Black hole effect and gravitational redshift in the post-Newtonian potential fields

Diana R. Constantin[1], Erika Varga-Verebélyi[2]

1. Astronomical Institute of the Romanian Academy
2. Konkoly Thege Miklós Observatory, Hungary

Logic, Relativity and Beyond
2nd international conference
August 9-13 2015, Budapest

Outline

- Introduction
- General Relativistic Approximation (GRA) & results for the subclasses of the post – Newtonian gravitational fields:

1. Black hole (BH) effect
2. Gravitational redshift

- References

Introduction

In the two-body problem for the **scaled Maneff – Schwarzschild – de Sitter** gravitational field, the force function is:

$$U = m\left(\frac{A}{r} + \frac{B}{r^2} + \frac{C}{r^3} + Dr^2\right) = G\frac{Mm}{r}\left(1 + \frac{3G(m+M)}{c^2 r} + \frac{L^2}{c^2 r^2}\right) + m\Lambda\frac{c^2}{6}r^2 \qquad (1)$$

where

$A = GM$, $B = 3G(m+M)/c^2$, $C = GML^2/c^2$, $D = \Lambda c^2/6$

and

G = Newtonian gravitational constant;
M, m = masses of two interacting bodies in the field (e.g. a massive cosmic object and a test particle);
r = the distance between M and m;
c = speed of light;
L = constant angular momentum, Λ= cosmological constant.

We analyze the gravitational redshift produced by the field described by the following **potential Φ**, which is attached to U given by (1) for $\boldsymbol{m \ll M}$**:**

$$\Phi(r) = -\frac{GM}{r} - \frac{3G^2M^2}{r^2} - \frac{GML^2}{c^2 r^3} - \Lambda\frac{c^2}{6}r^2 = -\frac{A}{r} - \frac{B}{r^2} - \frac{C}{r^3} - Dr^2 \qquad (2)$$

GRA & Results

The **general relativistic metric associated to the potential Φ** is:

$$ds^2 = (1+\frac{2\Phi}{c^2})c^2 dt^2 - \frac{dr^2}{(1+\frac{2\Phi}{c^2})} - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \qquad (3)$$

The relation between **the proper time τ** and **the time *t* of distant observer** is:

$$\tau = \sqrt{1+\frac{2\Phi}{c^2}}t \qquad (4)$$

The **wavelength λ is linked to λ_0 (for the distant observer)** by :

$$\lambda = \sqrt{1+\frac{2\Phi}{c^2}}\lambda_0 \qquad (5)$$

1. Black hole effect for the subclasses of Φ :

Considering the **existence condition** for (4), we obtain the **BH effect** as following**:**

- In the **scaled Maneff** gravitational field (C,D=0) :

$r > \rho_{Ms}$, $\boldsymbol{\rho_{Ms}}$ *– the BH effect radius of* ***scaled*** **Maneff** *field*
We obtain the result:

$$\rho_{Ms} = (1+\sqrt{7})\alpha > \rho_M > R_s = 2\alpha \tag{6}$$

(where $\alpha = A / c^2$ and $\rho_M = 3\alpha$ is *the gravitational radius of nonscaled* Maneff gravitational field.)

- In the Schwarzschild gravitational field (B,D=0):

$r > \rho_s$, ρ_s *– the gravitational radius of* ***Schwarzschild problem***
We obtain as result:

$$\rho_s > R_s \tag{7}$$

(Noted that for the Newton gravitational field (B=C=D=0) the existence condition is:
, $r > R_s = 2\alpha$ R_s – the **Schwarzschild gravitational radius**)

SRA & results

We consider a celestial body of mass M and the radius R and a photon with the mass $m = m_f$, the wavelength λ and the frequency ν at the surface of the body.

We use the index 0 for a distant observer. Taking in account the conservation of energy law for the considered photon, we have:

$$m_f c^2 + m_f \Phi = m_{f_0} c^2 + m_{f_0} \Phi_0 \qquad (3)$$

Also

$$m_f c^2 = h\nu \quad , \quad \nu = \frac{c}{\lambda} \qquad (4)$$

where h is Planck's constant.

Neglecting Φ_0 in (3), noting $\lambda_0 - \lambda = \Delta\lambda$, $z_g = \frac{\Delta\lambda}{\lambda}$ and developing (3) until the specific order term, we obtain the gravitational redshift of SRA

2. Gravitational Redshift for the subclasses of Φ :

Computing **differences** ΔS in Special Relativistic Approximation **(SRA)** and ΔG in **GRA** *between gravitational redshift* z_g in the **post - Newtonian** *vs* **Newton** gravitational fields:

$$\Delta S = z_{g\_pN} - z_{gN} \qquad \Delta G = z_g^{pN} - z_g^N$$

□ The corresponding results of **the Maneff problem (C, D=0),** respectively **the Schwarzschild problem (B,D=0):**

- The **differences** in the SRA and GRA **are the same**: (where $\gamma = C/c^2$)

$$\Delta S = \Delta G = \frac{3\alpha^2}{R^2} \quad \text{(C, D=0)} \quad , \qquad \Delta S = \Delta G = \frac{\gamma}{R^3} \quad \text{(B, D=0)}$$

* The **relative difference is**:

$$\Delta G / \left(z_g^N\right)^2 = 3 \quad \text{(C, D=0)} \quad \text{, respectively} \quad \Delta G / \left(z_g^N\right)^3 = \gamma / \alpha^3 \, \text{(B, D=0)}$$

□ The corresponding results of **the de Sitter problem (B, C=0) in the first order terms, namely for the very far objects (eg. ff galaxies),** are**:**

* The **differences** in the SRA and GRA are NOT the same: (where $\delta = D/c^2$)

$$\Delta S = -1 - \frac{\alpha}{R} \qquad \Delta G = -1 + \left(\frac{1}{\sqrt{-2\delta}} - \alpha \right) \frac{1}{R}$$

* We can consider a **relative difference** :

$$(\Delta G - \Delta S) / z_g^N \approx \frac{1}{\alpha \sqrt{-2\delta}}$$

References

1. Landau L.D. & Lifshitz E.M.: *The Classical Theory of Fields* (1971)
2. Constantin D., Verebeliy E.: *NicXIII Conference Proceedings,* **253** (2014)
3. Mioc V.: *Rom. Astron. J.,* **125** (1998)
4. Ureche V.: *Perihelion advance and Maneff's field* (2000)
5. Ureche V.: *Rom. Astron. J.,* **119** (1998)

THANK YOU!