Facts and conventions on Poincaré’s disc

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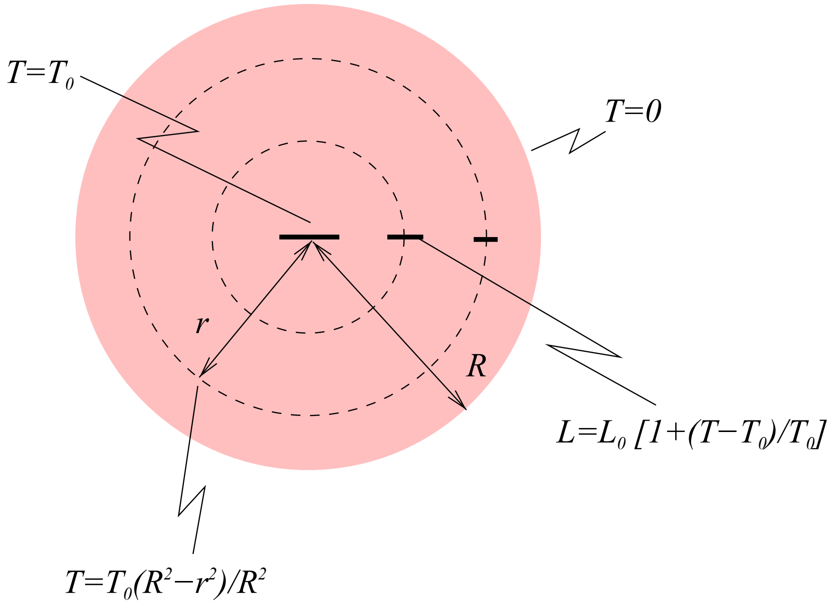
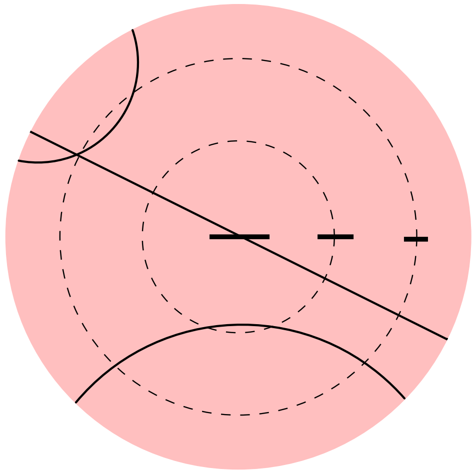
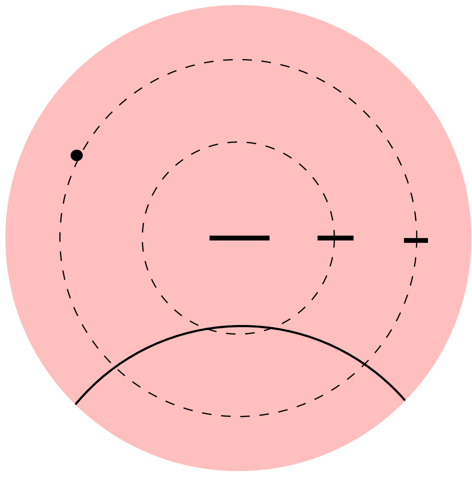
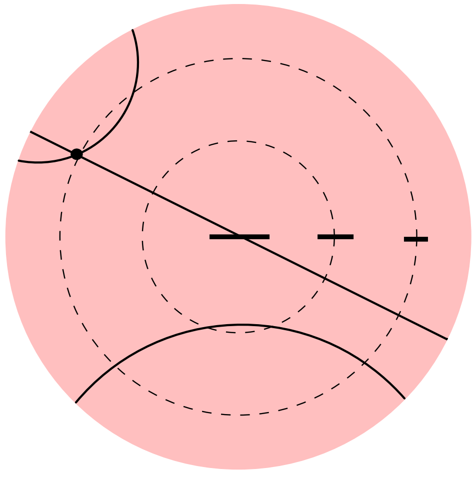
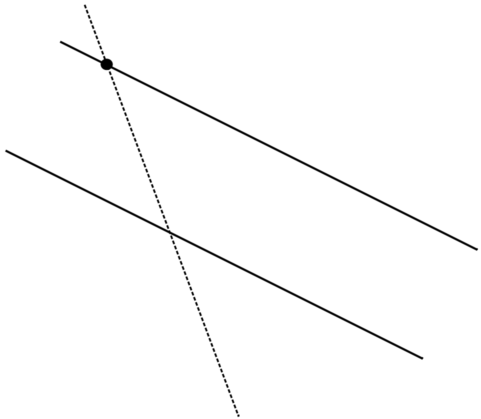


T=T0
T=0
r
R
L=L0 [1+(T−T0)/T0]
T=T0(R2−r2)/R2









$$\text{geometry} \quad + \quad \text{physics} \quad = \quad \text{empirical facts}$$

$$\begin{pmatrix} \text{Bolyai–} \\ \text{Lobachevskian} \\ \text{geometry} \end{pmatrix} + \begin{pmatrix} \text{the temperature} \\ \text{of the world is} \\ \text{constant} \end{pmatrix} = \begin{pmatrix} \text{empirical} \\ \text{facts} \end{pmatrix}$$

$$\begin{pmatrix} \text{Euclidean} \\ \text{geometry} \end{pmatrix} + \begin{pmatrix} \text{the temperature} \\ \text{of the world is:} \\ T(r) = T_0 \frac{R^2 - r^2}{R^2} \end{pmatrix} = \begin{pmatrix} \text{empirical} \\ \text{facts} \end{pmatrix}$$

Geometric conventionalism: we are free to choose the geometry we use to describe the world

Water boils at temperature 100°C.

Water boils at temperature 212°F.

Water boils at temperature 100.

Water boils at temperature 212.

Semantic conventionalism: we are free to choose the meanings of the terms in which we describe the world

Geometric conventionalism: we are free to choose the geometry we use to describe the world

Semantic conventionalism: we are free to choose the meanings of the terms in which we describe the world

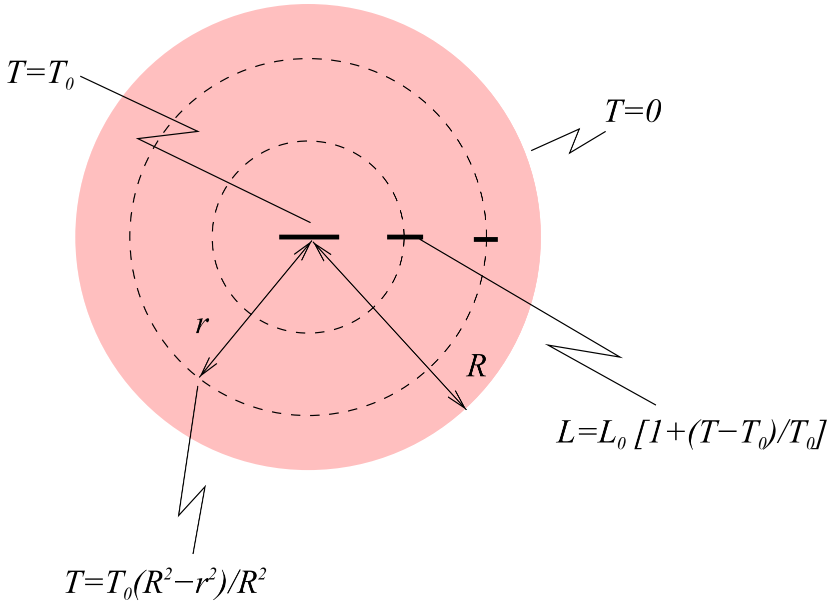
Definition 1

$$l \;:\; \begin{array}{c}\text{the measuring rod fits } n \text{ times}\\ \text{onto the line segment}\end{array} \;\Rightarrow\; l = n$$

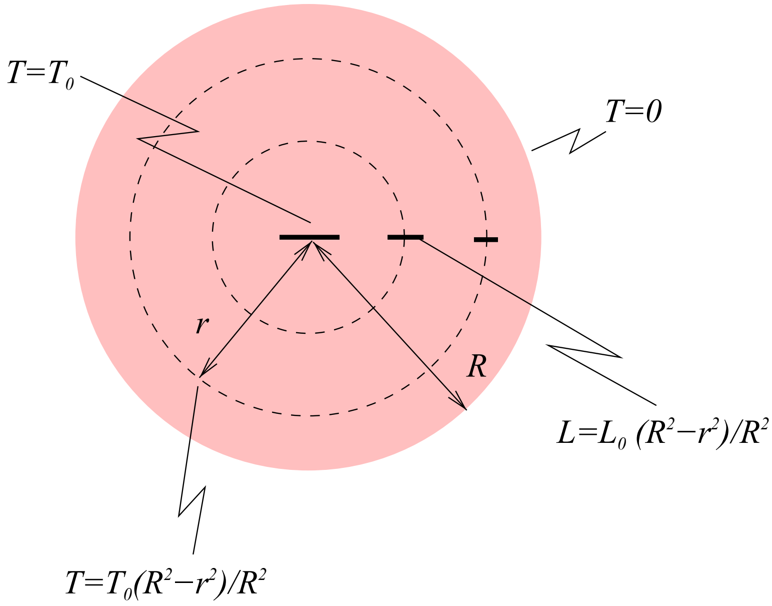
$$T \;:\; \begin{array}{c}\text{the measuring rod fits } n \text{ times}\\ \text{onto the mercury column}\\ \text{of the thermometer}\end{array} \;\Rightarrow\; T = n$$

Empirical law

$$\left(\begin{array}{c}\text{Bolyai–}\\ \text{Lobachevsky}\\ \text{geometry}\end{array}\right) + \left(\begin{array}{c}\text{the temperature of the world is}\\ \text{constant}\end{array}\right)$$



$T = T_0$
$T = 0$
r
R
$L = L_0\,[1 + (T - T_0)/T_0]$
$T = T_0(R^2 - r^2)/R^2$



$T=T_0$
$T=0$
r
R
$L=L_0\,(R^2-r^2)/R^2$
$T=T_0(R^2-r^2)/R^2$

Definition 2

l : the measuring rod fits n times onto the line segment $\Rightarrow$ $l = \sum_{i=1}^{n} \frac{R^2 - r_i^2}{R^2}$

T : the measuring rod fits n times onto the mercury column of the thermometer $\Rightarrow$ $T = n \frac{R^2 - r^2}{R^2}$

Definition 2

$$l \ : \ \begin{array}{c}\text{the measuring rod fits } n \text{ times} \\ \text{onto the line segment}\end{array} \ \Rightarrow \ l = \sum_{i=1}^{n} \frac{R^2 - r_i^2}{R^2}$$

$$T \ : \ \begin{array}{c}\text{the measuring rod fits } n \text{ times} \\ \text{onto the mercury column} \\ \text{of the thermometer}\end{array} \ \Rightarrow \ T = n \frac{R^2 - r^2}{R^2}$$

r, R, the center O of the disc?

Step 1

Choose a point O on the disc and a number R.

Step 2

Let r be the distance from O. Let r_n denote the (corrected) distance from O in case $r = n$. r_n is defined via successive approximation:

$$r_1 = 1$$

$$r_2 = r_1 + \frac{R^2 - r_1^2}{R^2}$$

$$r_3 = r_2 + \frac{R^2 - r_2^2}{R^2}$$

$$\vdots$$

$$r_n = r_{n-1} + \frac{R^2 - r_{n-1}^2}{R^2}$$

Step 2

$$r_n - r_{n-1} = \frac{R^2 - r_{n-1}^2}{R^2}$$

Take the quantity $r = n$ as a continuous variable. One obtains the following differential equation for the function $r(r)$:

$$\frac{dr}{dr} = \frac{R^2 - r^2}{R^2}$$

The solution satisfying $r(r = 0) = 0$ is

$$r(r) = R\text{th}\frac{r}{R}$$

$r \to R$, if $r \to \infty$, that is, the radius of the world is R.

Step 3

$l_{O,R}$: the measuring rod fits n times onto the line segment $\Rightarrow$ $l_{O,R} = \sum_{i=1}^{n} \frac{R^2 - r_i^2}{R^2}$

$T_{O,R}$: the measuring rod fits n times onto the mercury column of the thermometer $\Rightarrow$ $T_{O,R} = n \frac{R^2 - r^2}{R^2}$

Empirical law

There is a choice of O, R with which the geometry$_{O,R}$ is Euclidean.

Step 4

$$l \;:\; \begin{array}{c}\text{Euclidean}\\ \text{geometry}_{O,R}\end{array} \;\Rightarrow\; l = l_{O,R}$$

$$T \;:\; \begin{array}{c}\text{Euclidean}\\ \text{geometry}_{O,R}\end{array} \;\Rightarrow\; T = T_{O,R}$$

Empirical law

$$\begin{pmatrix} \text{Euclidean} \\ \text{geometry} \end{pmatrix} + \begin{pmatrix} \text{the temperature of the world is:} \\ T(r) = T_0 \frac{R^2 - r^2}{R^2} \end{pmatrix}$$

l, T l, T

$$\begin{pmatrix} \text{Bolyai–} \\ \text{Lobachevskian} \\ \text{geometry} \end{pmatrix} + \begin{pmatrix} \text{the temperature} \\ \text{of the world is} \\ \text{constant} \end{pmatrix} = \begin{pmatrix} \text{empirical} \\ \text{facts} \end{pmatrix}$$

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A rough philosophical thesis

> Criterion of nontriviality of conventions—*The nontriviality of a convention is a direct function of the physical content of the hypotheses that guarantee the existence of that convention.*
>
> According to this criterion, the conventions required by virtue of "trivial semantic conventionalism" are trivial because their existence depends upon purely semantic considerations, not upon physical facts of the world. At the same time, neither the conventionality of congruence nor the conventionality of simultaneity can be regarded as trivial, because the necessity for each of these conventions rests upon a significant physical hypothesis. (Salmon 1969)