constraint:

$$E^{(\mathcal{M})}_a = h^e{}_a n^f E_{ef} = D_e K^e{}_a - D_a K^e{}_e - \epsilon\,\mathfrak{p}_a = 0$$

back: $\nabla^a P_{ab}$

The splitting of the extrinsic curvature K_{ij}:

- $$K_{ij} = \boldsymbol{\kappa}\,\hat n_i \hat n_j + [\hat n_i\,\mathbf{k}_j + \hat n_j\,\mathbf{k}_i] + \mathbf{K}_{ij}$$

  where

  $$\boldsymbol{\kappa} = \hat n^k \hat n^l\,K_{kl}\,,\quad \mathbf{k}_i = \hat\gamma^k{}_i \hat n^l\,K_{kl} \quad \text{and} \quad \mathbf{K}_{ij} = \hat\gamma^k{}_i \hat\gamma^l{}_j\,K_{kl}$$

- the **trace** and **trace free** parts of $\mathbf{K}_{ij}$

  $$\mathbf{K}^l{}_l = \hat\gamma^{kl}\,\mathbf{K}_{kl} \quad \text{and} \quad \mathring{\mathbf{K}}_{ij} = \mathbf{K}_{ij} - \tfrac{1}{n-1}\,\hat\gamma_{ij}\mathbf{K}^l{}_l$$

- the independent components of (h_{ij}, K_{ij}) may be represented by the variables

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Constraints in new dress:

The momentum constraint:

$$E^{(\mathcal{M})}_a = h^e{}_a n^f E_{ef} = D_e K^e{}_a - D_a K^e{}_e - \epsilon\, \mathfrak{p}_a = 0$$

$$(\hat K^l{}_l)\,\mathbf{k}_i + \hat D^l \mathring{\mathbf{K}}_{li} + \boldsymbol{\kappa}\,\dot{\hat n}_i + \mathscr{L}_{\hat n}\mathbf{k}_i - \dot{\hat n}^l\,\mathbf{K}_{li} - \hat D_i \boldsymbol{\kappa} - \tfrac{n-2}{n-1}\,\hat D_i(\mathbf{K}^l{}_l) - \epsilon\,\mathfrak{p}_l\,\hat\gamma^l{}_i = 0$$

$$\boldsymbol{\kappa}\,(\hat K^l{}_l) + \hat D^l \mathbf{k}_l - \mathbf{K}_{kl}\hat K^{kl} - 2\,\dot{\hat n}^l\,\mathbf{k}_l - \mathscr{L}_{\hat n}(\mathbf{K}^l{}_l) - \epsilon\,\mathfrak{p}_l\,\hat n^l = 0$$

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$$\dot{\hat n}_k = \hat n^l D_l \hat n_k = -\hat D_k(\ln \hat N)$$

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The extrinsic curvature of the secondary foliation $\mathscr{S}_\rho$:

$$\hat K_{ij} = \hat\gamma^l{}_i\, D_l\, \hat n_j = \tfrac{1}{2}\,\mathscr{L}_{\hat n}\hat\gamma_{ij}$$

with trace

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$$\boldsymbol{\kappa}\, (\hat{K}^l{}_l) + \hat{D}^l \mathbf{k}_l - \mathbf{K}_{kl} \hat{K}^{kl} - 2\, \dot{\hat{n}}^l\, \mathbf{k}_l - \mathscr{L}_{\hat{n}} (\mathbf{K}^l{}_l) - \epsilon\, \mathfrak{p}_l\, \hat{n}^l = 0$$

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$$\hat{K}^l{}_l = \hat{\gamma}^{ij} \hat{K}_{ij} = \tfrac{1}{2}\, \hat{\gamma}^{ij} \mathscr{L}_{\hat{n}} \hat{\gamma}_{ij}$$

First order symmetric hyperbolic system:

The momentum constraint in local coordinates:

$$\mathscr{L}_{\hat n}\mathbf{k}_i - \tfrac{n-2}{n-1}\,\hat D_i(\mathbf{K}^l{}_l) - \hat D_i\boldsymbol{\kappa} + \hat D^l\mathring{\mathbf{K}}_{li} + (\hat K^l{}_l)\,\mathbf{k}_i + \boldsymbol{\kappa}\,\dot{\hat n}_i - \dot{\hat n}^l\,\mathbf{K}_{li} - \epsilon\,\mathfrak{p}_l\,\hat\gamma^l{}_i = 0 \quad (1)$$

back: str.hyp.sys.

$$\mathscr{L}_{\hat n}(\mathbf{K}^l{}_l) - \hat D^l\mathbf{k}_l - \boldsymbol{\kappa}\,(\hat K^l{}_l) + \mathbf{K}_{kl}\hat K^{kl} + 2\,\dot{\hat n}^l\,\mathbf{k}_l + \epsilon\,\mathfrak{p}_l\,\hat n^l = 0 \quad (2)$$

- notably, $\frac{n-1}{n-2}\,\hat N\,\hat\gamma^{ij}$ times of (1) and $\hat N$ times of (2) when writing them out in (local) coordinates $(\rho, x^2, \ldots, x^n)$, adopted to the foliation $\mathscr{S}_\rho$ and the vector field ρ^i,

$$\left\{\begin{pmatrix} \frac{n-1}{n-2}\,\hat\gamma^{AB} & 0 \\ 0 & 1 \end{pmatrix}\partial_\rho + \begin{pmatrix} -\frac{n-1}{n-2}\,\hat N^K\,\hat\gamma^{AB} & -\hat N\,\hat\gamma^{AK} \\ -\hat N\,\hat\gamma^{BK} & -\hat N^K \end{pmatrix}\partial_K\right\}\begin{pmatrix} \mathbf{k}_B \\ \mathbf{K}^E{}_E \end{pmatrix} + \begin{pmatrix} \mathscr{B}_{(\mathbf{k})}^A \\ \mathscr{B}_{(\mathbf{K})} \end{pmatrix} = 0$$

- indep. of ϵ: a first order **symmetric hyperbolic** system for the vector valued variable

$$(\mathbf{k}_B, \mathbf{K}^E{}_E)^T$$

  where **the 'radial coordinate'** ρ **plays the role of 'time'**.

- ... with characteristic cone (apart from the surfaces $\mathscr{S}_\rho$ with $\hat n^i\xi_i = 0$)

$$[\hat\gamma^{ij} - (n-1)\,\hat n^i\hat n^j]\,\xi_i\xi_j = 0$$

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The Hamiltonian constraint:

The Hamiltonian constraint in new dress:

$$E^{(\mathcal{H})} = n^e n^f E_{ef} = \tfrac{1}{2}\{-\epsilon\,{}^{(n)}\!R + (K^e{}_e)^2 - K_{ef}K^{ef} - 2\,\mathfrak{e}\} = 0$$

using $${}^{(n)}\!R = \hat R - \Big\{2\,\mathscr{L}_{\hat n}(\hat K^l{}_l) + (\hat K^l{}_l)^2 + \hat K_{kl}\hat K^{kl} + 2\,\hat N^{-1}\hat D^l \hat D_l \hat N\Big\}$$

$$-\epsilon\,\hat R + \epsilon\Big\{2\,\boxed{\mathscr{L}_{\hat n}(\hat K^l{}_l)} + (\hat K^l{}_l)^2 + \hat K_{kl}\,\hat K^{kl} + 2\,\boxed{\hat N^{-1}\hat D^l \hat D_l \hat N}\Big\} + 2\,\boxed{\boldsymbol{\kappa}}\,\mathbf{K}^l{}_l + \tfrac{n-2}{n-1}(\mathbf{K}^l{}_l)^2 - 2\,\mathbf{k}^l\mathbf{k}_l - \mathring{\mathbf{K}}_{kl}\,\mathring{\mathbf{K}}^{kl} - 2\,\mathfrak{e} = 0$$

$\hat R$ denotes the scalar curvature of $\hat\gamma_{ij}$

Two alternative choices that yield evolutionary systems for constraints:

- it is a **parabolic equation** for $\boxed{\hat N}$ if $\boxed{\tfrac{1}{2}\hat\gamma^{ij}\mathscr{L}_{\hat\rho}\hat\gamma_{ij} - \hat D_j \hat N^j}$ does not vanish
- it is an **algebraic equation** for $\boxed{\boldsymbol{\kappa}}$ provided that $\boxed{\mathbf{K}^l{}_l}$ does not vanish

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$$-\epsilon\,\hat{R} + \epsilon\left\{ 2\,\mathscr{L}_{\hat{n}}(\hat{K}^l{}_l) + (\hat{K}^l{}_l)^2 + \hat{K}_{kl}\,\hat{K}^{kl} + 2\,\hat{N}^{-1}\hat{D}^l\hat{D}_l\hat{N} \right\} + 2\,\boldsymbol{\kappa}\,\mathbf{K}^l{}_l + \tfrac{n-2}{n-1}\,(\mathbf{K}^l{}_l)^2 - 2\,\mathbf{k}^l\mathbf{k}_l - \mathring{\mathbf{K}}_{kl}\,\mathring{\mathbf{K}}^{kl} - 2\,\mathfrak{e} = 0$$

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The hyperbolic-parabolic system:

The Hamiltonian constraint:

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- in highly specialized cases of "quasi-spherical" foliations with $\hat{\gamma}_{ij} = r^2\,\mathring{\gamma}_{ij}$ and with time symmetric initial data $K_{ij} \equiv 0$ R. Bartnik (1993), R. Weinstein & B. Smith (2004)

The hyperbolic-parabolic system:

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The Hamiltonian constraint:

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The strongly hyperbolic system:

The Hamiltonian constraint as an algebraic equation for $\boldsymbol{\kappa}$:

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  - a **strongly hyperbolic** system for $(\mathbf{k}_i, \mathbf{K}^l{}_l)$ and an algebraic relation for $\boldsymbol{\kappa}$
    - with freely specifiable variables on Σ_0 and on $\mathscr{S}_0$:

$$(\hat{N}, \hat{N}^i, \hat{\gamma}_{ij}; \boxed{\boldsymbol{\kappa}}, \mathbf{k}_i|_{\mathscr{S}_0}, \mathbf{K}^l{}_l|_{\mathscr{S}_0}, \mathring{\mathbf{K}}_{ij})$$

    - by choosing the free data properly $\boldsymbol{\kappa}\cdot\mathbf{K}^l{}_l < 0$ can be guaranteed (locally)
  - **!!! (local) existence and uniqueness** of C^∞ solutions is guaranteed I.R. (2015)
  - **!!! some global results** apply for the hyperbolic-parabolic formulation I.R. (2015)

Summary and final remarks:

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1. $\mathbf{n+1}$-dimensional $(\mathbf{n \geq 3})$ Riemannian and Lorentzian spaces satisfying the Einstein equations, and some mild topological assumptions, were considered
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Final remarks:

- hyperbolicity and causality: $\tilde h_{ij} = h_{ij} - (1+\alpha)\,\hat n_i \hat n_j$ where α is a positive real function
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