Universität
Konstanz

What is a possible case in branching space-times?

Thomas Müller

Thomas.Mueller@uni-konstanz.de
Department of Philosophy
University of Konstanz

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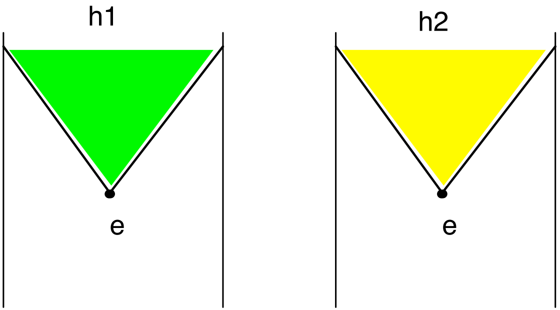
Branching space-times and possible cases



$$??? \models \phi$$

Branching space-times (BST)

A theory of indeterminism compatible with relativity theory:

- BST incorporates several space-times into one model
- Space-times as possible total courses of events (histories)
- Histories overlap in the common past and branch toward alternative futures
- Splitting points between histories represent local indeterminism

Possible cases

A general expression for what truth/extensions are relative to:

- A standard first-order logical model describes one possible case; truth is (just) relative to a model
- Possible worlds framework:
  One world as a possible case in a model;
  truth relative to model + world
- Linear temporal logic:
  One moment of time as a possible case in a model;
  truth relative to model + moment of time
- Branching time logic: moment/history pairs; ...
- Branching space-times: truth relative to ???

Overview

Branching space-times (BST)

Possible cases

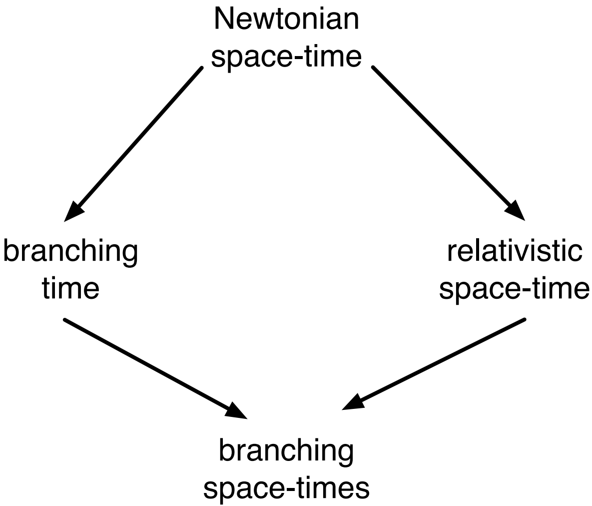
- Possible cases as parameters of truth
- Case-intensional first order logic

Possible cases in BST: Three options

Conclusions and open questions

Branching space-times

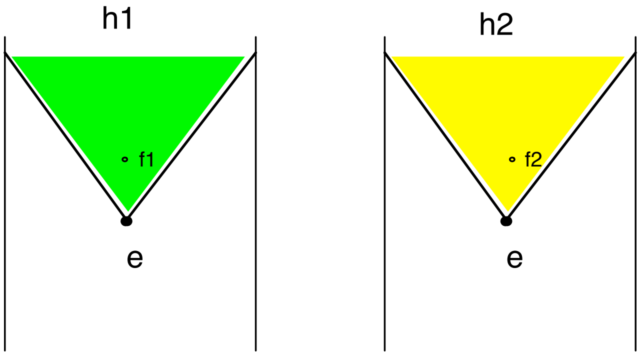
Belnap: From Newtonian space-time to BST



Motivating BST: Partial orderings

- Our world as a partial ordering of possible point events
- A downward fork ($x < z$, $y < z$, x, y incomparable) is read spatio-temporally (as common in relativistic space-time): x and y are in the common past of z
- An upward fork ($x > z$, $y > z$, x, y incomparable) has *two* readings:
  - spatio-temporally (as common in relativistic space-time): x and y are in the common future of z
  - modally (in order to represent indeterminism): x and y are in alternative possible futures of z
- Which reading of the upward fork applies, depends on whether x and y have a common upper bound or not
- A common upper bound signals spatio-temporal relation; thus a *history* h (a complete possible course of events) is a maximal directed subset of Our world

BST: Two histories



BST: a framework for modal alternatives in space-time

The axioms of BST92 (Belnap 1992)

- $\langle W, \leq \rangle$ is a nonempty, dense partial order without maxima.
- A *history* is a subset $h \subseteq W$ that is maximal upward directed, i.e., maximal w.r.t. the property that for any $x, y \in h$ there is some $z \in h$ s.t. $x \leq z$ and $y \leq z$.
- Each lower bounded chain $C \subseteq W$ has an infimum in W.
- Each upper bounded chain $C \subseteq W$ has a supremum-in-h ($\sup_h C$) for each history $h \in H$ for which $C \subseteq h$.
- (Prior choice principle.) If $C \in h - h'$ is a lower bounded chain in h none of whose elements is an element of h', then there is a choice point $c \in h \cap h'$ such that c is maximal in $h \cap h'$, and $c < C$ (i.e., for all $e \in C$, we have $c < e$).

There are other developments of the main ideas (e.g., BST based on non-Hausdorff manifolds rather than on partial orderings); details do not influence the conceptual question of possible cases.

Possible cases

Cases in English

"In any case, I won't go to that meeting."

"In case it rains, the brown horse will win."

"It's possible that John will join us, for example, in case his flight is delayed."

Quantification over cases is idiomatic in English.

- Possibility as truth in some case.
- Necessity as truth in all cases.
- Plausibly, simple truth as truth in the actual case.

Cases as anchors for extensions

"What needs to be specified such that a piece of language has a semantic value?"

Several possible answers. Consider two questions:

- Which kind of semantic value? $\Rightarrow$ Extension vs. intension. We consider *extensions* (more local, fine-grained semantic value); intensions are derivative (pattern of extensions as the case varies).
- Which assumptions about the language? Propositional or first order (with terms)? Are there indexicals? Modal expressions? We leave that open for now: look at the *general* situation.

A piece of language has an extension in a case. Key examples:

- A sentence has a truth value in a case.
- A term has an extension in a case. (But what *is* that?)

Cases and the representation of things

Propositional languages:

- only sentences have extensions;

$\Rightarrow$ (more or less) anything could be a case.

First-order languages:

- *individual terms* have to have extensions

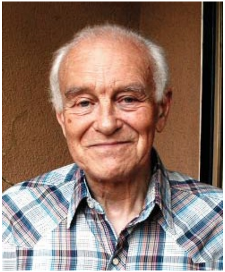
$\Rightarrow$ the question of cases is connected with the representation of individual things.

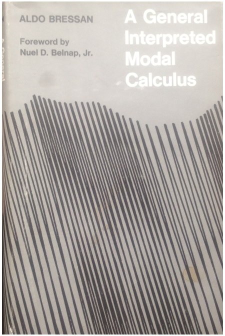
- Once time enters the picture, the persistence of things needs to be represented as well.

The question of cases is best approached in a time-friendly framework of intensional logic: *Case-intensional logic*

Case-intensional logic: Bressan and CIFOL

The most persistently overlooked important contribution to quantified modal logic





First order part: Case-intensional first order logic
N. Belnap & T. Müller, CIFOL, BH-CIFOL, *J Phil Logic* 2014.

CIFOL semantics (very briefly)

- Cases $\gamma \in \Gamma$; extensional domain D
- Individual term α (constant, variable, ...) has
  - extension in each case: $ext_\gamma(\alpha) \in D$
  - intension: pattern of extensions, $int(\alpha) \in (\Gamma \mapsto D)$
- Predication is intensional:
  - Standard conception: extensional predication, $int(P) \in \Gamma \mapsto (D \mapsto \mathbf{2})$
  - Here: intensional predication, $int(P) \in \Gamma \mapsto ((\Gamma \mapsto D) \mapsto \mathbf{2})$
- No need to ask what's in D — only cardinality is important.
- Alethic modality: simple **S5**:
  $\gamma \models \Box\phi$ iff for all $\gamma' \in \Gamma$: $\gamma' \models \phi$
- Identity is extensional:
  $\gamma \models \alpha = \beta$ iff $ext_\gamma\alpha = ext_\gamma\beta$
- Only necessary identity $\Box\alpha = \beta$ allows replacement
- Existence $Ex \Leftrightarrow_{df} x \neq *$ via "throwaway" $* \in D$

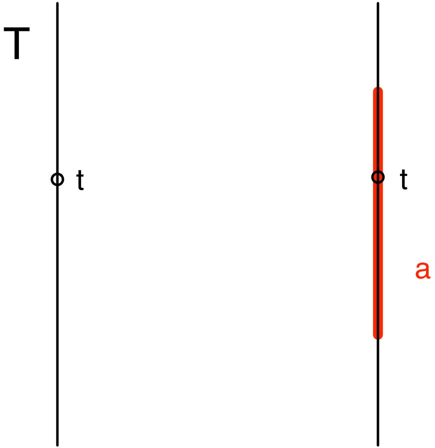
CIFOL and the structure of cases

In basic CIFOL, the cases are just a set Γ with no structure (not even an "accessibility relation").

Additional structure can ground additional modal operators.

- Cases in linear time (linear ordering of times $\langle T, \leq \rangle$):
  Temporal operators *Will*, *Was* (and their duals)
- Cases in branching time (based on a left-linear partial ordering of moments $\langle M, \leq \rangle$):
  Temporal operators *Will*, *Was* (and their duals)
  and settledness (and its dual, historical possibility)
- Cases in branching space-times: ???

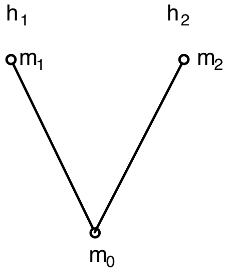
Cases and things in linear time



Red: $\alpha \neq *$

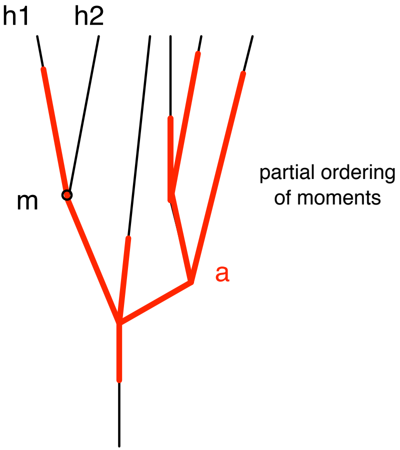
$t \models \alpha \neq * \wedge \mathit{Will} : \alpha \neq *$

Branching: Cases as moment/history pairs



- three moments m_0, m_1, and m_2, partial ordering;
- two histories $h_1 = \{m_0, m_1\}$ and $h_2 = \{m_0, m_2\}$;
- four moment/history cases m/h with $m \in h$:
  $\gamma_1 = m_0/h_1$, $\gamma_2 = m_0/h_2$, $\gamma_3 = m_1/h_1$, and $\gamma_4 = m_2/h_2$
  (Occamism)

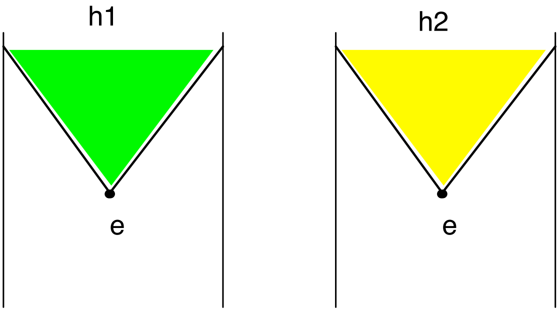
Individual things in branching histories



Red: $\alpha \neq *$. $m/h_1 \models \mathit{Will} : \alpha \neq *$; $m/h_2 \models \neg \mathit{Will} : \alpha \neq *$

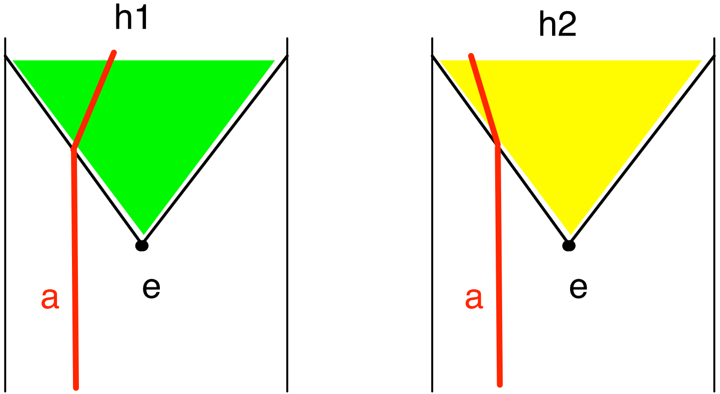
Possible cases in BST

Branching space-times and possible cases

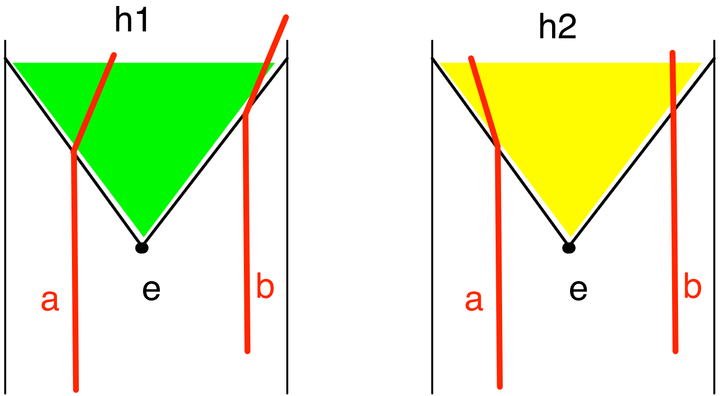


$$??? \models \phi$$

Representing individual things in BST (1)



Representing individual things in BST (2)



Representing individual things in BST (3)

Formally, we have, for any thing a in a BST model $\langle W, \leq \rangle$:

- a set of space-time locations $L_a \subseteq W$, such that for each history h, the set $L_a \cap h$ is a gapless chain (n.b. this means we are idealising things to be point-thin, and it naturally implements a necessity of origin thesis);
- an assignment of extensions from D, e.g., momentary states or stages, to the thing at its various space-time locations: $S_a : L_a \mapsto D$.

This representation of the things and their locations/states is prior to settling what the cases in BST look like.
The extension of a term α denoting a thing a in a case γ has to be a member of D; this should match the information in S_a.

Cases as pairs e/h

(We disregard as obviously inappropriate anything without a history parameter; this lesson we take from Occamism.)

e/h (with $e \in h$) initially looks good:

- It matches the m/h cases from branching time:
  like m there, e is a member of the basic partial ordering.
- It's simple, and as the representation of things (idealised to be point-thin) shows, e/h can be mapped to the state of a thing, thus providing enough information for "what is so in a case".

But it won't do:

- We will have $ext_{e/h}\alpha \neq *$ only if the thing denoted by α is present at e.

$\Rightarrow$ A difference in spatial location counts as relevant for existence.

$\Rightarrow$ There can be no coexistence of two different, non-overlapping things in any case. But that is inappropriate.

Cases as pairs Σ/h

A case as a Cauchy surface Σ plus a history (with $\Sigma \subseteq h$)

- Given the representation of things via L_a and S_a, we know that

$$L_a \cap \Sigma = \emptyset \text{ or } L_a \cap \Sigma = \{e\}.$$

- So, given a term α referring to the thing a:
  - in the former case, set $ext_{\Sigma/h}\alpha = *$;
  - in the latter case, set $ext_{\Sigma/h}\alpha = S_a(e)$.
- This makes it possible to have a case s.t.

$$\Sigma/h \models a \neq * \wedge b \neq *$$

  for different, non-overlapping things a and b.
- But where is Σ supposed to come from?

Cases as pairs $\mathfrak{F}/h$

Third option: a case as a frame $\mathfrak{F}$ and a history h

The frame determines

- an origin $O(\mathfrak{F})$; we demand $O(\mathfrak{F}) \in h$
- a Cauchy surface $\Sigma(\mathfrak{F})$ as the frame-dependent Now; we demand $\Sigma(\mathfrak{F}) \subseteq h$
- also a spatial orientation anchoring indexicals "left", "right" etc.—we won't use those here.

Go for (abstract) frames rather than (concrete) observers, since we want to keep the ontology simple;
the possibilities for the existence of things does not match a notion of possible observers (there are too many of those).

(Of course, frames could be observers' rest frames.)

Cases as pairs $\mathfrak{F}/h$, and things

As previously, we can determine terms' extensions:

$$ext_{\mathfrak{F}/h}\alpha = \begin{cases} *, & \text{if } L_a \cap \Sigma(\mathfrak{F}) = \emptyset, \\ S_a(e), & \text{if } L_a \cap \Sigma(\mathfrak{F}) = \{e\}. \end{cases}$$

So we allow for cases in which different things coexist.

We can also introduce a (quasi-logical) predicate, *Here*:

$$\mathfrak{F}/h \models Here(\alpha) \text{ iff } O(\mathfrak{F}) \in L_a$$

for α a term denoting the thing a.

Modal operators: general frame-changes, perhaps groups of those

Conclusions and open questions

Conclusions

- BST promises to deliver a rich picture of things in space-time, and a basis for a spatio-temporal predicate logic
- Like branching time, BST needs a two-part notion of a case (including a history)
- Things in BST are explicitly modeled to have a spatial location, which should not have direct ontological significance
- So, a case in BST should single out "space as of now"
- The best option for that seems to be:
  cases as pairs $\mathfrak{F}/h$, with $\mathfrak{F}$ compatible with h

Open questions

All of this is work in progress, and there are many open issues. For example:

- Metaphysically:
  - Is it a good idea to incorporate a full frame into the logical cases? Too fine-grained? Link with story of possible observers?
  - Are the demands on representing things really appropriate? Need distinction substances / other things? (Esp. artifacts?)
- Logically:
  - What is an appropriate set of modal operators? Should we have a full group of Poincaré transformation-indexed operators, or can we bundle them? Causal future/past?
  - The representation of things is now a two-stage affair; terms need to be assocated with things represented separately. Can we start with general intensions and restrict those axiomatically?

Thanks for your attention!

CIFOL semantics (i)

- Cases $\gamma \in \Gamma$; extensional domain D
- Individual term α (constant, variable, ...) has
  - extension in each case: $ext_\gamma(\alpha) \in D$
  - intension: pattern of extensions, $int(\alpha) \in (\Gamma \mapsto D)$
- Assignment δ: $Var \mapsto (\Gamma \mapsto D)$ (intensional variables)
- General link extensions / intension for expressions ξ:

$$ext_{\gamma,\delta}(\xi) = (int_\delta(\xi))(\gamma); \qquad int_\delta(\xi) = \lambda\gamma(ext_{\gamma,\delta}(\xi))$$

- Predication is intensional:
  - Standard conception: extensional predication, $int(P) \in \Gamma \mapsto (D \mapsto \mathbf{2})$
  - Here: intensional predication, $int(P) \in \Gamma \mapsto ((\Gamma \mapsto D) \mapsto \mathbf{2})$. Uniform clause:

$$ext_{\gamma,\delta}(P\alpha) = (ext_{\gamma,\delta}P)(int_\delta\alpha) \in \mathbf{2}$$

CIFOL semantics (ii): intensional predication

Four cases, $\gamma_1, \ldots, \gamma_4$; domain $D = \{$a,b,c,d,e,f,g,h,j,k,l,m,n,*$\}$;
terms / intensions: "Bas" (abcd), "Bess" (*fg*), "Lumpi" (bbbb).

Property \ Case	γ_1	γ_2	γ_3	γ_4
Basil	abcd *fg*	abcd *fg*	abcd *fg*	abcd *fg*
Lump	aaaa bbbb	aaaa bbbb	aaaa bbbb	aaaa bbbb
Green	a---	-b-- -f--	--c- --g-	---d
Blooming	$\varnothing$	$\varnothing$	--g-	---d

$\gamma_4 \models$ Blooming(Bas), i.e., ext_{γ_4}(Blooming(Bas)) $= T$
$\gamma_1 \models \neg$Blooming(Bas), i.e., ext_{γ_1}(Blooming(Bas)) $= F$
$\gamma_1 \models$ Basil(Bas) $\wedge\neg$Basil(Lumpi)

No need to ask what's in D — only cardinality is important.

CIFOL semantics (iii)

- Alethic modality: simple **S5**:
  $\gamma, \delta \models \Box\phi$ iff for all $\gamma' \in \Gamma$: $\gamma', \delta \models \phi$
- Quantification: variables for individual intensions:
  $\gamma, \delta \models \forall x \phi$ iff for all $\bar{z} \in (\Gamma \mapsto D)$: $\gamma, \delta[\bar{z}/x] \models \phi$
  $\Rightarrow$ BF and CBF are valid
  N.B.: Can't read "$\forall x$" as "for all things x"

- Identity is extensional:
  $\gamma, \delta \models \alpha = \beta$ iff $ext_{\gamma,\delta}\alpha = ext_{\gamma,\delta}\beta$
  Thus, $\gamma_2 \models \text{Bas} = \text{Lumpi}$; $\gamma_3 \models \text{Bas} \neq \text{Lumpi}$
- Only necessary identity $\Box\alpha = \beta$ allows replacement
- Existence $Ex \Leftrightarrow_{df} x \neq *$ via "throwaway" $* \in D$
  E.g., $\gamma_1 \models \text{Bess} = *$

- Easy handling of λ-predicates/-predications, λ-operators/-terms, definite descriptions

CIFOL's interface for for sortals

- Interface to metaphysical/scientific discussion via *definable properties of properties*, not via rigid designators

- EXT: P is *extensional* $\Leftrightarrow_{df} \Box\forall x\forall y\,(x = y \rightarrow (Px \leftrightarrow Py))$
- MC: P is *modally constant* $\Leftrightarrow_{df}$ $\forall x\,(\Diamond Px \rightarrow \Box Px)$
- MS: P is *modally separated* $\Leftrightarrow_{df}$ $\Box\forall x\forall y\,(Px \wedge Py \rightarrow (\Diamond(x \neq * \wedge x = y) \rightarrow \Box x = y))$
- ABS: P is an *absolute property* $\Leftrightarrow_{df}$ P is MC and MS

- Every ABS property has extensional companions:
  $P^e x \Leftrightarrow_{df}\ \exists y\,(Py \wedge x = y)$; $\quad P^{e!}x \Leftrightarrow_{df}\ P^e x \wedge x \neq *$

Slogan: Sortal properties are absolute