Operationalization of relativistic energy, momentum and inertial mass

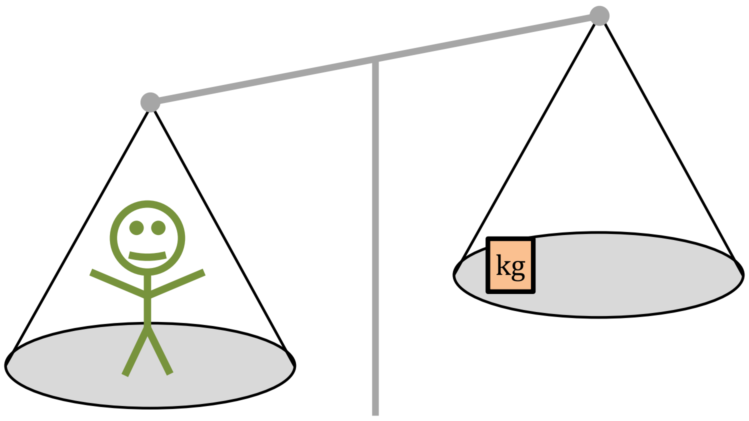
Bruno Hartmann (Humboldt Univ.)

LRB’15, Budapest 12.08.2015

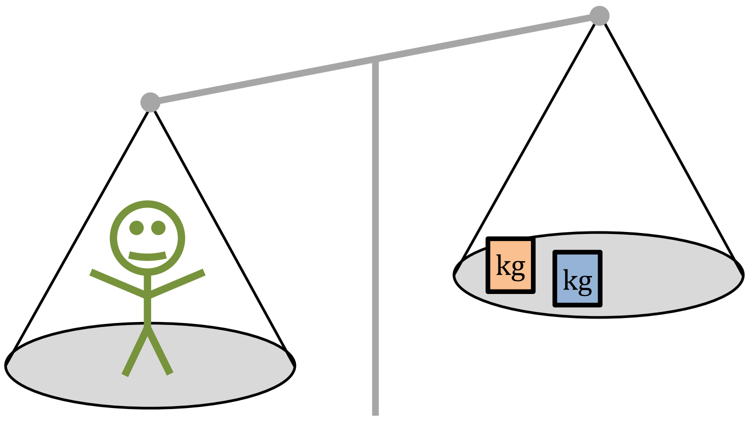
Basic Measurement

$>_l$ $>_m$ $>_{\boldsymbol{p}}$

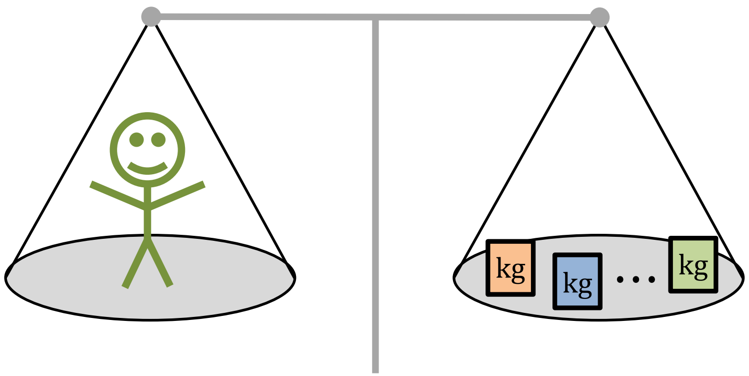
Basic Measurement



Basic Measurement



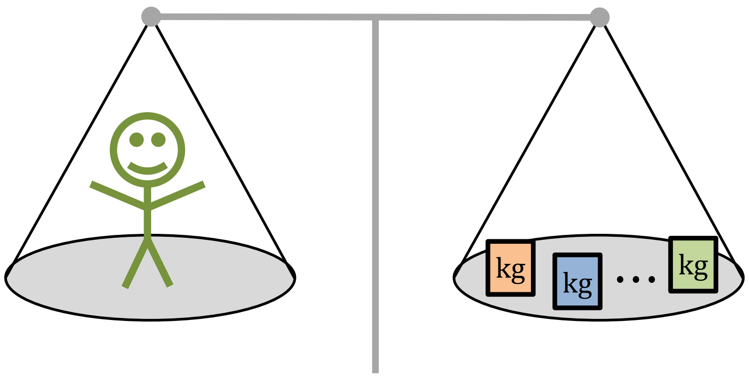
Basic Measurement



arithmetic formulation

$m[$  $]\ '='\ 1\text{kg}\ '+'\ 1\text{kg}\ '+'\ \ldots$

Basic Measurement



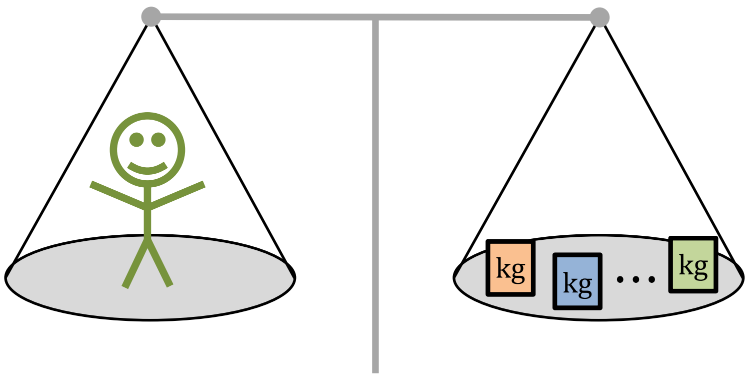
arithmetic formulation

$m[$  $]\ {'}{=}{'}\ 1\mathrm{kg}\ {'}{+}{'}\ 1\mathrm{kg}\ {'}{+}{'}\ \ldots$

physical operations

$$>_m \qquad *_m$$

Basic Measurement



arithmetic formulation

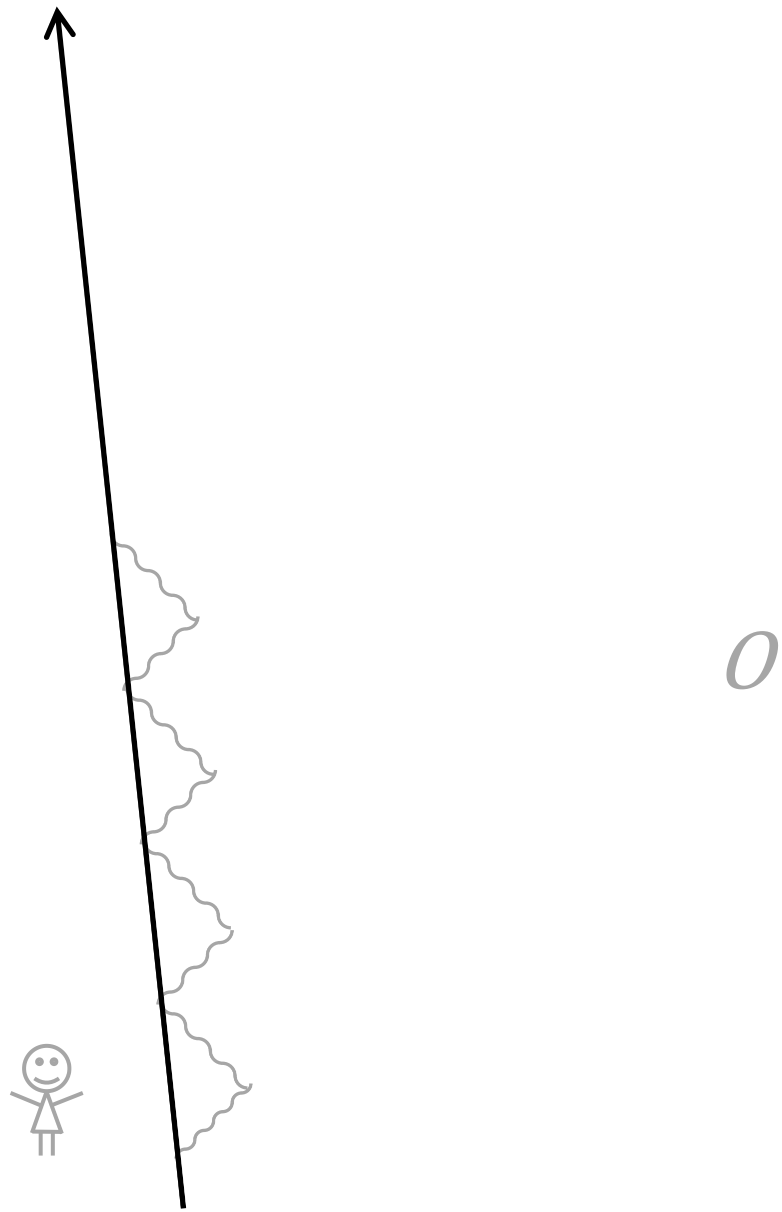
$m[$  $]\ '='\ 1\text{kg}\ '+'\ 1\text{kg}\ '+'\ \ldots$ $\quad 1\text{m}\ '+'\ 1\text{m}$ $\quad 1\text{s}\ '+'\ 1\text{s}$

physical operations

$>_m \qquad *_m$

Kinematics

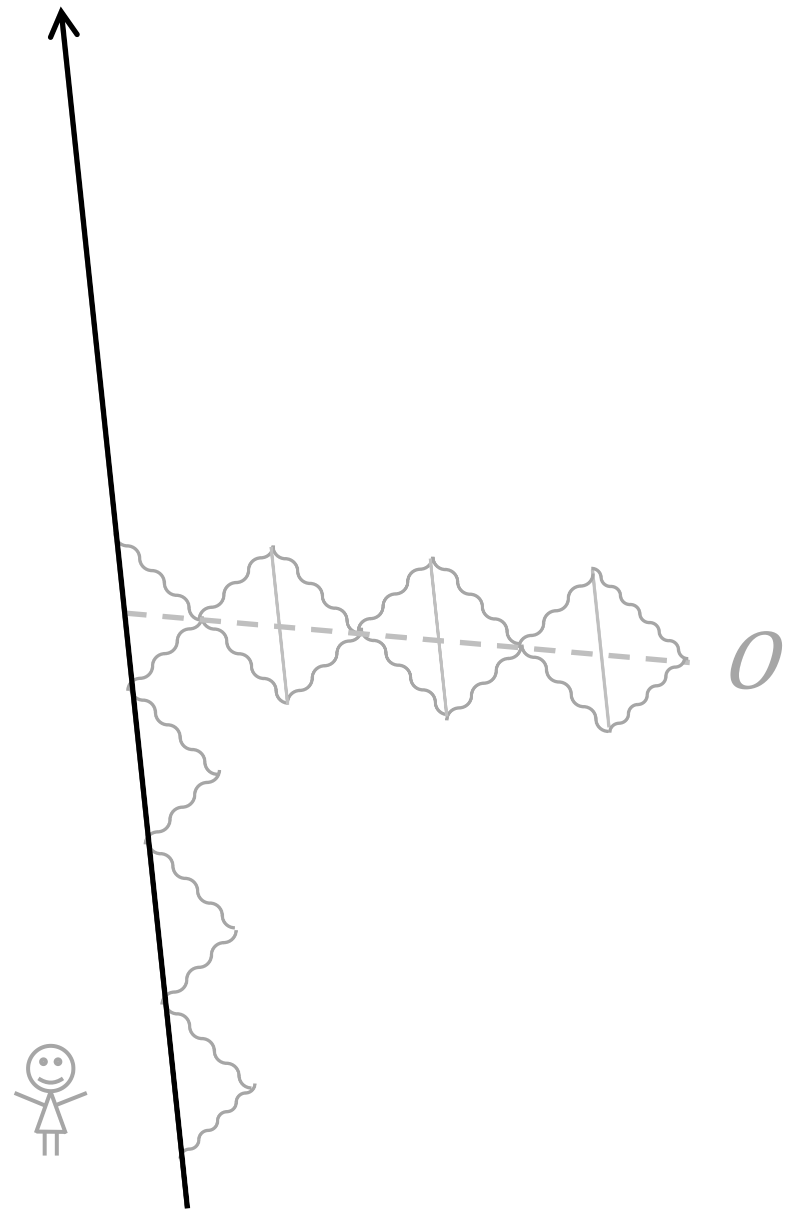
$>_{t,s}$



Kinematics

$>_{t,s}$

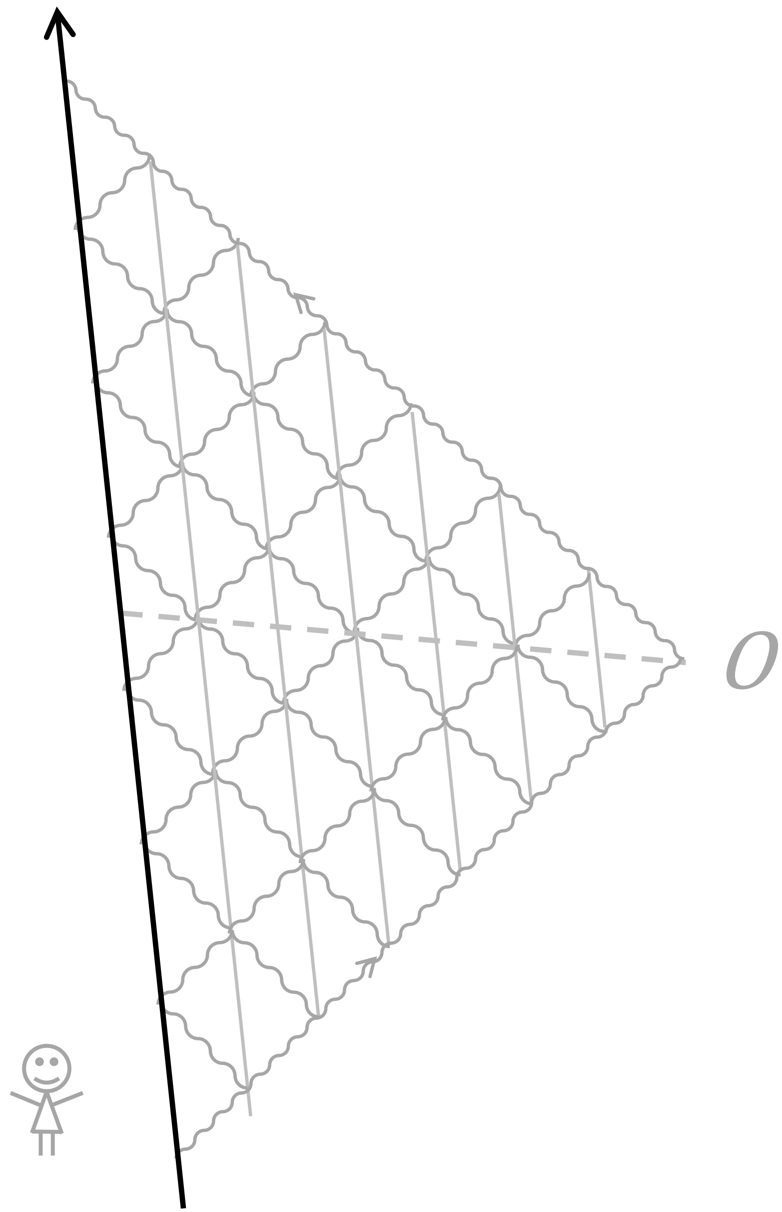
$L * \cdots * L$



Kinematics

$>_{t,s}$

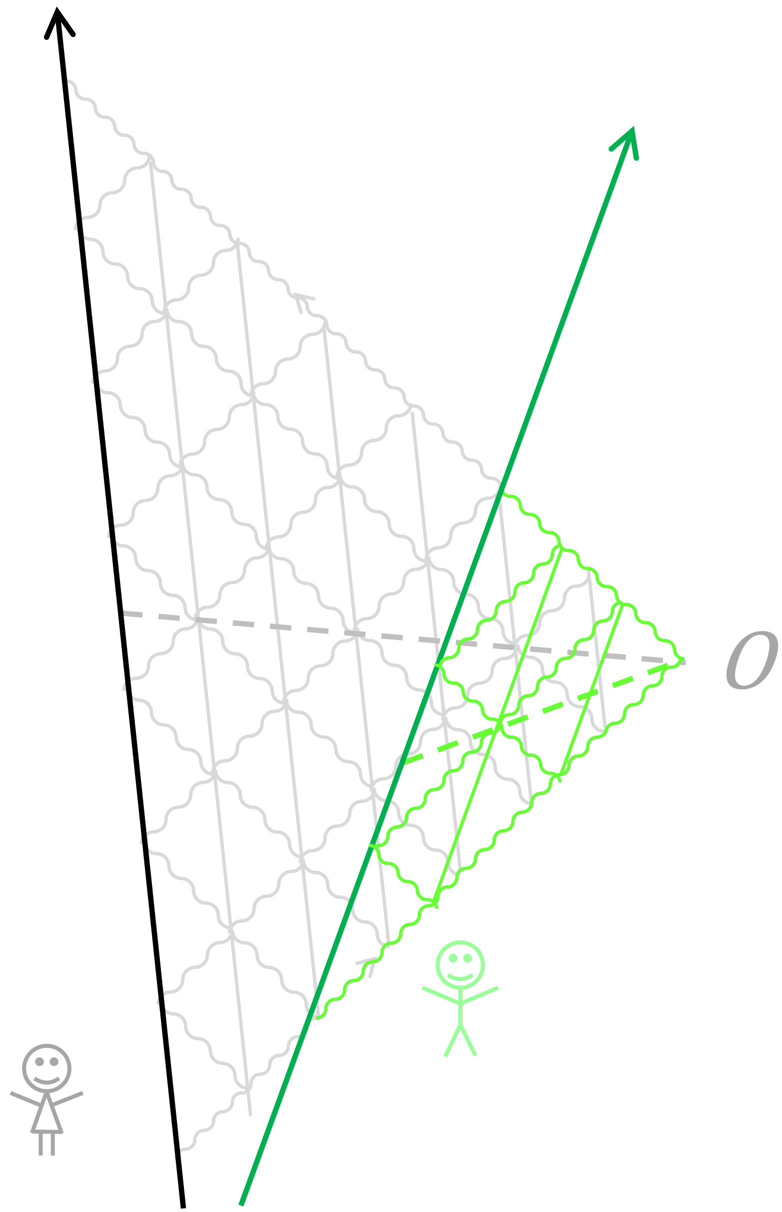
$L * \cdots * L$



Kinematics

$>_{t,s}$

$L * \cdots * L$



Dynamics

Dynamics

Hertz outline

energy as basic observable

Dynamics

Hertz outline

energy as basic observable

define elementary comparison $>_E$ (Leibniz)

basic measurement $*_E$ (Helmholtz)

Dynamics

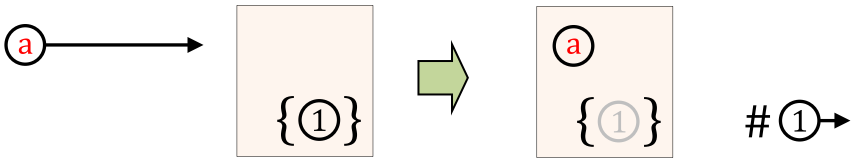
Hertz outline

energy as basic observable

define elementary comparison $>_E$ (Leibniz)

basic measurement $*_E$ (Helmholtz)

material model



Dynamics

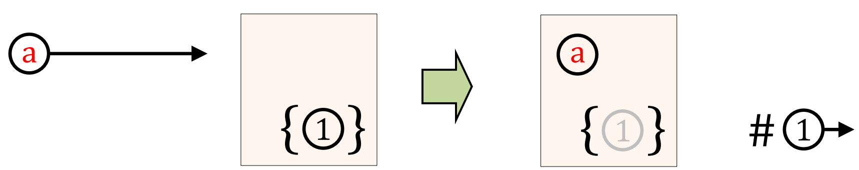
Hertz outline

energy as basic observable

define elementary comparison $>_E$ (Leibniz)

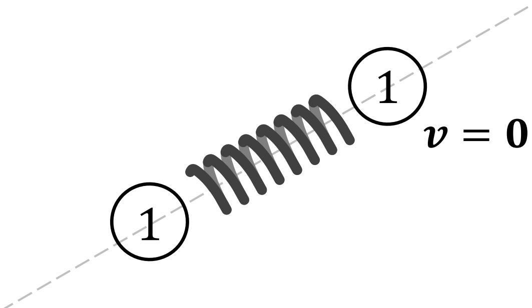
basic measurement $*_E$ (Helmholtz)

material model



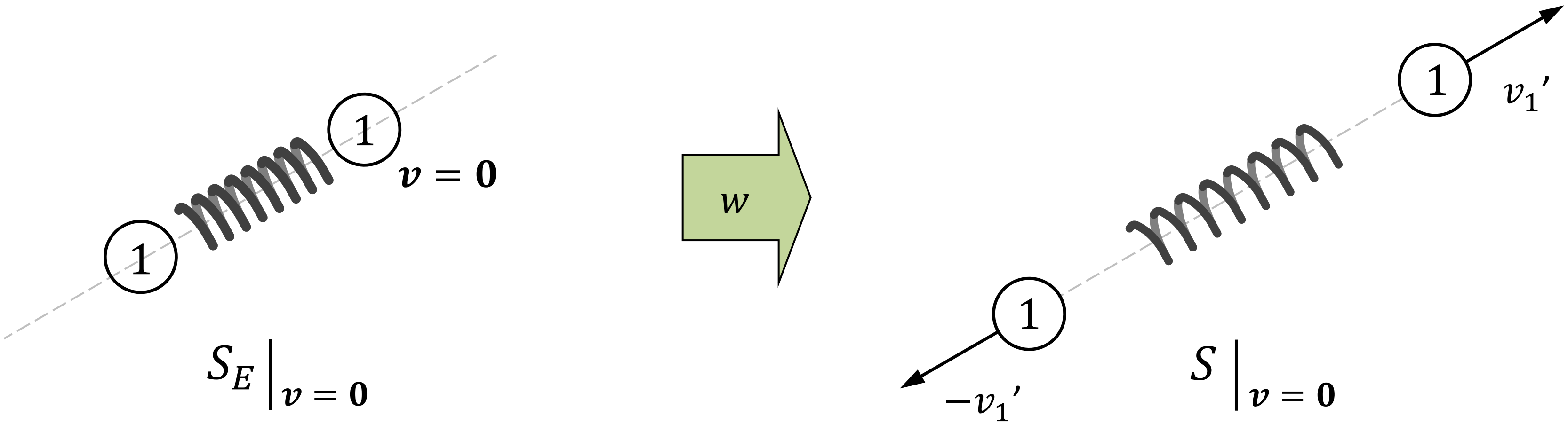
abstract physical perspective

Reference Process

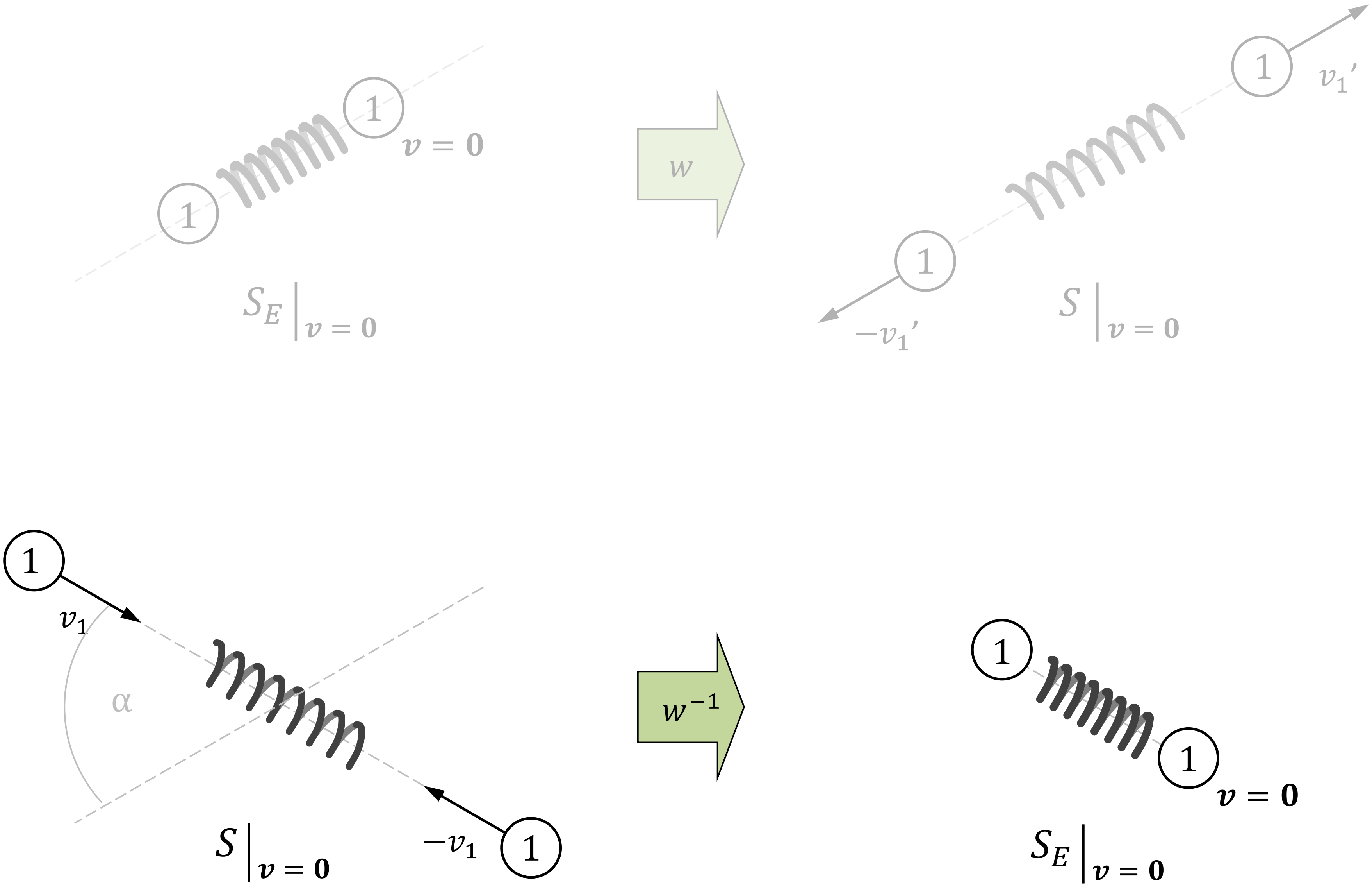


$$S_E \Big|_{v=0}$$

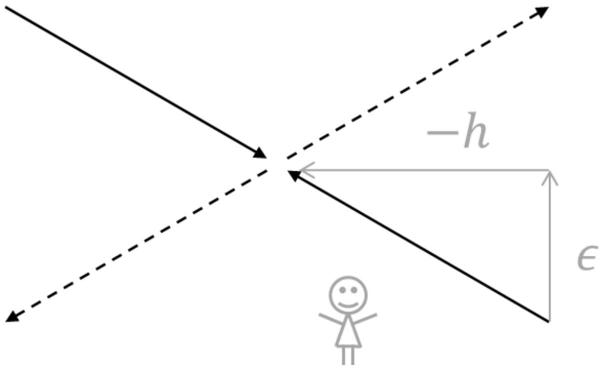
Reference Process



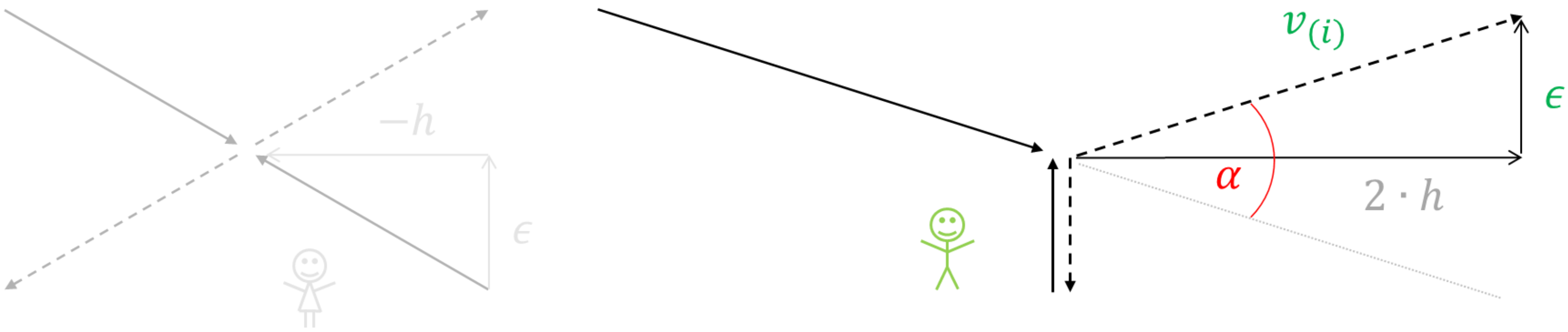
Reference Process



Eccentric Elastic Collision



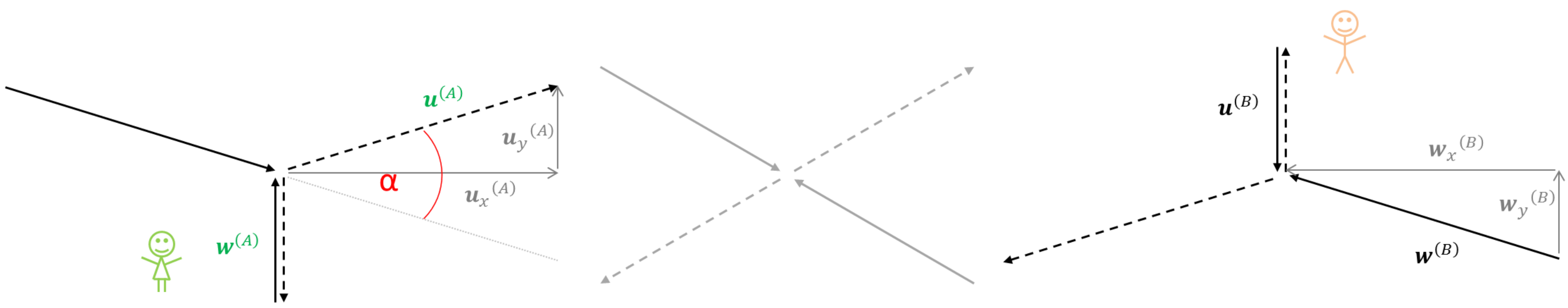
Eccentric Elastic Collision



Transversal Kick

$$\sin\frac{\alpha}{2} = \frac{\epsilon}{v}$$

Eccentric Elastic Collision



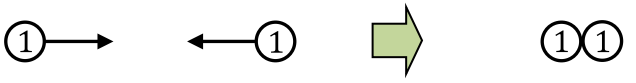
Transversal Kick

$$\sin\frac{\alpha}{2} = \frac{\sqrt{1-\frac{{v_x}^2}{c^2}}}{v}\cdot w$$

Assemble Calorimeter

inelastic collision

w_1



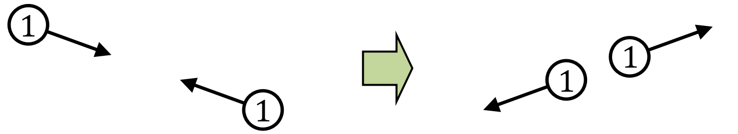
Assemble Calorimeter

inelastic collision

$$w_1$$

eccentric elastic collision

$$w_1{}^{-1} * w_1$$



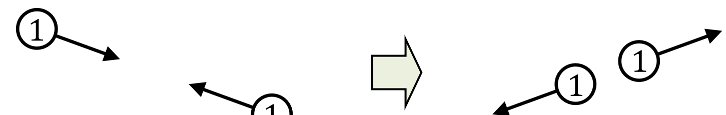
Assemble Calorimeter

inelastic collision

$$w_1$$

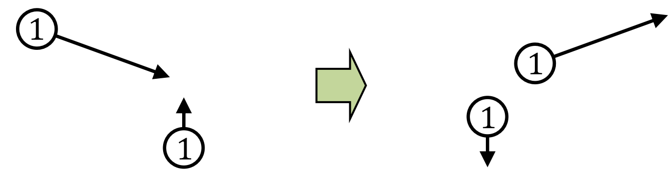
eccentric elastic collision

$$w_1{}^{-1} * w_1$$



transversal kick

$$w_T := (w_1{}^{-1} * w_1)^{(B)}$$



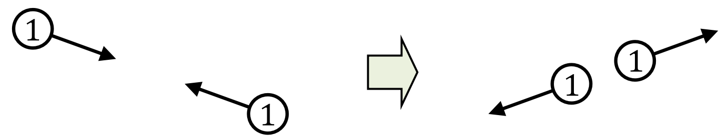
Assemble Calorimeter

inelastic collision

$$w_1$$

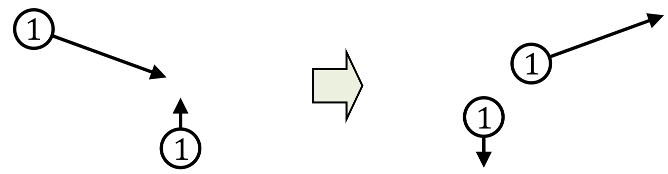
eccentric elastic collision

$$w_1{}^{-1} * w_1$$



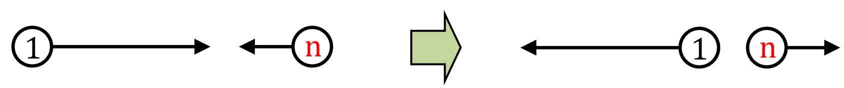
transversal kick

$$w_T := (w_1{}^{-1} * w_1)^{(B)}$$



generic head-on collision

$$w_H := w_T * \cdots * w_T$$



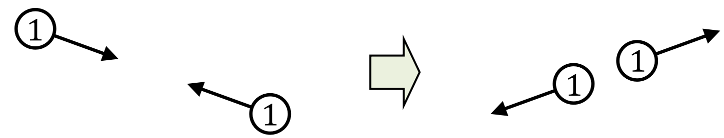
Assemble Calorimeter

inelastic collision

$$w_1$$

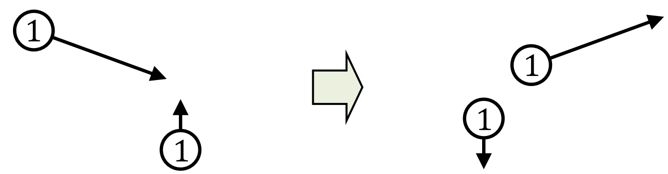
eccentric elastic collision

$$w_1{}^{-1} * w_1$$



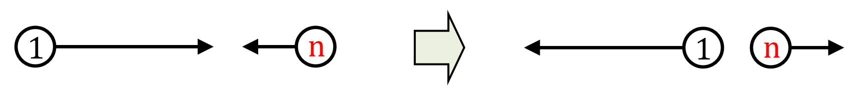
transversal kick

$$w_T := (w_1{}^{-1} * w_1)^{(B)}$$



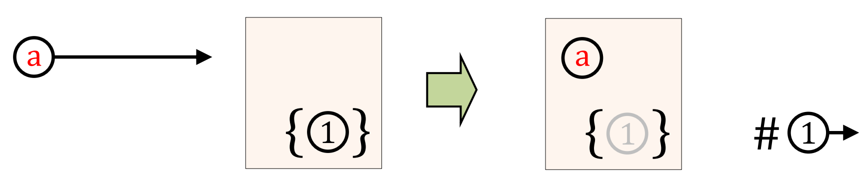
generic head-on collision

$$w_H := w_T * \cdots * w_T$$



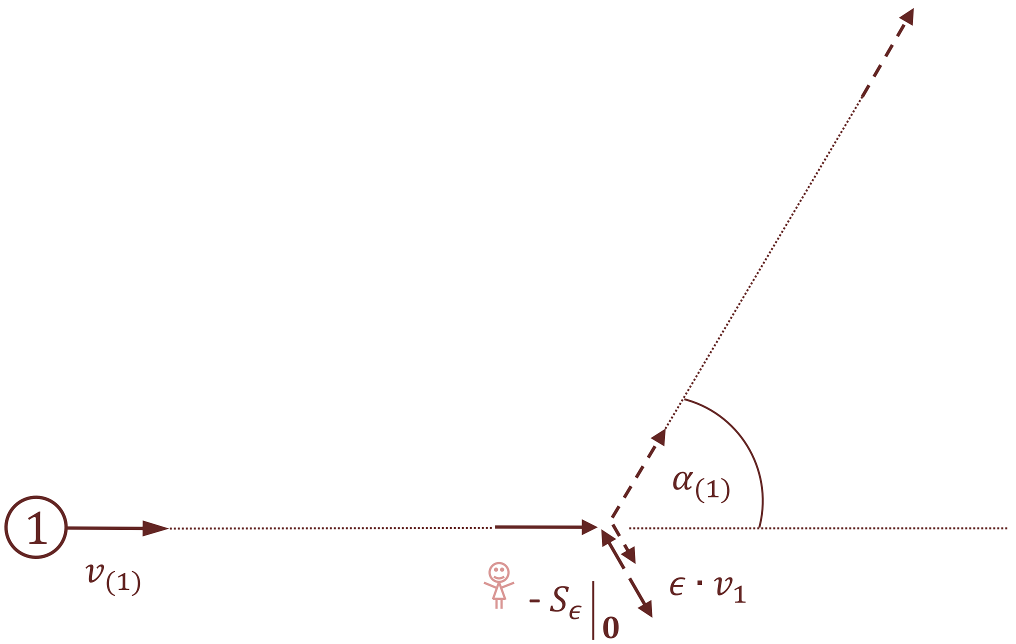
absorption in calorimeter

$$W_{\mathrm{cal}} := w_L{}^{(A)} * w_L{}^{(B)} * \cdots$$



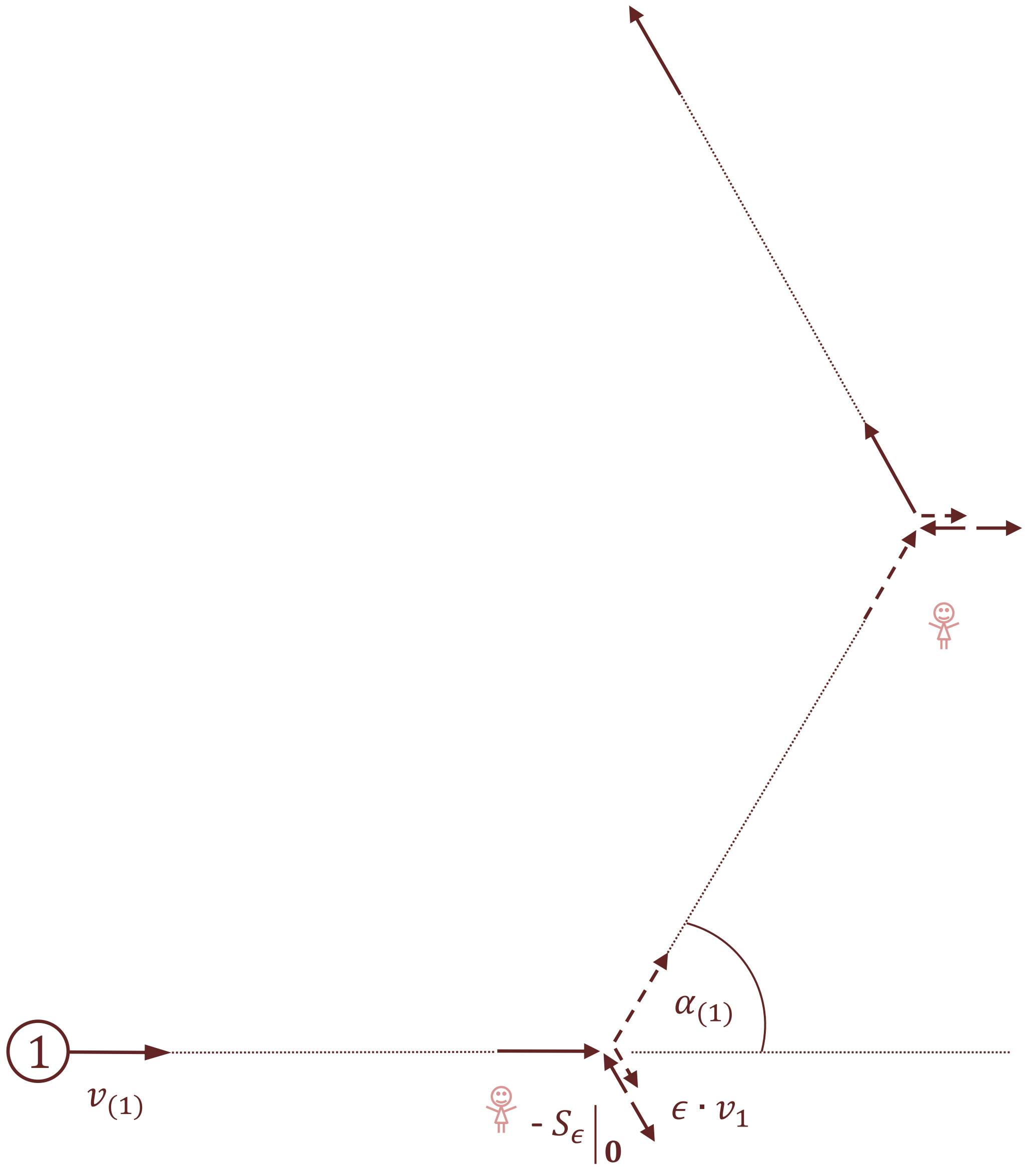
Reversion Process

w_{T}

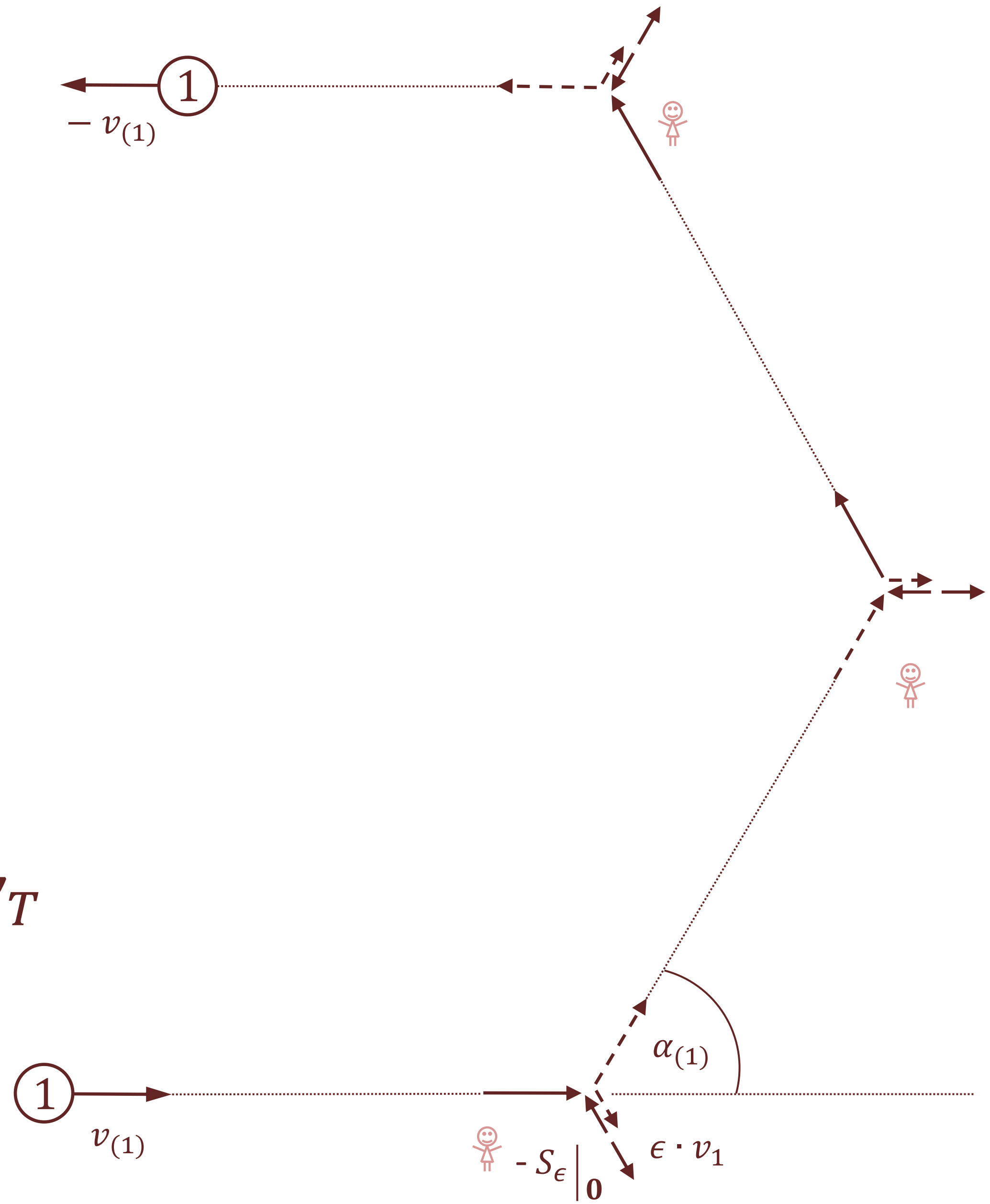


Reversion Process

$w_{\mathrm{T}} * w_{\mathrm{T}}$

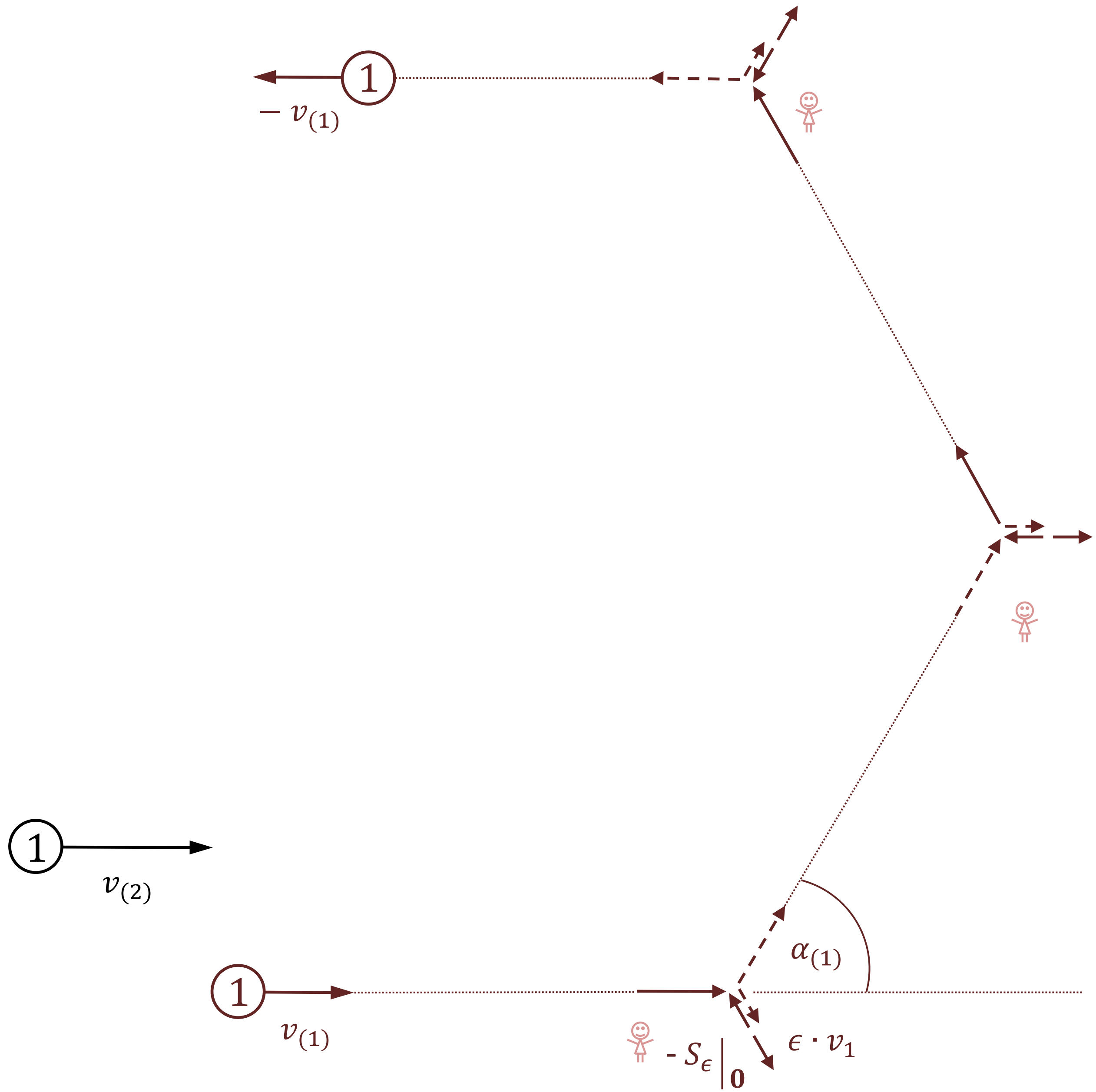


Reversion Process

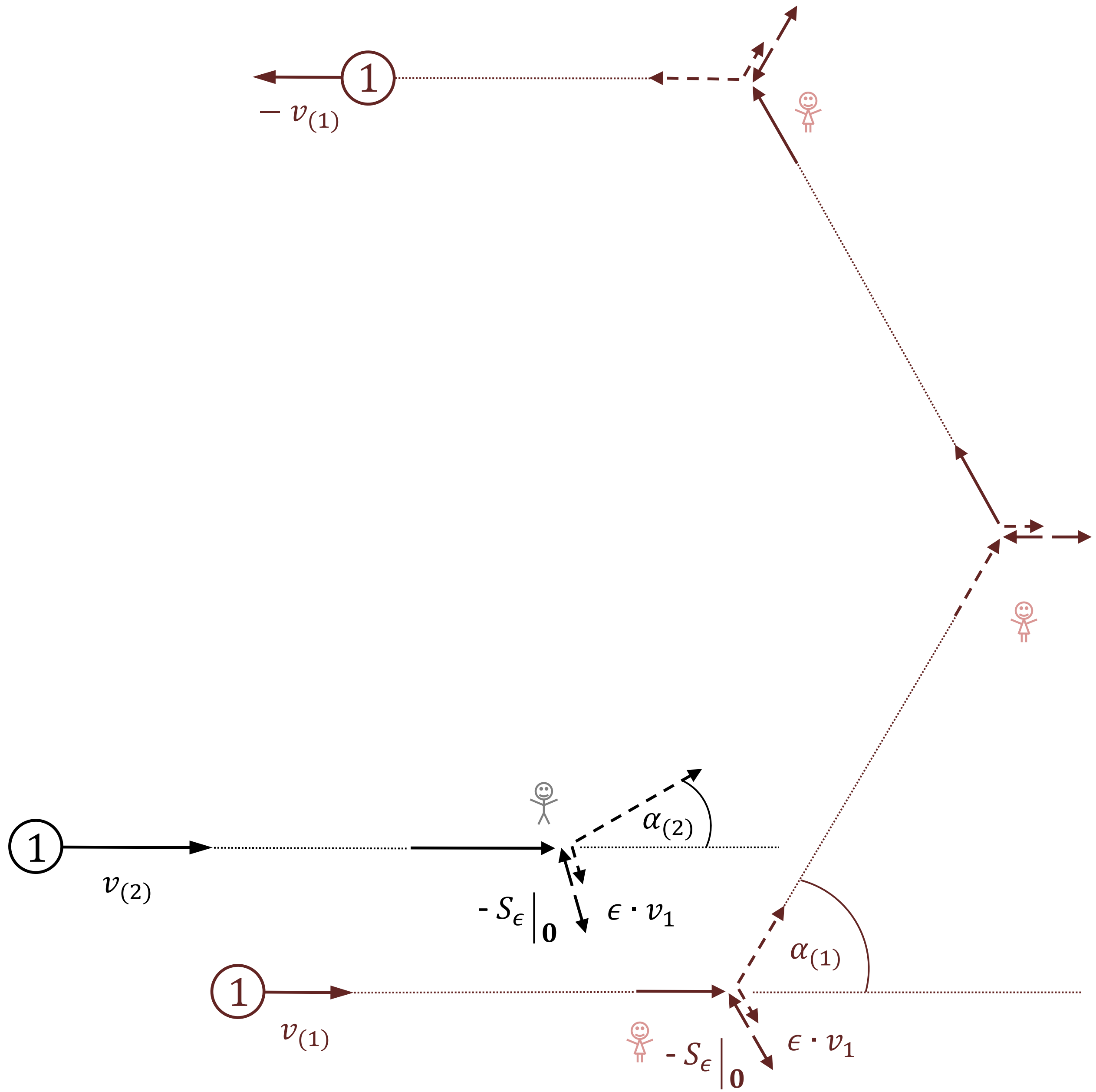


$$w_{\mathrm{T}} * w_{\mathrm{T}} * w_T$$

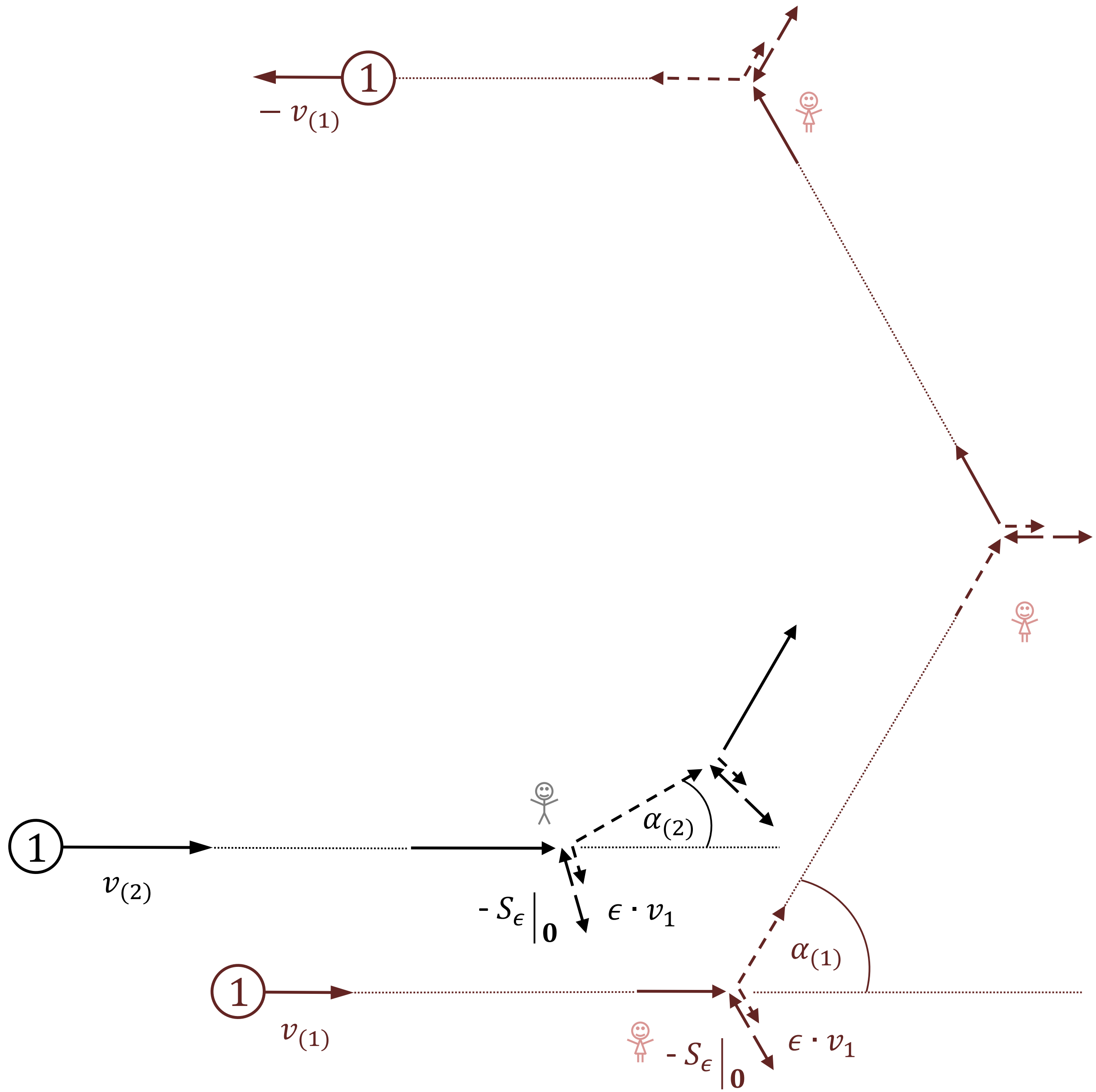
Reversion Process



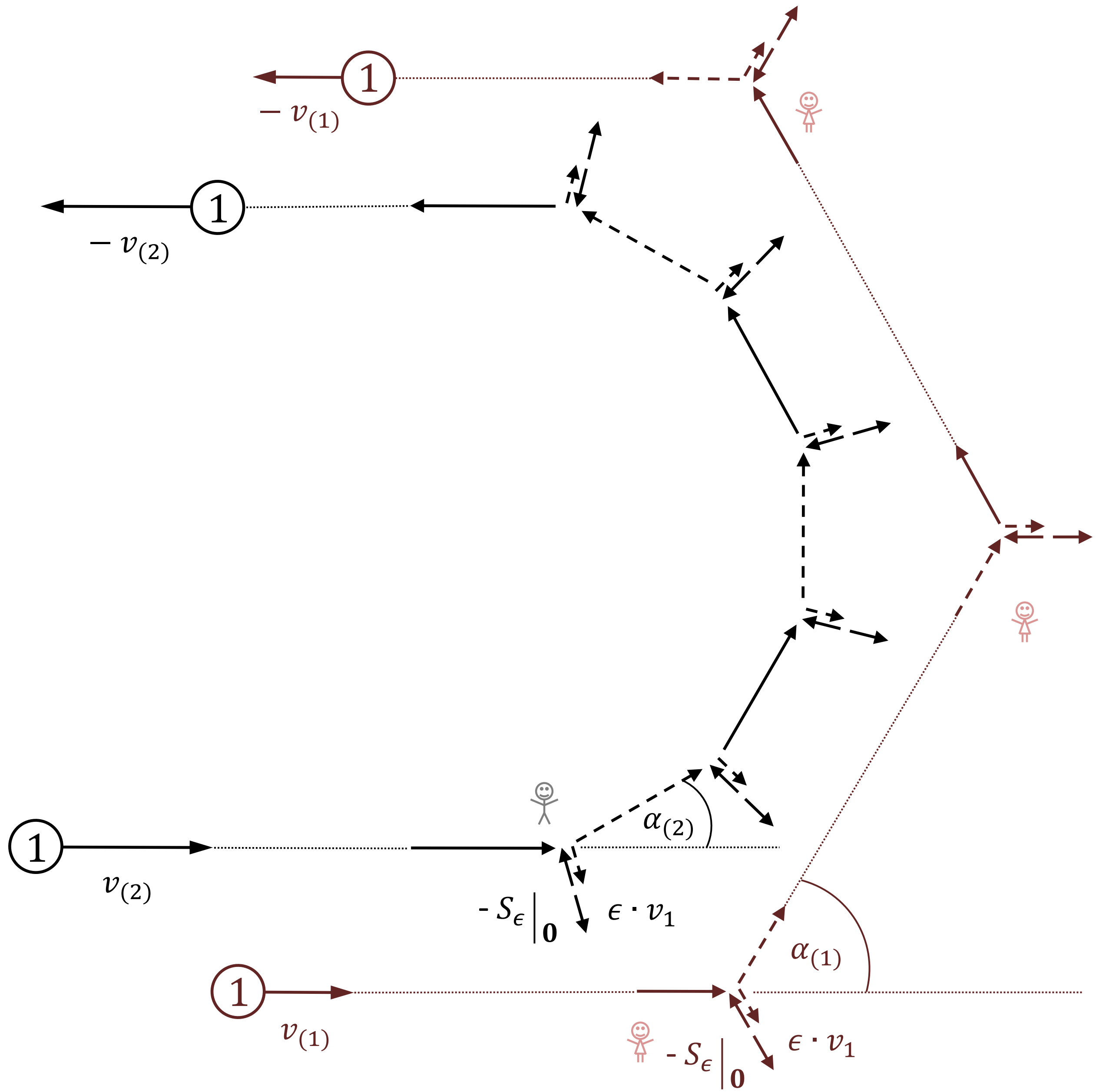
Reversion Process



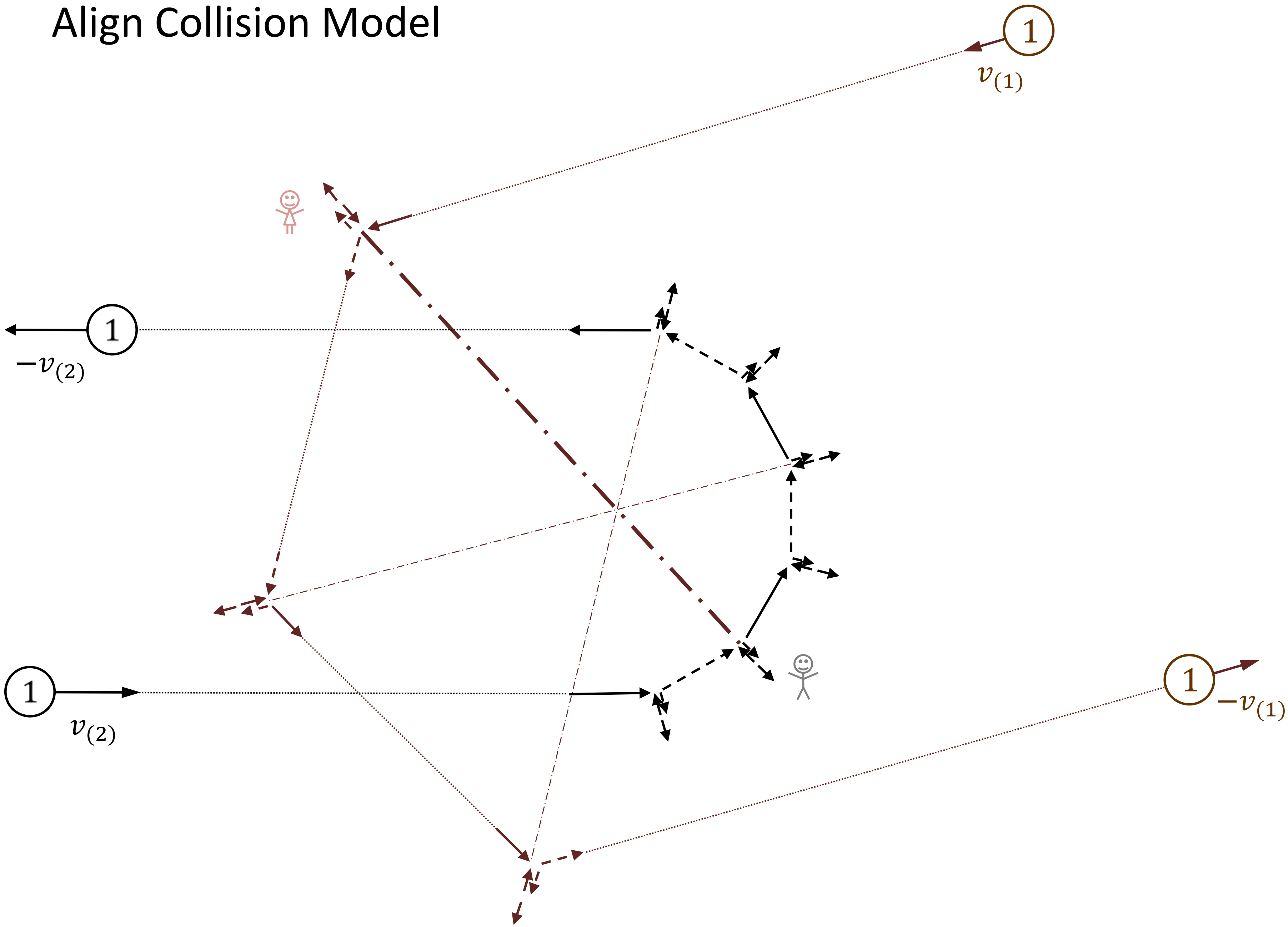
Reversion Process



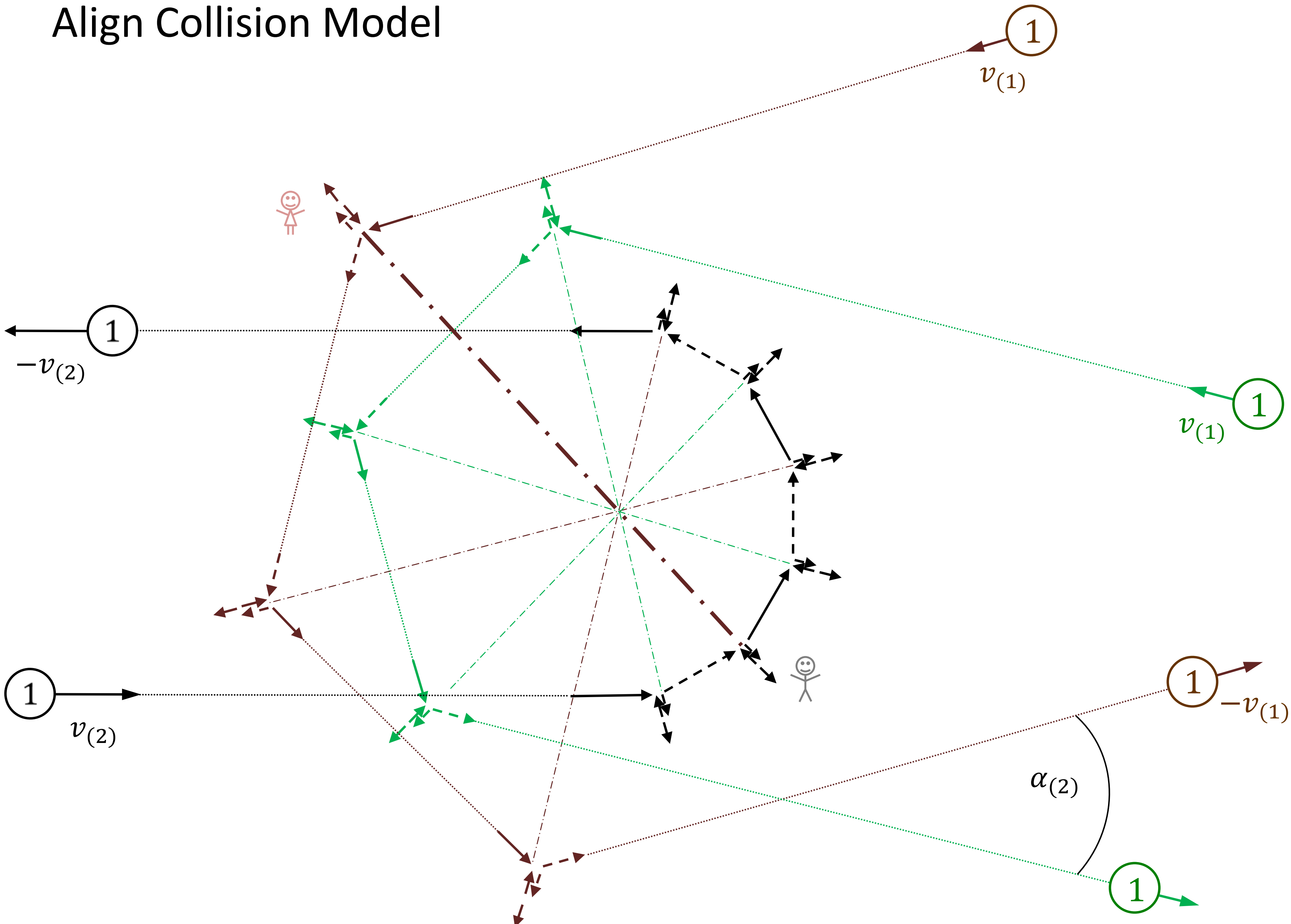
Reversion Process



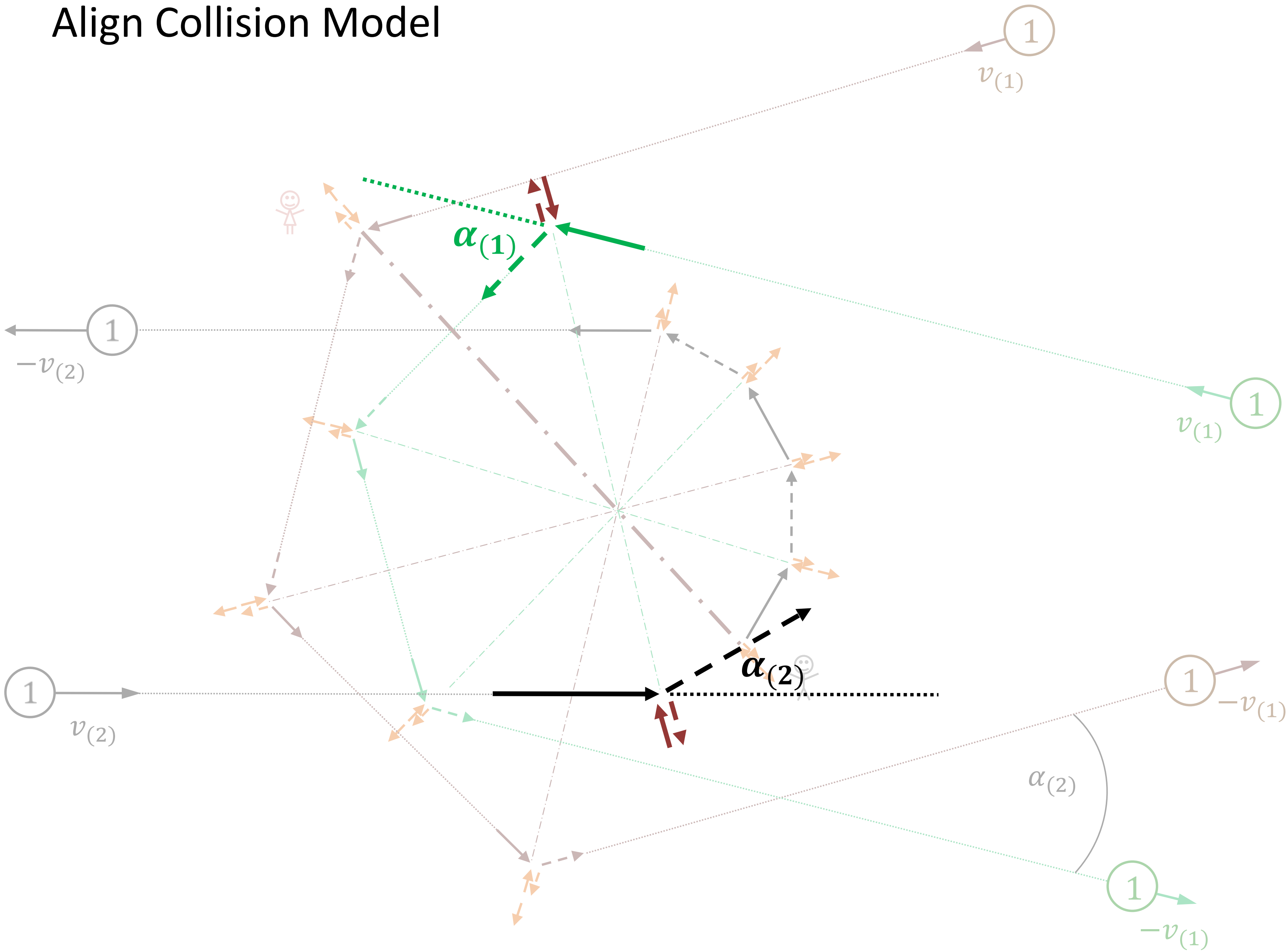
Align Collision Model



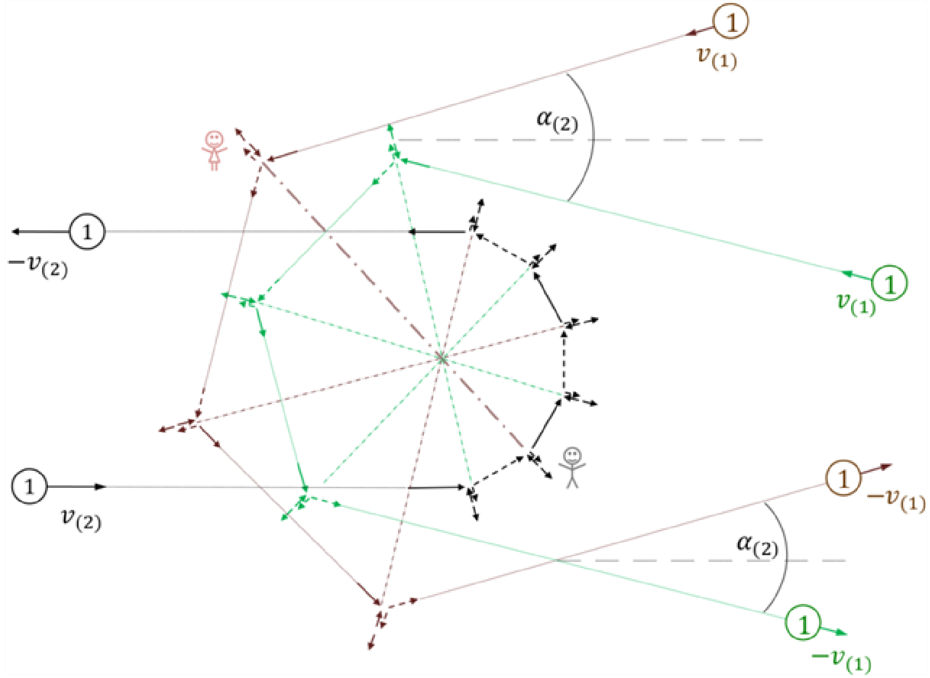
Align Collision Model



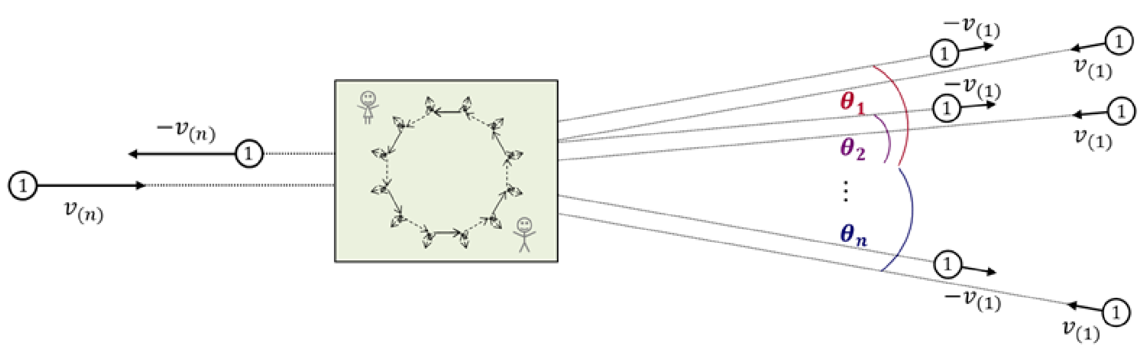
Align Collision Model



Refinement

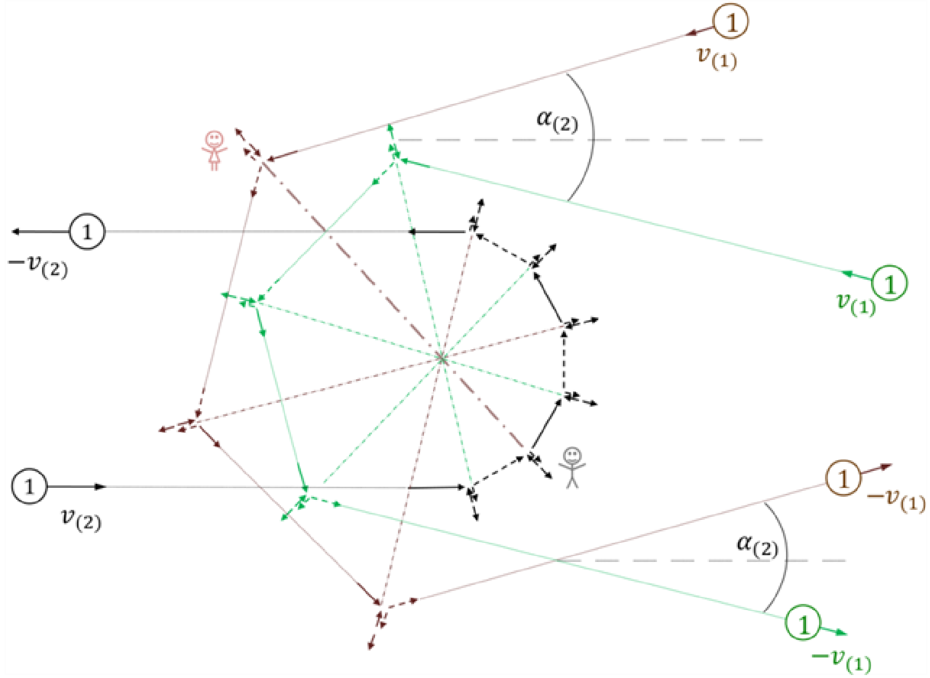


$v_{(1)}$
$\alpha_{(2)}$
$-v_{(2)}$
$v_{(1)}$
$v_{(2)}$
$-v_{(1)}$
$\alpha_{(2)}$
$-v_{(1)}$

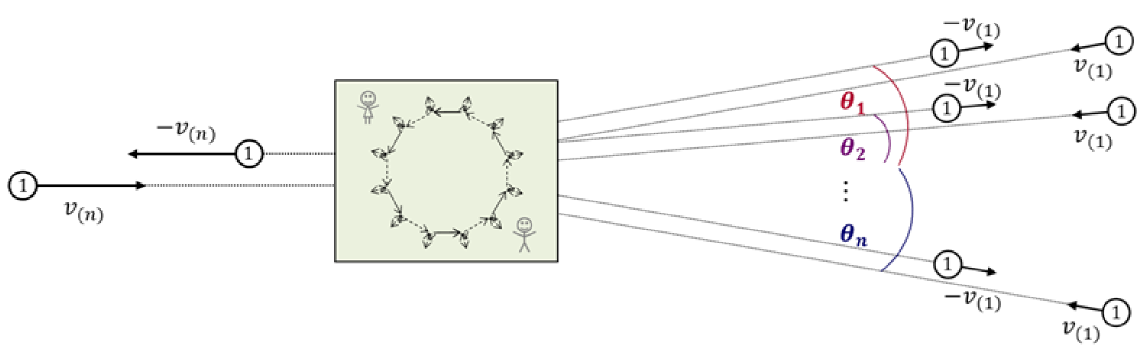


$-v_{(1)}$
$v_{(1)}$
θ_1
$-v_{(1)}$
$-v_{(n)}$
θ_2
$v_{(1)}$
$v_{(n)}$
θ_n
$-v_{(1)}$
$v_{(1)}$

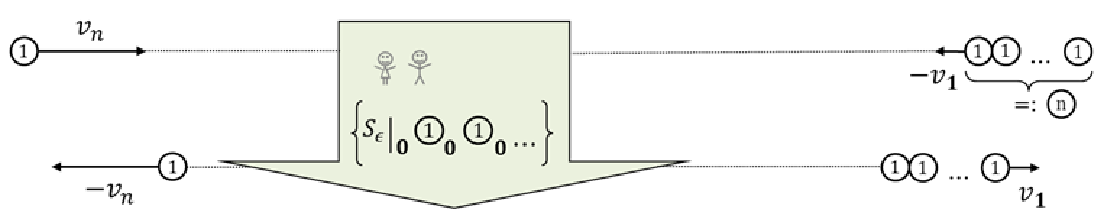
Refinement



1
$v_{(1)}$
$\alpha_{(2)}$
1
$-v_{(2)}$
1
$v_{(1)}$
1
$v_{(2)}$
1
$-v_{(1)}$
$\alpha_{(2)}$
1
$-v_{(1)}$

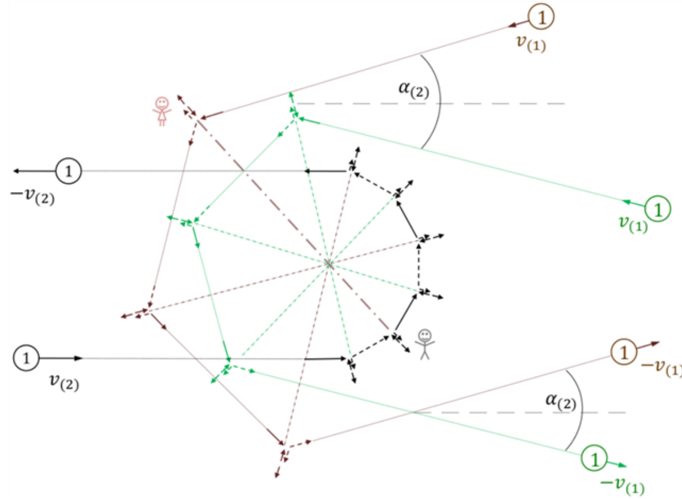


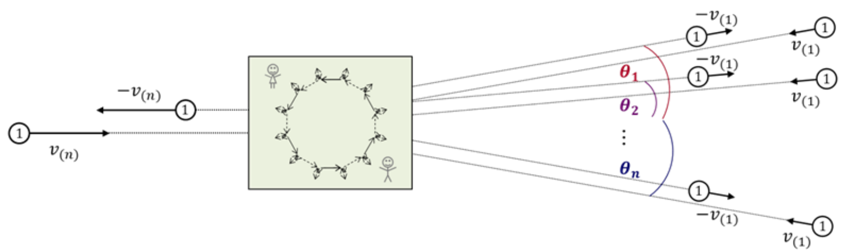
$-v_{(1)}$
1
1
$v_{(1)}$
$-v_{(1)}$
1
1
$v_{(1)}$
θ_1
θ_2
θ_n
$-v_{(n)}$
1
1
$v_{(n)}$
1
$-v_{(1)}$
1
$v_{(1)}$

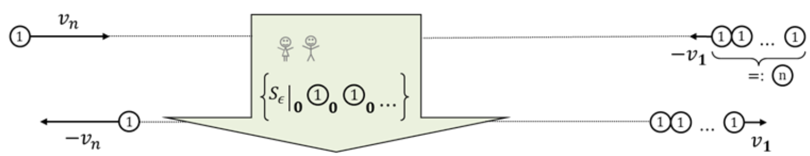


v_n
1
1 1 … 1
$-v_1$
=: n
$\{S_\epsilon|_0 ①_0 ①_0 \dots\}$
1
$-v_n$
1 1 … 1
v_1

Refinement



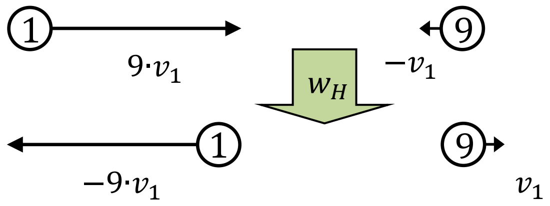




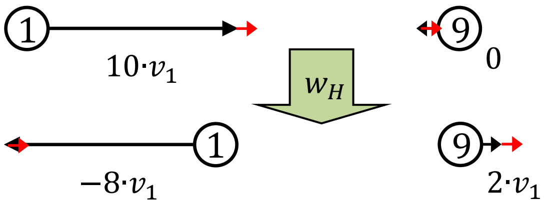
generic elastic collision

$$\textcircled{n}_{1\cdot \mathbf{v}}\,,\ \textcircled{1}_{-\mathrm{n}\cdot \mathbf{v}} \Longrightarrow \textcircled{n}_{-1\cdot \mathbf{v}}\,,\ \textcircled{1}_{\mathrm{n}\cdot \mathbf{v}}$$

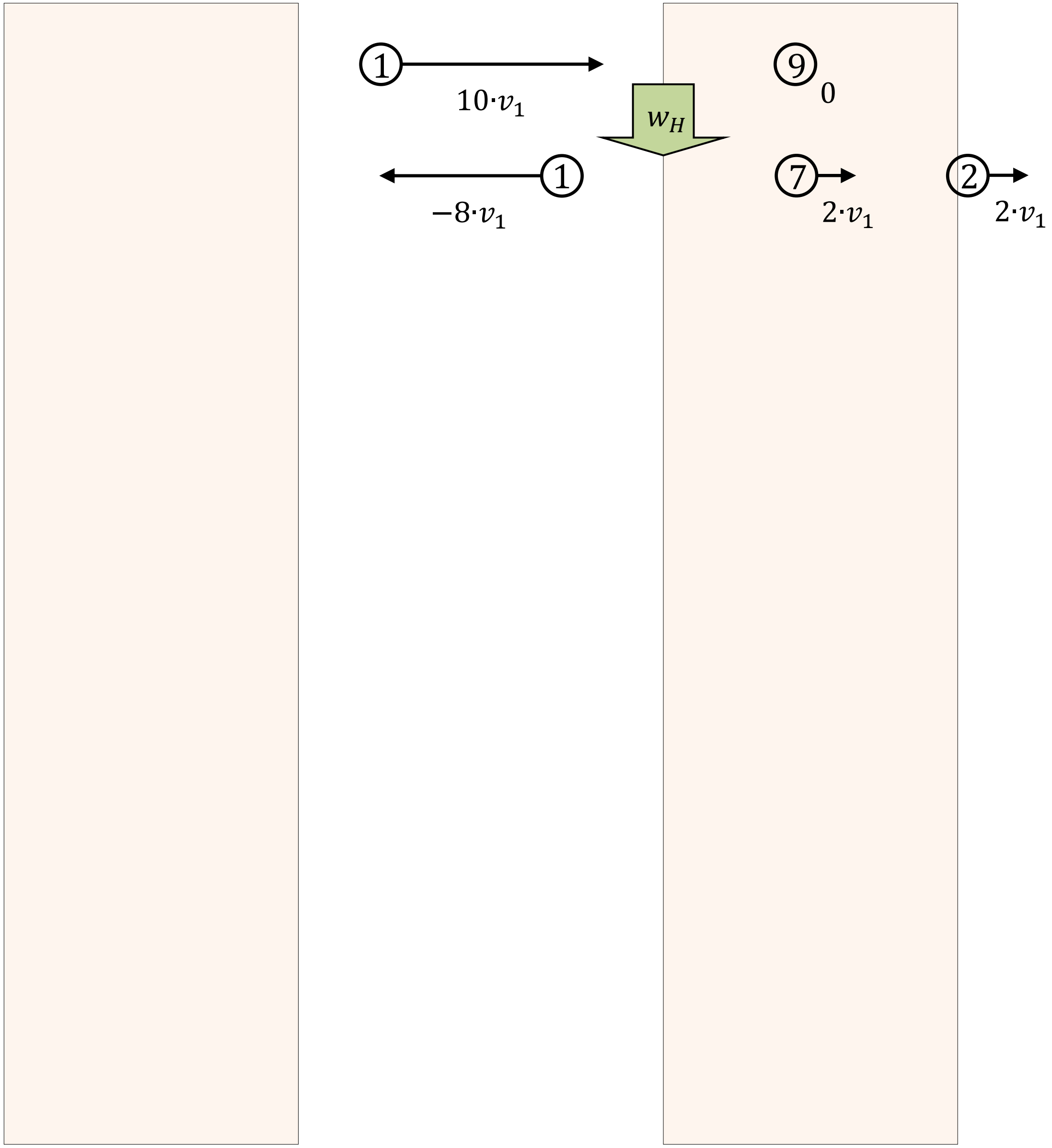
Calorimeter Model



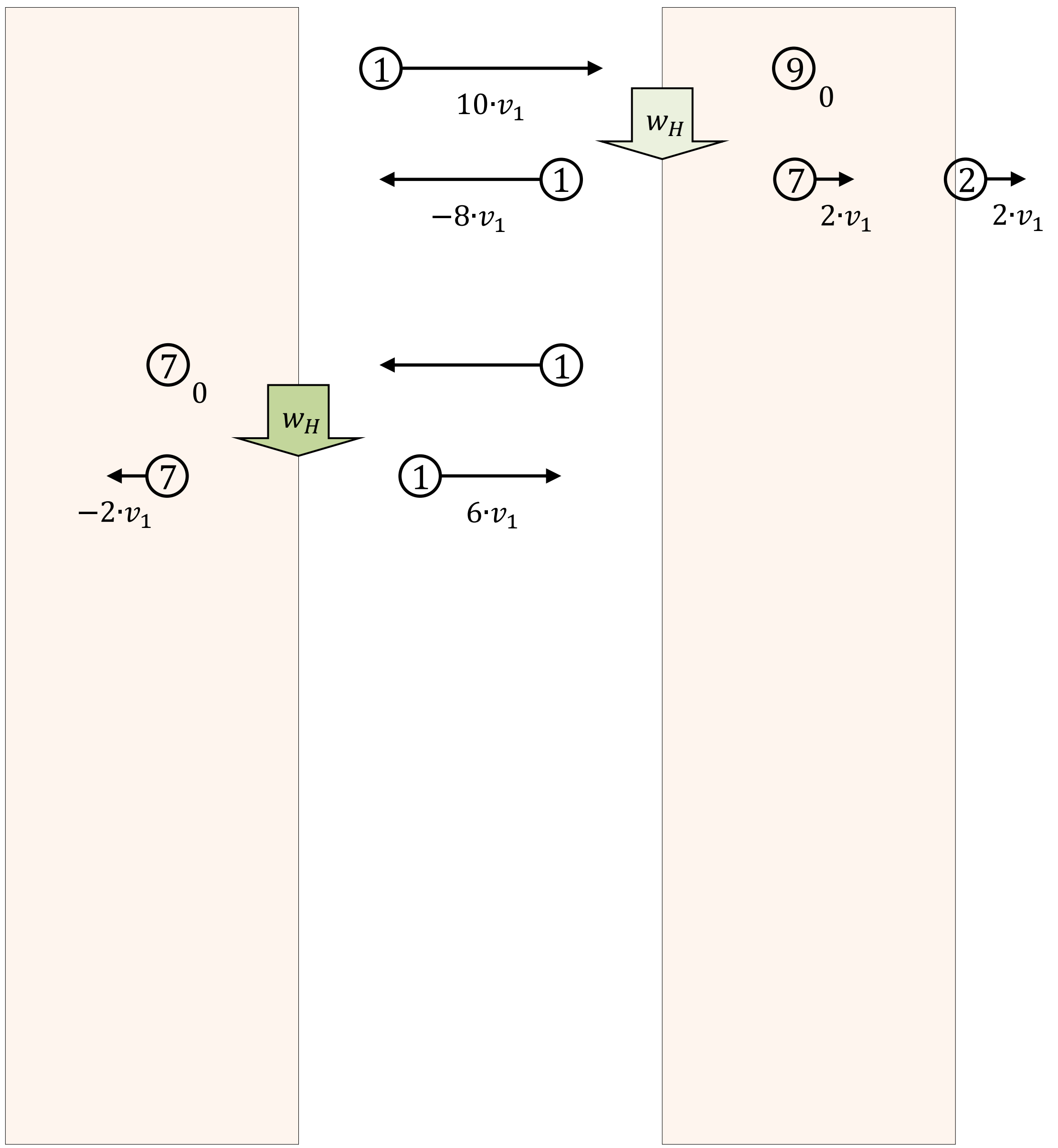
Calorimeter Model



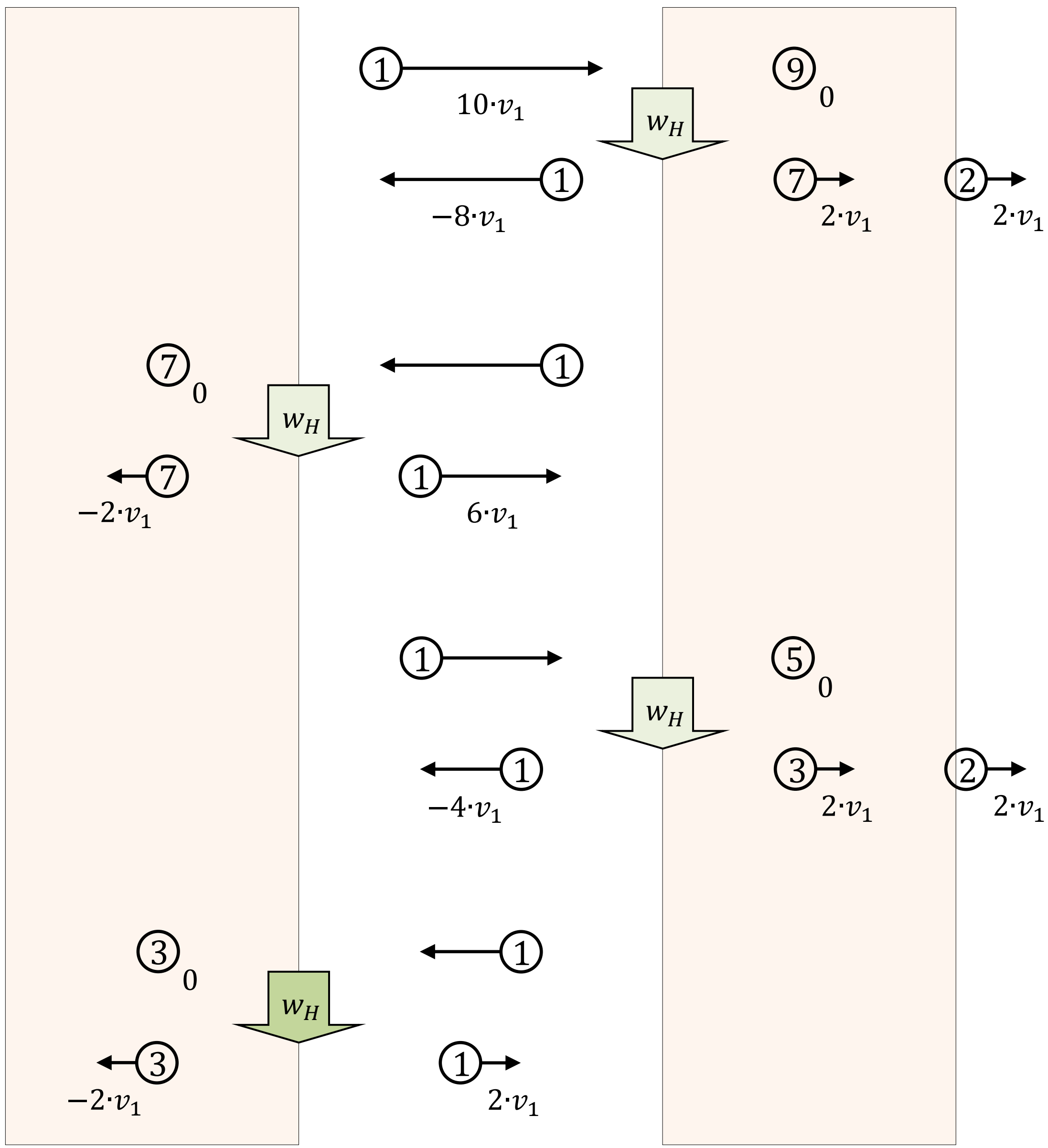
Calorimeter Model

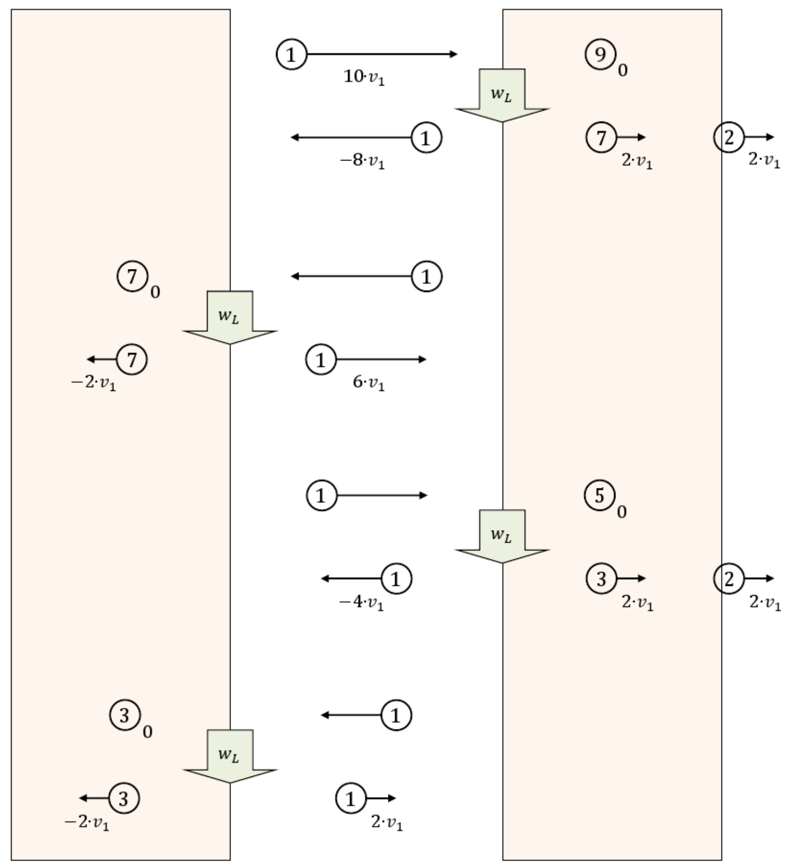


Calorimeter Model



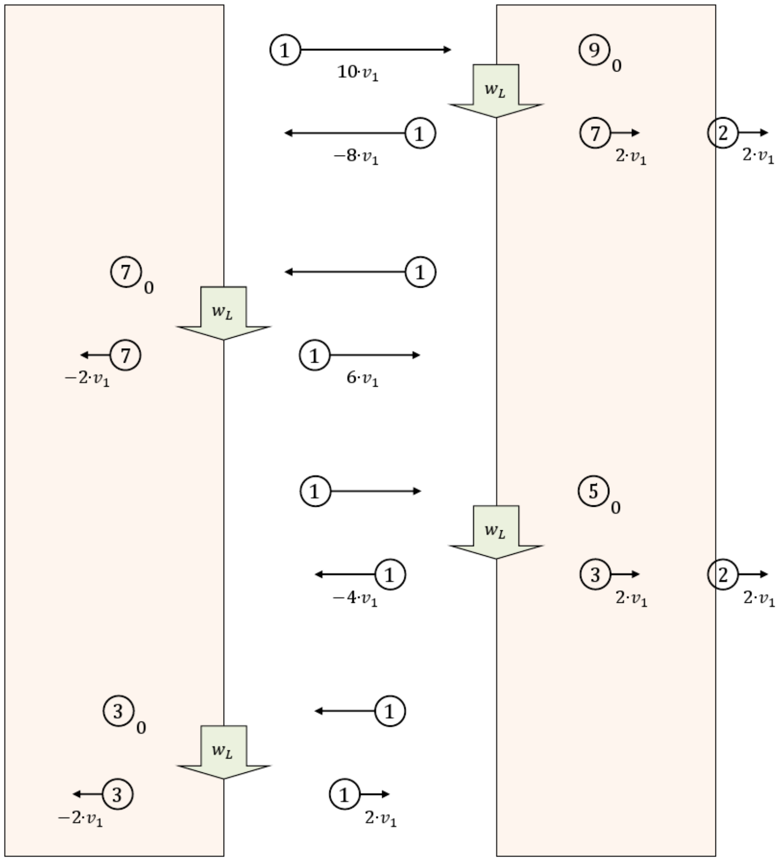
Calorimeter Model





Total balance

$$①_{\color{red}{10}\cdot\mathbf{v}},\ \color{#4a7ebb}{25}\cdot①_{\mathbf{0}} \Longrightarrow ①_{\mathbf{0}},\ \underbrace{\color{#4a7ebb}{10\cdot\{}①_{2\cdot\mathbf{v}}\color{#4a7ebb}{,}\,①_{-2\cdot\mathbf{v}}\color{#4a7ebb}{\}},\ \color{#4a7ebb}{5\cdot}①_{2\cdot\mathbf{v}}}_{\text{calorimeter extract}}$$



Total balance

$$①_{10\cdot\mathbf{v}},\ 25\cdot ①_{\mathbf{0}} \Longrightarrow ①_{\mathbf{0}},\ 10\cdot\{①_{2\cdot\mathbf{v}},①_{-2\cdot\mathbf{v}}\},\ 5\cdot ①_{2\cdot\mathbf{v}}$$

$$①_{16\cdot\mathbf{v}},\ 64\cdot ①_{\mathbf{0}} \Longrightarrow ①_{\mathbf{0}},\ 28\cdot\{①_{2\cdot\mathbf{v}},①_{-2\cdot\mathbf{v}}\},\ 8\cdot ①_{2\cdot\mathbf{v}}$$

Abstraction

Definition: *In an* ***abstraction*** *we regard the common quality of two empirical objects for itself without needing to consider the dissimilarity (in other regards).*

Abstraction

Definition: *In an <u>abstraction</u> we regard the common quality of two empirical objects for itself without needing to consider the dissimilarity (in other regards).*

$>_{l}$ if two extended objects lie on top of each other: one will *cover* the other

$>_{t}$ if two processes begin simultaneously: one will *outlast* the other

$>_{\boldsymbol{E}}$ if against the same system $\{G_I\}$: the effect of one source *exceeds* the other

$>_{\boldsymbol{p}}$ in an head-on collision: one body *overruns* the other

$$>_m \; := \; >_{\boldsymbol{p}} \big|_{\boldsymbol{v}_a = -\boldsymbol{v}_b}$$

Abstract Characterization

of reference objects

$$E[\,①_{-v_1}, ①_{v_1}\,] = E[\,S_1|_0\,] \qquad E[①_{v_1}] = \frac{1}{2} \cdot E[\,S_1|_0\,]$$

$$\boldsymbol{p}[\,S_1|_0\,] = 0 \qquad \boldsymbol{p}[①_{v_1}]$$

of calorimeter extract

$$ⓐ_{v_a} \quad \sim_{E,\boldsymbol{p}} \quad k \cdot S_1|_0\,,\; l \cdot ①_{v_1}$$

Quantification

$$E\left[ⓐ_{\boldsymbol{v}_a} \right] \overset{\text{(Equip.)}}{=} E\left[\mathrm{S}_1|_{\mathbf{0}} * \cdots * \mathrm{S}_1|_{\mathbf{0}} \right] \overset{\text{(Congr.)}}{=:} E \cdot E\left[\mathrm{S}_1|_{\mathbf{0}} \right]$$

Quantification

$$E\,[\,ⓐ_{\boldsymbol{v}_a}\,] \overset{\text{(Equip.)}}{=} E\,[\,\mathrm{S}_1|_{\mathbf{0}} * \cdots * \mathrm{S}_1|_{\mathbf{0}}\,] \overset{\text{(Congr.)}}{=:} E \cdot E[\,\mathrm{S}_1|_{\mathbf{0}}\,]$$

$$\boldsymbol{p}\,[\,ⓐ_{\boldsymbol{v}_a}\,] \overset{\text{(Perp.Mob.)}}{=} \boldsymbol{p}\,[①_{\boldsymbol{v}_1} * \cdots * ①_{\boldsymbol{v}_1}] \overset{\text{(Congr.)}}{=:} \boldsymbol{p} \cdot \boldsymbol{p}\,[①_{\boldsymbol{v}_1}\,]$$

$$m\,[\,ⓐ\,] \overset{\text{(Galilei)}}{=} m\,[① * \cdots * ①\,] \overset{\text{(Congr.)}}{=:} m \cdot m[\,①\,]$$

$$\boldsymbol{v}_a =: \boldsymbol{v} \cdot \boldsymbol{v}_{\mathbf{1}}$$

Quantity Equations

count congruent units

$$\#\{S_1|_0\} \qquad \#\{①\} \qquad \#\{①_{v_1}\} \qquad \#\{\boldsymbol{v_1}\}$$

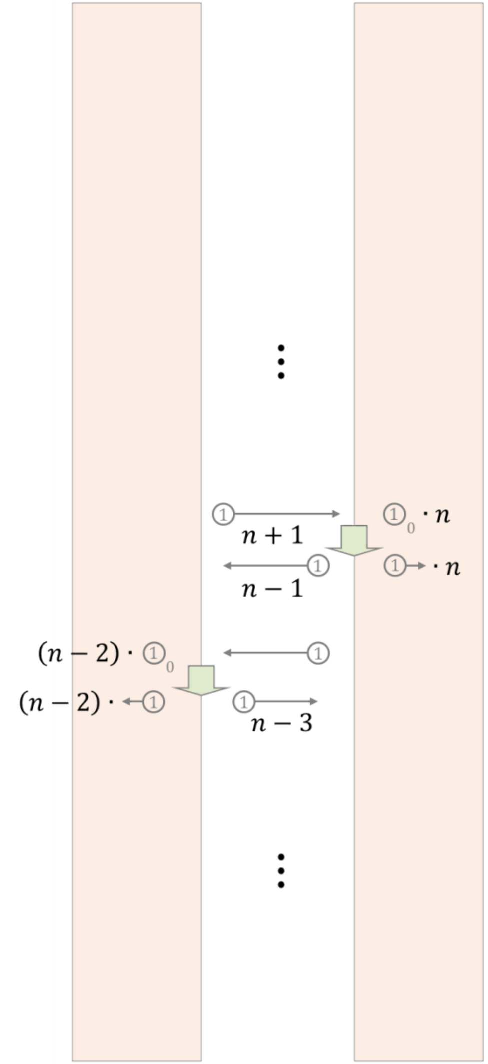
Quantity Equations

count congruent units

$$\#\{S_1|_0\} \qquad \#\{①\} \qquad \#\{①_{\boldsymbol{v_1}}\} \qquad \#\{\boldsymbol{v_1}\}$$

in Galilei-kinematics accumulate

$$(n-2)\cdot ①_{-\boldsymbol{v_1}} \;,\;\; n\cdot ①_{\boldsymbol{v_1}} \qquad \Delta v = 4\cdot v_1$$



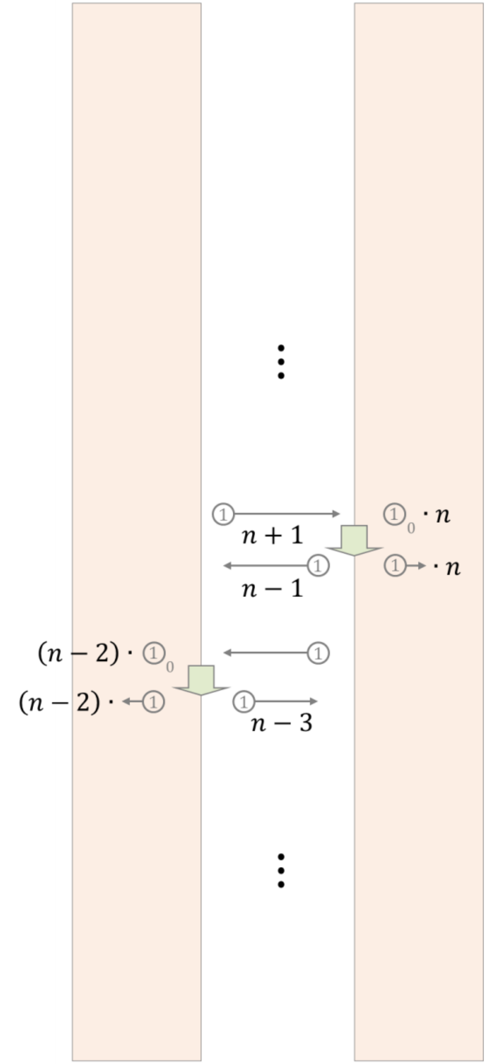
Quantity Equations

count congruent units

$$\#\{S_1|_0\} \qquad \#\{\textcircled{1}\} \qquad \#\{\textcircled{1}_{\boldsymbol{v_1}}\} \qquad \#\{\boldsymbol{v_1}\}$$

in Galilei-kinematics accumulate

$$(n-2)\cdot\textcircled{1}_{-\boldsymbol{v_1}} \ , \quad n\cdot\textcircled{1}_{\boldsymbol{v_1}} \qquad \Delta v = 4\cdot v_1$$



derive (tailored) quantity equations

$$\left\{\frac{E\,[\textcircled{a}_{\boldsymbol{v_a}}]}{E\,[S_1|_0]}\right\} = \frac{1}{2}\cdot\left\{\frac{m\,[\textcircled{a}_{\boldsymbol{v_a}}]}{m\,[\textcircled{1}_{\boldsymbol{v_1}}]}\right\}\cdot\left\{\frac{v_a}{v_1}\right\}^2 \qquad \left\{\frac{\boldsymbol{p}\,[\textcircled{a}_{\boldsymbol{v_a}}]}{\boldsymbol{p}\,[\textcircled{1}_{\boldsymbol{v_1}}]}\right\} = \left\{\frac{m\,[\textcircled{a}_{\boldsymbol{v_a}}]}{m\,[\textcircled{1}_{\boldsymbol{v_1}}]}\right\}\cdot\left\{\frac{\boldsymbol{v_a}}{\boldsymbol{v_1}}\right\}$$

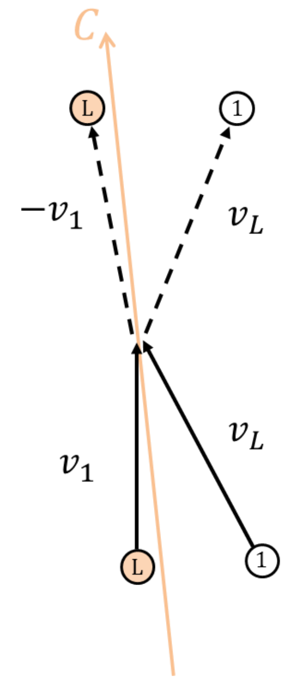
Quantity Equations

count congruent units

$\#\{S_1|_0\}$ $\#\{①\}$ $\#\{①_{v_1}\}$ $\#\{\boldsymbol{v_1}\}$

in Poincare-kinematics

$$\frac{v_L}{\sqrt{1-\frac{{v_L}^2}{c^2}}} = L \cdot \frac{v_1}{\sqrt{1-\frac{{v_1}^2}{c^2}}}$$

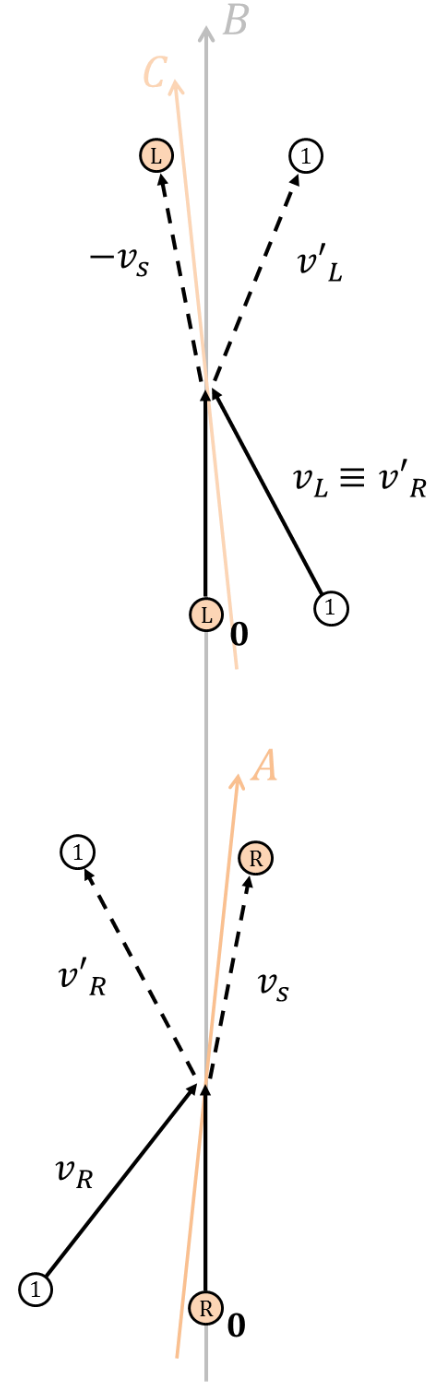


Quantity Equations

count congruent units

$\#\{S_1|_0\}$ $\#\{①\}$ $\#\{①_{\boldsymbol{v}_1}\}$ $\#\{\boldsymbol{v}_1\}$

in Poincare-kinematics



Quantity Equations

count congruent units

$$\#\{S_1|_0\} \qquad \#\{①\} \qquad \#\{①_{v_1}\} \qquad \#\{\boldsymbol{v_1}\}$$

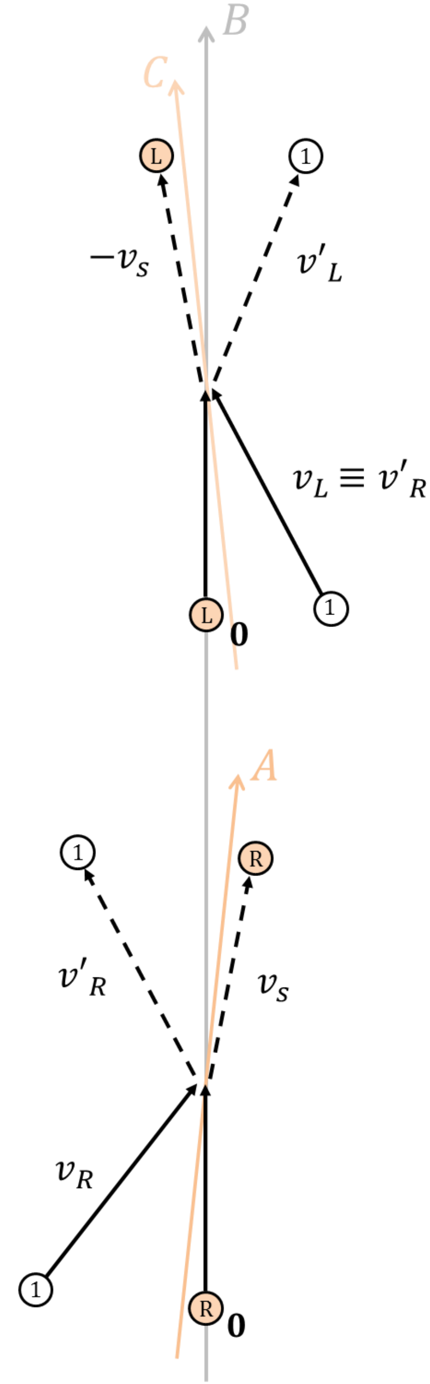
in Poincare-kinematics integrate

$$L \cdot ①_{-v_s} \quad , \quad R \cdot ①_{v_s} \qquad\qquad \Delta v = v'_L - v_R$$

derive quantity equations

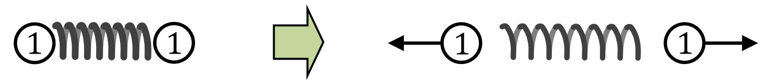
$$E[ⓐ_{v_a}] = \{m \cdot c^2 \cdot (\gamma - 1)\} \cdot E[S_1|_0]$$

$$\boldsymbol{p}[ⓐ_{v_a}] = \{\gamma \cdot m \cdot \boldsymbol{v}\} \cdot \boldsymbol{p}[①_{v_1}]$$



Measurement and Steering *Tool*

elementary standard interaction w_1 :



eccentric elastic collision

$$w_1{}^{-1} * w_1$$

transversal kick

$$w_T := (w_1{}^{-1} * w_1)^{(B)}$$

generic head-on collision

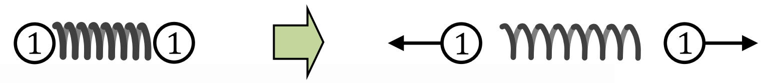
$$w_H := w_T * \cdots * w_T$$

absorption in calorimeter

$$W_{\text{cal}} := w_L{}^{(A)} * w_L{}^{(B)} * \cdots$$

Measurement and Steering *Tool*

elementary standard interaction w_1 :



eccentric elastic collision

$w_1{}^{-1} * w_1$

transversal kick

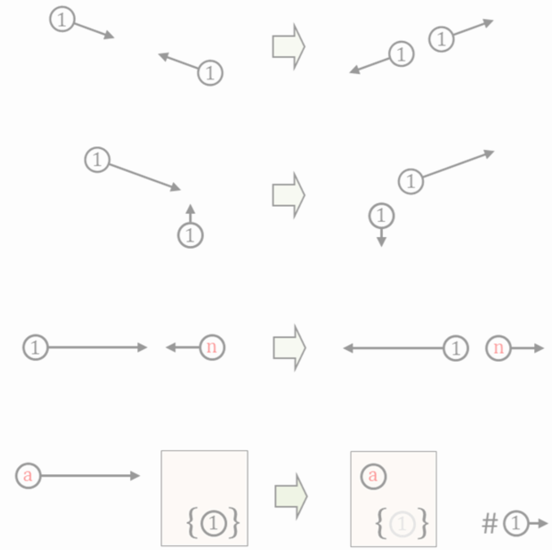
$w_T := (w_1{}^{-1} * w_1)^{(B)}$

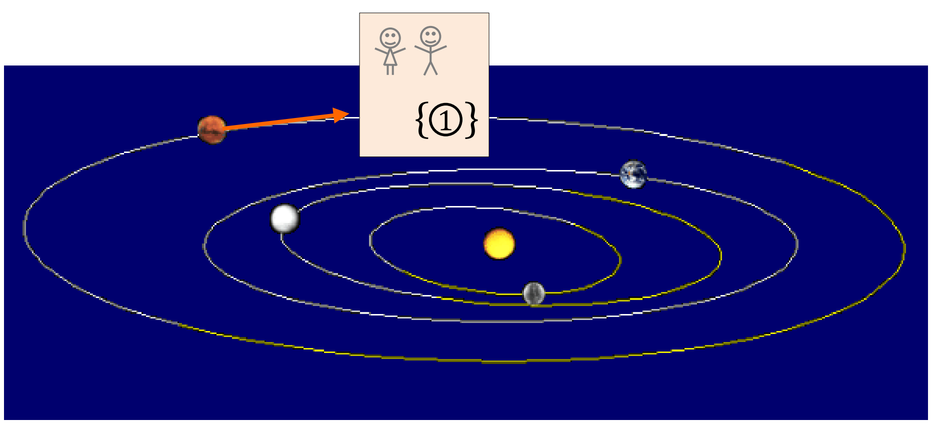
generic head-on collision

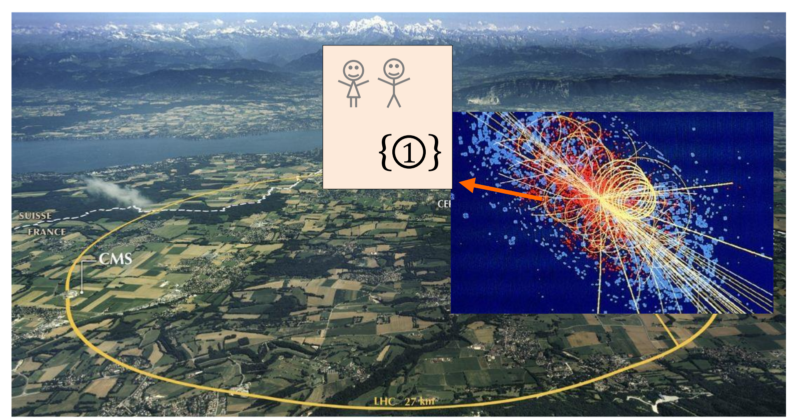
$w_H := w_T * \cdots * w_T$

absorption in calorimeter

$W_{\mathrm{cal}} := w_L{}^{(A)} * w_L{}^{(B)} * \cdots$







gravitational interaction

nuclear interaction

Physical Principles

Principle of Causality

Principle of Inertia

Impossibility of a Perpetuum Mobile

Principle of Sufficient Reason

Relativity Principle

Superposition Principle

Methodical Principles

Basic measurement: as doubling of physical measures

Congruence Principle: for reliable quantification

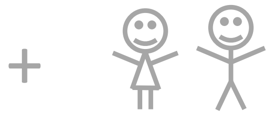
Equipollence Principle: of measuring the cause of an action by its effect

Methodical Principles

Basic measurement: as doubling of physical measures

Congruence Principle: for reliable quantification

Equipollence Principle: of measuring the cause of an action by its effect



Development

$$0 < \delta S_{\mathrm{Ham}}[\gamma]$$

Principle of Least Action (external steering effort)

steer Hamilton type variation $\delta\gamma^{(\mathrm{Ham})}$ of free course γ of intrinsic action w

derived physical quantities & EOM

$$V_{\mathrm{pot}}[x_I \to x'_I] := V_{\mathrm{pot}}[\gamma] \big/ \mathrm{mod}\ \gamma \qquad \boldsymbol{F}_i := \frac{\Delta \boldsymbol{p}_i}{\Delta t}\left[w|_{x_I,v_I}\right] \big/ \mathrm{mod}\ \boldsymbol{v}_I \qquad m_i \cdot \frac{\mathrm{d}^2 \boldsymbol{s}_i}{\mathrm{d}t^2}\left[w|_{x_I,v_I}\right] = -\nabla^{(i)} V_{\mathrm{pot}}$$

$$\forall\, i \in I$$

analyse course of intrinsic action w by external steering action $\mathrm{RB}^{(i)}$

$$E = \frac{1}{2} \cdot m \cdot v^2 \qquad \boldsymbol{p} = m \cdot \boldsymbol{v}$$

quantity equations

$$\#\{S_1|_0\} \quad \#\{①_{v_1}\} \quad \#\{①\}$$

equivalent elements in calorimeter model W_{cal}

quantified observable $\quad E\,[ⓐ_{v_a}] = E \cdot E[\,S_1|_0\,] \qquad \boldsymbol{p}\,[ⓐ_{v_a}] = \boldsymbol{p} \cdot \boldsymbol{p}\,[①_{v_1}]$

physical quantities

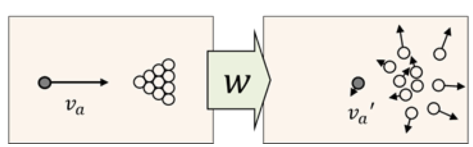
basic measurement

$$w \sim_{E,\boldsymbol{p}} W_{\mathrm{cal}} := w_1 * \ldots * w_1$$

basic observable $\quad E\,[ⓐ_{v_a}] \qquad \boldsymbol{p}\,[ⓐ_{v_a}]$

Definition: $\sim_E \qquad \sim_{\boldsymbol{p}}$

empirical basis



Planck - *Wege zur physikalischen Erkenntnis,* 1944

’The axiomatic way of thinking is useful and necessary but therein also hides the dubious danger of one-sidedness, that the physical world view loses its meaning and degenerates into an empty formalism. Because if the connection with reality is detached then a physical law *appears* - not anymore as relation between quantities which can all be measured independently from one another but - *as a definition*, by means of which one of those quantities is reduced to the others. Such *reinterpretation* is particularly tempting because a physical quantity can be defined much more exactly by an equation than by a *measurement*; however that fundamentally represents *abandonment of its true meaning*.’